Quantum AI and Error Correction

Chi-Kwong Li
Department of Mathematics
The College of William and Mary

Introduction

- M_n (H_n): the set of $n \times n$ complex (Hermitian) matrices.

Introduction

- M_n (H_n): the set of $n \times n$ complex (Hermitian) matrices.
- For example: $A = \begin{bmatrix} i & 2+i \\ 6-4i & 2-i \end{bmatrix}$, $B = \begin{bmatrix} 9 & 1+i \\ 1-i & 2 \end{bmatrix} = B^\dagger = (\overline{B})^t$.

Introduction

- M_n (H_n): the set of $n \times n$ complex (Hermitian) matrices.
- For example: $A = \begin{bmatrix} i & 2+i \\ 6-4i & 2-i \end{bmatrix}$, $B = \begin{bmatrix} 9 & 1+i \\ 1-i & 2 \end{bmatrix} = B^\dagger = (\overline{B})^t$.
- Quantum states are represented as density matrices,
  i.e., Hermitian matrices with nonnegative eigenvalues summing up to 1.

Introduction

- M_n (H_n): the set of $n \times n$ complex (Hermitian) matrices.
- For example: $A = \begin{bmatrix} i & 2+i \\ 6-4i & 2-i \end{bmatrix}$, $B = \begin{bmatrix} 9 & 1+i \\ 1-i & 2 \end{bmatrix} = B^\dagger = (\overline{B})^t$.
- Quantum states are represented as density matrices,
  i.e., Hermitian matrices with nonnegative eigenvalues summing up to 1.
- Denote by D_n the set of density matrices in M_n.

Introduction

- M_n (H_n): the set of $n \times n$ complex (Hermitian) matrices.
- For example: $A = \begin{bmatrix} i & 2+i \\ 6-4i & 2-i \end{bmatrix}$, $B = \begin{bmatrix} 9 & 1+i \\ 1-i & 2 \end{bmatrix} = B^\dagger = (\overline{B})^t$.
- Quantum states are represented as density matrices,

  i.e., Hermitian matrices with nonnegative eigenvalues summing up to 1.
- Denote by D_n the set of density matrices in M_n.
- A quantum channel (operation) $\mathcal{E} : M_n \to M_n$ is a trace preserving completely positive map admitting an operator sum representation

$$\rho \to \boxed{\text{Quantum Channel } \mathcal{E}} \to \mathcal{E}(\rho), \qquad \mathcal{E}(\rho) = E_1 \rho E_1^\dagger + \cdots + E_r \rho E_r^\dagger,$$

for some $E_1, \ldots, E_r \in M_n$ such that $E_1^\dagger E_1 + \cdots + E_r^\dagger E_r = I_n$.

Quantum information and AI

- A quantum state (system) has m measurable or observable states is represented as an $m \times m$ density matrix (positive semi-definite matrix with trace 1).

Quantum information and AI

- A quantum state (system) has m measurable or observable states is represented as an $m \times m$ density matrix (positive semi-definite matrix with trace 1).
- A 2-dimensional quantum state ρ such as a photon is a 2×2 density matrix and is called a qubit (quantum bit).

Quantum information and AI

- A quantum state (system) has m measurable or observable states is represented as an $m \times m$ density matrix (positive semi-definite matrix with trace 1).
- A 2-dimensional quantum state ρ such as a photon is a 2×2 density matrix and is called a qubit (quantum bit).
- Every qubit ρ has the form $\frac{1}{2}\begin{bmatrix} 1+a & b+ic \\ b-ic & 1-a \end{bmatrix}$ with $1 \geq a^2 + b^2 + c^2$.

Quantum information and AI

- A quantum state (system) has m measurable or observable states is represented as an $m \times m$ density matrix (positive semi-definite matrix with trace 1).
- A 2-dimensional quantum state ρ such as a photon is a 2×2 density matrix and is called a qubit (quantum bit).
- Every qubit ρ has the form $\frac{1}{2}\begin{bmatrix} 1+a & b+ic \\ b-ic & 1-a \end{bmatrix}$ with $1 \geq a^2 + b^2 + c^2$.
- Measurable states are $|0\rangle\langle 0| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $|1\rangle\langle 1| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, which are the alive and dead Schrodinger's cat in classical world.

