

Fitch Cheney Card Trick

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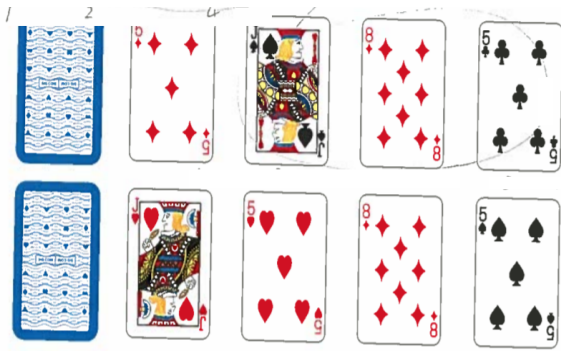
Basic Problem

A show of magic or telepathy

Take five cards out of a deck of cards with 52 cards (no joker).

Put down 4 cards in a row.

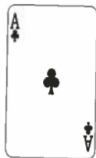
Your partner will be able to tell what is the hidden card.



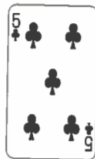
Hidden card:
Jack of Diamonds

Hidden card:
King of Hearts

More examples



Hidden Card:
Two of Clubs



Hidden Card:
Two of Spades



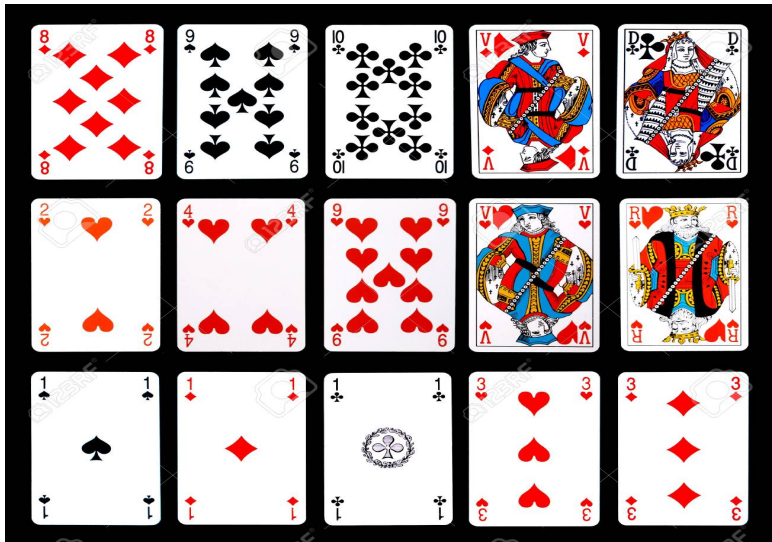
Hidden Card:
Jack of Diamonds

- Because you're dealt five cards, and there are only four suits, there must be (at least) two cards that have the same suit in your hand.
- Additionally, one of those two cards must be ahead of the other in the cycle of cards in that suit.
- The hidden card should be of the same suit, and ahead of the first card you reveal in fewer than 6 steps.
- Now, the second, third and fourth revealed cards will tell how many steps you have to move forward using the rule

$$(SML, SLM, MSL, MLS, LSM, LMS) = (1, 2, 3, 4, 5, 6),$$

where S, M, L are the order determined by $A \leq 2 \leq 3 \leq \dots \leq K$, and the suit Club \leq Diamond \leq Heart \leq Spade in case the numbers are the same.

Practice



- We can improve the scheme.
- There is a clear upper bound on the deck size based on the number of possible ways you can reveal cards, and the number of revealed cards.
- There are $5!$ ways to reveal four of your five cards, and the predicting player knows the hidden card is not any of the four revealed cards. This places an upper bound of $5! + 4 = 124$ cards in the deck.
- How close to that upper bound can we get? All the way.
 - * Arrange the 124 cards into a cycle.
 - * Eliminate positions based on the revealed cards.
 - * When you reveal a card, eliminate it and the 24 positions ahead of it in the cycle.
 - * Propagate elimination instead of stacking it.
 - * When a position would be eliminated twice, eliminate the next position instead.
 - * There is exactly one choice of card to not reveal, out of the five dealt cards, such that revealing the other four won't eliminate the chosen card. Use it.
 - * Given four revealed cards, there will be 24 positions remaining after elimination (and it doesn't matter what order you do it in).
 - * You can encode the hidden card's position, amongst those $24 = 4!$, into the ordering of the revealed cards.
- See <https://algassert.com/puzzle/2014/03/16/Encode-Hidden-Card-Puzzle.html>

The end!