Quantum information and AI

- A quantum state (system) has m measurable or observable states is represented as an $m \times m$ density matrix (positive semi-definite matrix with trace 1).
- A 2-dimensional quantum state ρ such as a photon is a 2×2 density matrix and is called a qubit (quantum bit).
- Every qubit ρ has the form $\frac{1}{2}\begin{bmatrix} 1+a & b+ic \\ b-ic & 1-a \end{bmatrix}$ with $1 \geq a^2 + b^2 + c^2$.
- Measurable states are $|0\rangle\langle 0| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $|1\rangle\langle 1| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, which are the alive and dead Schrodinger's cat in classical world.
- A diagonal matrix $\begin{bmatrix} a & 0 \\ 0 & 1-a \end{bmatrix}$ is a classical state.

Quantum information and AI

- A quantum state (system) has m measurable or observable states is represented as an $m \times m$ density matrix (positive semi-definite matrix with trace 1).
- A 2-dimensional quantum state ρ such as a photon is a 2×2 density matrix and is called a qubit (quantum bit).
- Every qubit ρ has the form $\frac{1}{2}\begin{bmatrix} 1+a & b+ic \\ b-ic & 1-a \end{bmatrix}$ with $1 \geq a^2 + b^2 + c^2$.
- Measurable states are $|0\rangle\langle 0| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $|1\rangle\langle 1| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, which are the alive and dead Schrodinger's cat in classical world.
- A diagonal matrix $\begin{bmatrix} a & 0 \\ 0 & 1-a \end{bmatrix}$ is a classical state.
- The matrix ρ is the Schrödinger's cat in the quantum environment.

- A system of n-qubits is represented as an $2^n \times 2^n$ density matrix.

Quantum AI

- A system of n-qubits is represented as an $2^n \times 2^n$ density matrix.
- Upon measurement, ρ will collapse to one of the $N = 2^n$ measurable states $E_{11}, \ldots, E_{NN}$.

Quantum AI

- A system of n-qubits is represented as an $2^n \times 2^n$ density matrix.
- Upon measurement, ρ will collapse to one of the $N = 2^n$ measurable states $E_{11}, \ldots, E_{NN}$.
- For a two qubit system, $\rho = [\rho_{ij}] \in M_4$, the four measurable states are:

$$|00\rangle\langle 00| = E_{11}, \ |01\rangle\langle 01| = E_{22}, \ |10\rangle\langle 10| = E_{33}, \ |11\rangle\langle 11| = E_{44}.$$

Quantum AI

- A system of n-qubits is represented as an $2^n \times 2^n$ density matrix.
- Upon measurement, ρ will collapse to one of the $N = 2^n$ measurable states $E_{11}, \ldots, E_{NN}$.
- For a two qubit system, $\rho = [\rho_{ij}] \in M_4$, the four measurable states are:

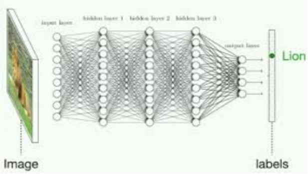
$$|00\rangle\langle 00| = E_{11}, \; |01\rangle\langle 01| = E_{22}, \; |10\rangle\langle 10| = E_{33}, \; |11\rangle\langle 11| = E_{44}.$$

- In quantum information science, one would apply suitable quantum operations to "influence" (the evolution environment of) the system so that the measurement will yield some useful conclusions.

- A system of n-qubits is represented as an $2^n \times 2^n$ density matrix.
- Upon measurement, ρ will collapse to one of the $N = 2^n$ measurable states $E_{11}, \ldots, E_{NN}$.
- For a two qubit system, $\rho = [\rho_{ij}] \in M_4$, the four measurable states are:

$$|00\rangle\langle 00| = E_{11}, \ |01\rangle\langle 01| = E_{22}, \ |10\rangle\langle 10| = E_{33}, \ |11\rangle\langle 11| = E_{44}.$$

- In quantum information science, one would apply suitable quantum operations to "influence" (the evolution environment of) the system so that the measurement will yield some useful conclusions.
- Recently, researchers tried to use this idea to determine whether a certain input (state) can be "correctly" identified as a certain outcome $y_j \in T$.

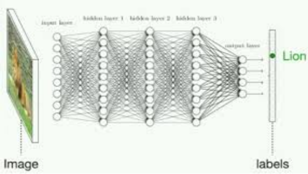


Quantum AI

- A system of n-qubits is represented as an $2^n \times 2^n$ density matrix.
- Upon measurement, ρ will collapse to one of the $N = 2^n$ measurable states $E_{11}, \ldots, E_{NN}$.
- For a two qubit system, $\rho = [\rho_{ij}] \in M_4$, the four measurable states are:

$$|00\rangle\langle 00| = E_{11}, \ |01\rangle\langle 01| = E_{22}, \ |10\rangle\langle 10| = E_{33}, \ |11\rangle\langle 11| = E_{44}.$$

- In quantum information science, one would apply suitable quantum operations to "influence" (the evolution environment of) the system so that the measurement will yield some useful conclusions.
- Recently, researchers tried to use this idea to determine whether a certain input (state) can be "correctly" identified as a certain outcome $y_j \in T$.



$$\rho \to \boxed{\text{Quantum Operation } \mathcal{E}} \to \mathcal{E}(\rho).$$

Quantum Error Correction

Basic Problem

How to recover useful/important information (data bits) after it passes through a noisy quantum channel $\mathcal{E}$?

Basic Problem

How to recover useful/important information (data bits) after it passes through a noisy quantum channel $\mathcal{E}$?

Correction scheme Encode the data bits $\tilde{\rho} \in D_k$ as $\rho \in D_n$ for a larger n, and send it through the channel $\mathcal{E}$ to get $\mathcal{E}(\rho)$. Then do one of the following.

Basic Problem

How to recover useful/important information (data bits) after it passes through a noisy quantum channel $\mathcal{E}$?

Correction scheme Encode the data bits $\tilde{\rho} \in D_k$ as $\rho \in D_n$ for a larger n, and send it through the channel $\mathcal{E}$ to get $\mathcal{E}(\rho)$. Then do one of the following.

- Apply a syndrome measurement to $\mathcal{E}(\rho)$ using additional qubits;

Basic Problem

How to recover useful/important information (data bits) after it passes through a noisy quantum channel $\mathcal{E}$?

Correction scheme Encode the data bits $\tilde{\rho} \in D_k$ as $\rho \in D_n$ for a larger n, and send it through the channel $\mathcal{E}$ to get $\mathcal{E}(\rho)$. Then do one of the following.

- Apply a syndrome measurement to $\mathcal{E}(\rho)$ using additional qubits; then construct a recovery channel $\mathcal{R}$ so that $\mathcal{R} \circ \mathcal{E}(\rho) = \rho$.

Basic Problem

How to recover useful/important information (data bits) after it passes through a noisy quantum channel $\mathcal{E}$?

Correction scheme Encode the data bits $\tilde{\rho} \in D_k$ as $\rho \in D_n$ for a larger n, and send it through the channel $\mathcal{E}$ to get $\mathcal{E}(\rho)$. Then do one of the following.

- Apply a syndrome measurement to $\mathcal{E}(\rho)$ using additional qubits; then construct a recovery channel $\mathcal{R}$ so that $\mathcal{R} \circ \mathcal{E}(\rho) = \rho$.

$$\boxed{\tilde{\rho} \to \rho \to \mathcal{E}(\rho) \to \mathcal{E}(\rho) \otimes \sigma \to (\mathcal{E}(\rho), \tilde{\sigma}) \to \mathcal{R} \circ \mathcal{E}(\rho) = \rho \to \tilde{\rho}.}$$

$$\tilde{P} \to P_1P_2P_2 \to Q_1Q_2Q_3 \to (Q_1Q_2Q_3) \otimes (R_1R_2) \to ((Q_1Q_2Q_3), (M_1M_2)) \to (P_1P_2P_3) \to \tilde{P}.$$

Quantum Error Correction

Basic Problem

How to recover useful/important information (data bits) after it passes through a noisy quantum channel $\mathcal{E}$?

Correction scheme Encode the data bits $\tilde{\rho} \in D_k$ as $\rho \in D_n$ for a larger n, and send it through the channel $\mathcal{E}$ to get $\mathcal{E}(\rho)$. Then do one of the following.

- Apply a syndrome measurement to $\mathcal{E}(\rho)$ using additional qubits; then construct a recovery channel $\mathcal{R}$ so that $\mathcal{R} \circ \mathcal{E}(\rho) = \rho$.

$$\boxed{\tilde{\rho} \to \rho \to \mathcal{E}(\rho) \to \mathcal{E}(\rho) \otimes \sigma \to (\mathcal{E}(\rho), \tilde{\sigma}) \to \mathcal{R} \circ \mathcal{E}(\rho) = \rho \to \tilde{\rho}.}$$

$$\tilde{P} \to P_1 P_2 P_2 \to Q_1 Q_2 Q_3 \to (Q_1 Q_2 Q_3) \otimes (R_1 R_2) \to ((Q_1 Q_2 Q_3), (M_1 M_2)) \to (P_1 P_2 P_3) \to \tilde{P}.$$

- Use operator algebra techniques to determine an error avoiding subspace to construct a recovery channel $\mathcal{R}$.

$$\boxed{\tilde{\rho} \to \rho \to \mathcal{E}(\rho) \to \mathcal{R} \circ \mathcal{E}(\rho) = \hat{\rho} \to \tilde{\rho}.}$$

$$\tilde{P} \to P_1 P_2 P_2 \to Q_1 Q_2 Q_3 \to \tilde{P} \otimes (R_1 R_2)) \to \tilde{P}.$$

Example 1: Bit-flip channel

Consider the bit-flip error $\tilde{\rho} \mapsto X\tilde{\rho}X^\dagger$ with $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ that will exchange the two classical states $|0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $|1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

Example 1: Bit-flip channel

Consider the bit-flip error $\tilde{\rho} \mapsto X\tilde{\rho}X^\dagger$ with $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ that will exchange the two classical states $|0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $|1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

We can encode one qubit $\tilde{\rho} \in M_2$ into a 3-qubit system $\rho \in M_{2^3}$, i.e., we will use two qubits to protect one qubits.

Example 1: Bit-flip channel

Consider the bit-flip error $\tilde{\rho} \mapsto X\tilde{\rho}X^\dagger$ with $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ that will exchange the two classical states $|0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $|1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

We can encode one qubit $\tilde{\rho} \in M_2$ into a 3-qubit system $\rho \in M_{2^3}$, i.e., we will use two qubits to protect one qubits. The channel on three qubits has the form:

$$\rho \mapsto p_0\rho + p_1 E_1 \rho E_1^\dagger + p_2 E_2 \rho E_2^\dagger + p_3 E_3 \rho E_3^\dagger,$$

where $p_0, p_1, p_2, p_3 \geq 0$ summing up to 1,

$$E_1 = X \otimes I_2 \otimes I_2, E_2 = I_2 \otimes X \otimes I_2, E_3 = I_2 \otimes I_2 \otimes X.$$

Example 1: Bit-flip channel

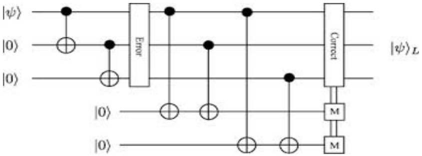
Consider the bit-flip error $\tilde{\rho} \mapsto X\tilde{\rho}X^\dagger$ with $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ that will exchange the two classical states $|0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $|1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

We can encode one qubit $\tilde{\rho} \in M_2$ into a 3-qubit system $\rho \in M_{2^3}$, i.e., we will use two qubits to protect one qubits. The channel on three qubits has the form:

$$\rho \mapsto p_0\rho + p_1 E_1 \rho E_1^\dagger + p_2 E_2 \rho E_2^\dagger + p_3 E_3 \rho E_3^\dagger,$$

where $p_0, p_1, p_2, p_3 \geq 0$ summing up to 1,

$$E_1 = X \otimes I_2 \otimes I_2, E_2 = I_2 \otimes X \otimes I_2, E_3 = I_2 \otimes I_2 \otimes X.$$



QECC with syndrome measurement.

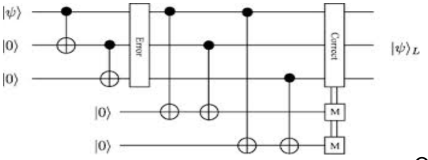
Consider the bit-flip error $\tilde{\rho} \mapsto X\tilde{\rho}X^\dagger$ with $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ that will exchange the two classical states $|0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $|1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.

We can encode one qubit $\tilde{\rho} \in M_2$ into a 3-qubit system $\rho \in M_{2^3}$, i.e., we will use two qubits to protect one qubits. The channel on three qubits has the form:

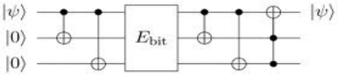
$$\rho \mapsto p_0\rho + p_1 E_1 \rho E_1^\dagger + p_2 E_2 \rho E_2^\dagger + p_3 E_3 \rho E_3^\dagger,$$

where $p_0, p_1, p_2, p_3 \geq 0$ summing up to 1,

$$E_1 = X \otimes I_2 \otimes I_2, E_2 = I_2 \otimes X \otimes I_2, E_3 = I_2 \otimes I_2 \otimes X.$$



QECC with syndrome measurement.



QECC without syndrome measurement.

Example 2: Fully correlated channels

Denote the Pauli's matrices by

$$\sigma_0 = I_2, \ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Example 2: Fully correlated channels

Denote the Pauli's matrices by

$$\sigma_0 = I_2,\ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

A fully correlated channel $\mathcal{E}$ on n-qubit states are defined by

$$\mathcal{E}(\rho_1 \otimes \cdots \otimes \rho_n) = \sum_{j=0}^{3} p_j (\sigma_j \rho_1 \sigma_j^\dagger) \otimes \cdots \otimes (\sigma_j \rho_n \sigma_j^\dagger).$$

Example 2: Fully correlated channels

Denote the Pauli's matrices by

$$\sigma_0 = I_2,\ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

A fully correlated channel $\mathcal{E}$ on n-qubit states are defined by

$$\mathcal{E}(\rho_1 \otimes \cdots \otimes \rho_n) = \sum_{j=0}^{3} p_j(\sigma_j \rho_1 \sigma_j^\dagger) \otimes \cdots \otimes (\sigma_j \rho_n \sigma_j^\dagger).$$

Theorem [Li, Lyles and Poon, 2019]

For the fully correlated channels on n-qubits with error operators X_n, Y_n, Z_n, there are efficient error correction schemes.

Example 2: Fully correlated channels

Denote the Pauli's matrices by

$$\sigma_0 = I_2, \ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

A fully correlated channel $\mathcal{E}$ on n-qubit states are defined by

$$\mathcal{E}(\rho_1 \otimes \cdots \otimes \rho_n) = \sum_{j=0}^{3} p_j (\sigma_j \rho_1 \sigma_j^\dagger) \otimes \cdots \otimes (\sigma_j \rho_n \sigma_j^\dagger).$$

Theorem [Li, Lyles and Poon, 2019]

For the fully correlated channels on n-qubits with error operators X_n, Y_n, Z_n, there are efficient error correction schemes.

1. When $n = 2k + 1$ is odd, one can use one arbitary qubit to protect $2k$-qubits of data.

Example 2: Fully correlated channels

Denote the Pauli's matrices by

$$\sigma_0 = I_2, \ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

A fully correlated channel $\mathcal{E}$ on n-qubit states are defined by

$$\mathcal{E}(\rho_1 \otimes \cdots \otimes \rho_n) = \sum_{j=0}^{3} p_j (\sigma_j \rho_1 \sigma_j^\dagger) \otimes \cdots \otimes (\sigma_j \rho_n \sigma_j^\dagger).$$

Theorem [Li, Lyles and Poon, 2019]

For the fully correlated channels on n-qubits with error operators X_n, Y_n, Z_n, there are efficient error correction schemes.

1. When $n = 2k+1$ is odd, one can use one arbitary qubit to protect $2k$-qubits of data.
2. When $n = 2k+2$, one can transmit two classical bits and $2k$-qubits of data without error!

The two classical bits encoded as qubits will be used to protect the $2k$-qubits.

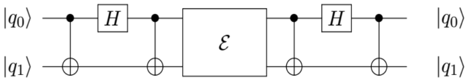
Implementation

The scheme was implemented using Matlab, Mathematica, Python, and the IBM's quantum computing framework `qiskit`.

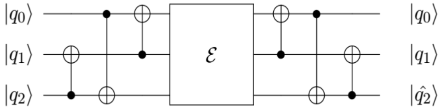
Implementation

The scheme was implemented using Matlab, Mathematica, Python, and the IBM's quantum computing framework `qiskit`.

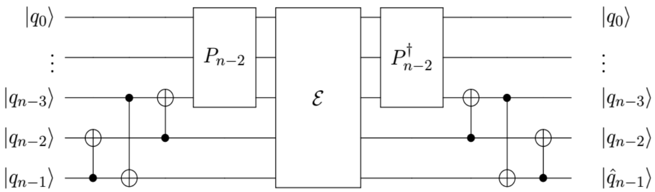
For $n = 2$, if $|q_1 q_0\rangle \in \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, then circuit diagram will be:



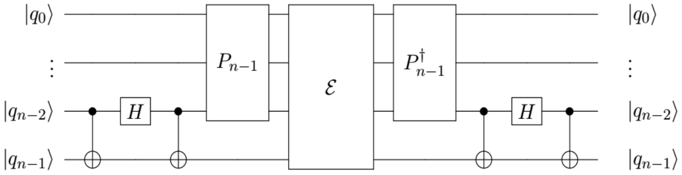
For $n = 3$,



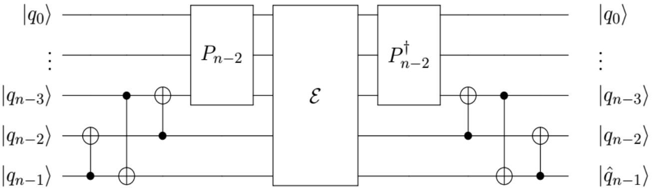
For odd n, the circuit diagram will be



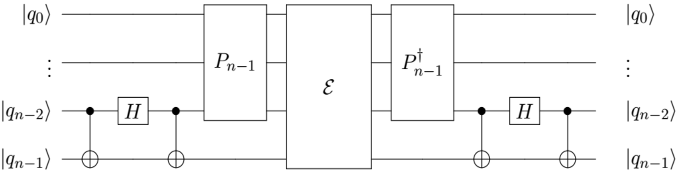
For even n, if $|q_{n-1}q_{n-2}\rangle \in \{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}$, then the circuit diagram will look like:



For odd n, the circuit diagram will be



For even n, if $|q_{n-1}q_{n-2}\rangle \in \{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}$, then the circuit diagram will look like:



Note that our scheme is good for multiple times of quantum error correction without syndrome measurement.

Experimental results

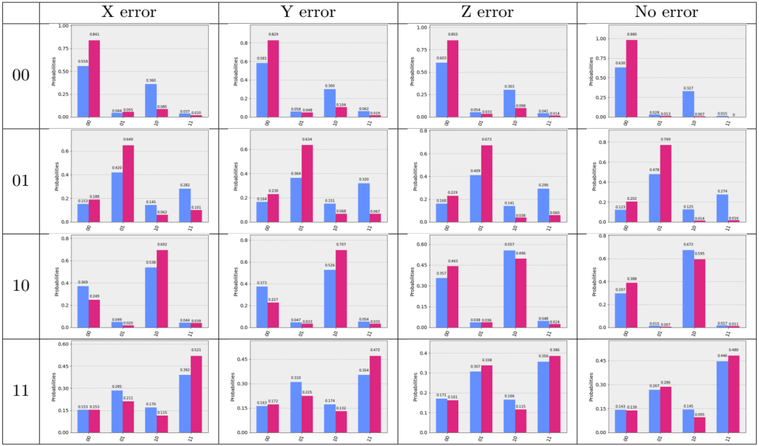


Table 1: Inputs and Errors on sigma = 0, Legend: Tenerife (pink) and Yorktown (blue)

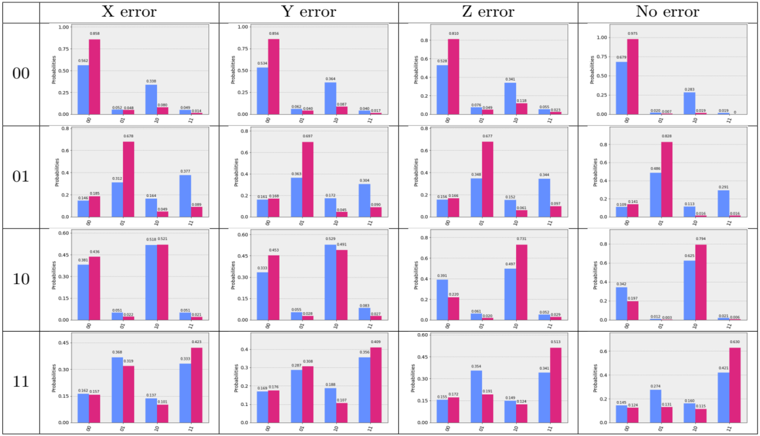


Table 2: Inputs and Errors on sigma = 1, Legend: Tenerife (pink) and Yorktown (blue)

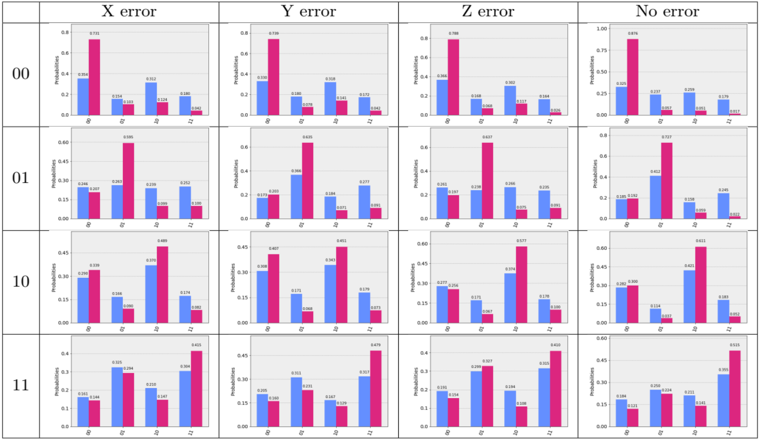


Table 3: Inputs and Errors on random sigma, Legend: Tenerife (pink) and Yorktown (blue)

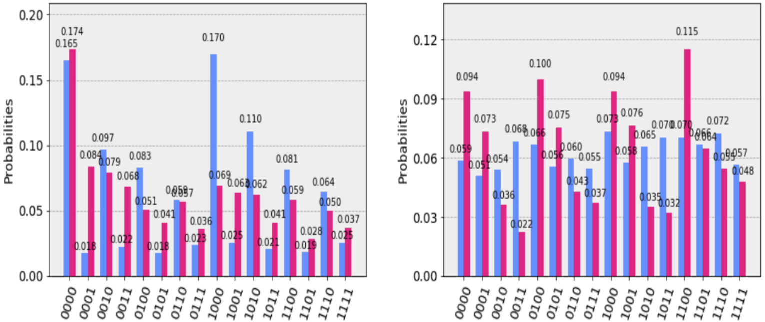


Figure 1: QECC on 4 and 5 qubits

Current research

- IBM quantum computers have improved; better results have been obtained.

Current research

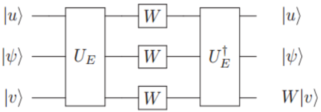
- IBM quantum computers have improved; better results have been obtained.
- Results have been obtained for the general fully correlated quantum channels on n-qubits with error operators $W^{\otimes n}$ for unitary $W \in M_2$.

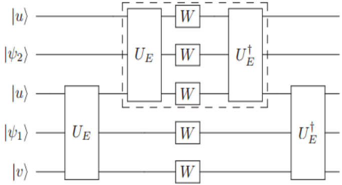
Current research

- IBM quantum computers have improved; better results have been obtained.
- Results have been obtained for the general fully correlated quantum channels on n-qubits with error operators $W^{\otimes n}$ for unitary $W \in M_2$.
- In [C.K. Li, M. Nakahara, Y.T. Poon, N.S. Sze, H. Tomita, 2011] we used $n = 2k + 1$ physical qubits protect k logical qubits.

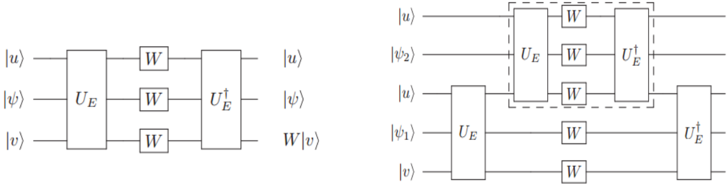
Current research

- IBM quantum computers have improved; better results have been obtained.
- Results have been obtained for the general fully correlated quantum channels on n-qubits with error operators $W^{\otimes n}$ for unitary $W \in M_2$.
- In [C.K. Li, M. Nakahara, Y.T. Poon, N.S. Sze, H. Tomita, 2011] we used $n = 2k + 1$ physical qubits protect k logical qubits.





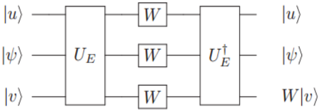
- IBM quantum computers have improved; better results have been obtained.
- Results have been obtained for the general fully correlated quantum channels on n-qubits with error operators $W^{\otimes n}$ for unitary $W \in M_2$.
- In [C.K. Li, M. Nakahara, Y.T. Poon, N.S. Sze, H. Tomita, 2011] we used $n = 2k + 1$ physical qubits protect k logical qubits.

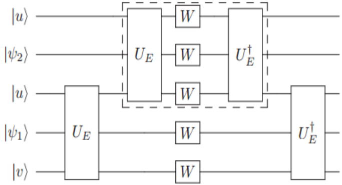


- Note that in the above encoding, $|u\rangle = |0\rangle$ and $|v\rangle$ is arbitrary.

Current research

- IBM quantum computers have improved; better results have been obtained.
- Results have been obtained for the general fully correlated quantum channels on n-qubits with error operators $W^{\otimes n}$ for unitary $W \in M_2$.
- In [C.K. Li, M. Nakahara, Y.T. Poon, N.S. Sze, H. Tomita, 2011] we used $n = 2k + 1$ physical qubits protect k logical qubits.





- Note that in the above encoding, $|u\rangle = |0\rangle$ and $|v\rangle$ is arbitrary.
- The recursive scheme is useful because of its efficiency in encoding and decoding. We will study whether it can protect classical information.

Any questions?

Any questions?

Thank you for your attention!