

A large, stylized number '24' in a bold, blocky font. The number is white with a thick yellow outline and a purple shadow. The '2' and '4' are connected. The '2' has a purple diagonal bar. The '4' has a purple diagonal bar. The number is set against a white background with a subtle shadow.

24-puzzle

Tony Zheng

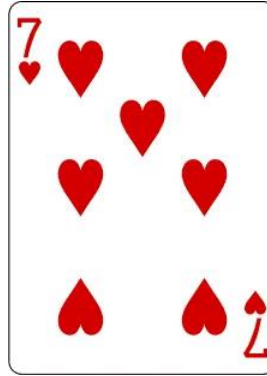
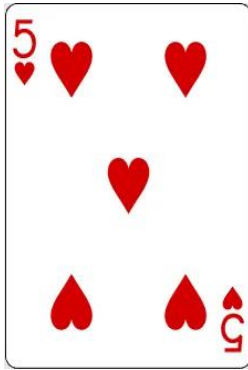
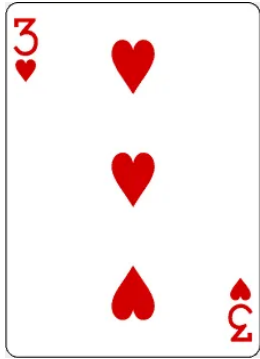
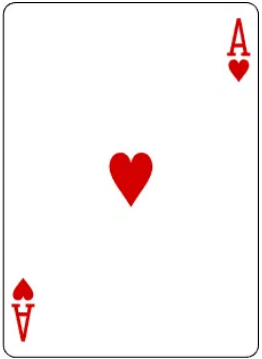
Introduction

The problem has been played as a card game in Shanghai since the 1960s, using poker cards. It has been known by other names, including 24-game and Math24.

Randomly draw 4 cards from a deck of poker cards, the first player who come up with 24 using all values on the cards wins. (A = 1, J = 11, Q = 12 and K = 13)

Only elementary arithmetic operations (+ - $\times$ and /) are allowed.

Examples?



$$(1+5)*(7-3) = 6*4 = 24$$

More Complicated examples

$\{7, 9, 3, K\}$

$\{A, 6, J, K\}$

What about $\{7, 7, 7, 7\}$?

In the original version of the game played with a standard 52-card deck, there are 1820 four-card combinations. Not all of them have solutions. 1362 or 74.84% of them are solvable. (3017 distinct solutions)

515 solvables have 1 solutions each

427 solvables have 2 solutions each

216 solvables have 3 solutions each

125 solvables have 4 solutions each

31 solvables have 5 solutions each

17 solvables have 6 solutions each

17 solvables have 7 solutions each

8 solvables have 8 solutions each

$\{2, 8, a, b\}$

2 solvables have 9 solutions each

$\{3, 9, a, b\}$

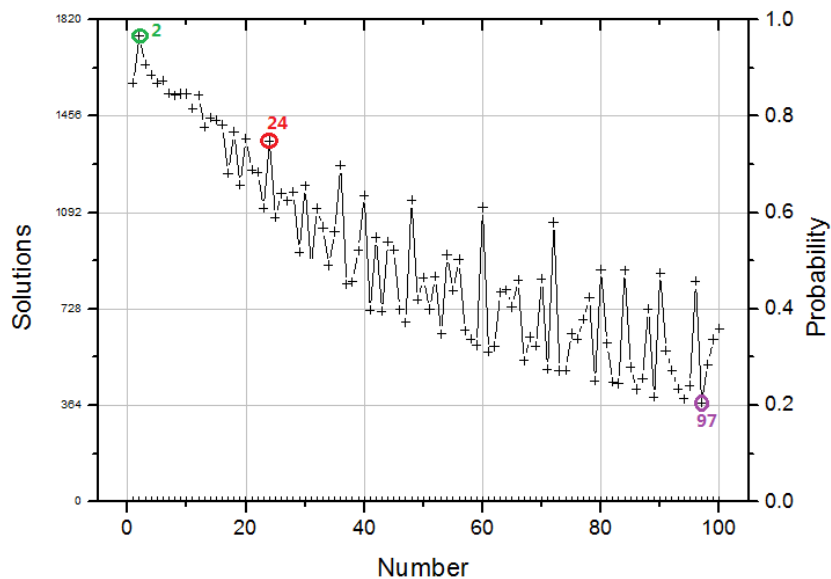
3 solvables have 10 solutions each

And finally, {2,4,8,10) has 11 distinct solutions.

Why 24?

Why 24?

24 is the smallest natural number with at least 8 divisors (1, 2, 3, 4, 6, 8, 12, 24). It is a **Highly Composite Number**. This means that if we randomly draw 4 cards from the deck, there is a decent chance that it's solvable.



Expansion to more complex operations?

$\{1, 5, 11, 13\}$

$5^{(13 - 11)} - 1 = 24$ (exponential allowed)

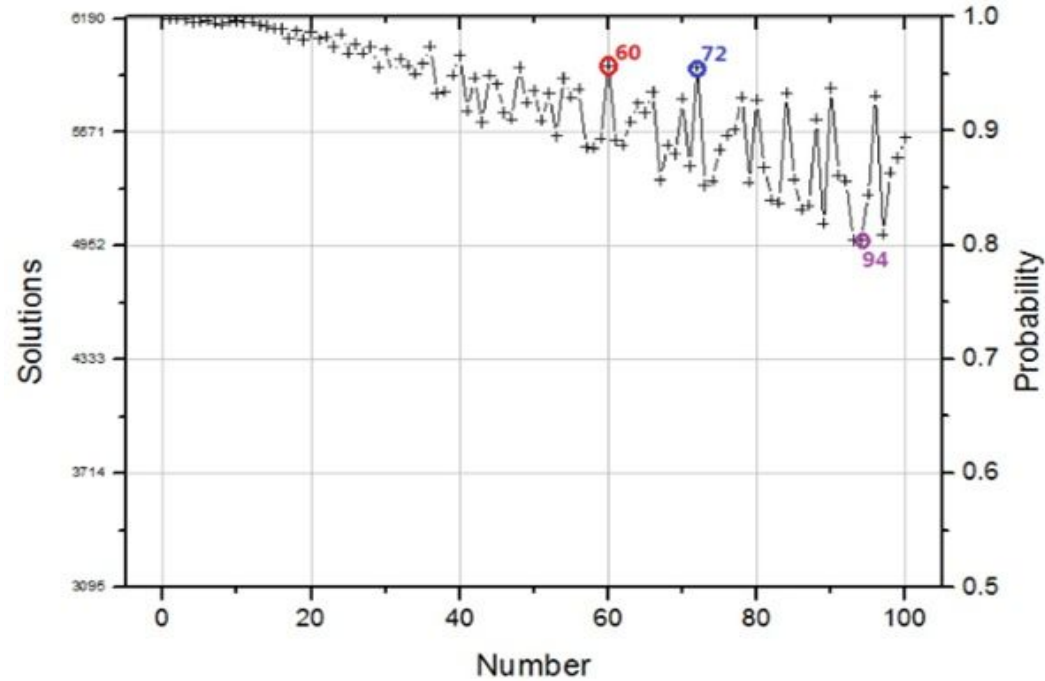
$\{1, 1, 1, 1\}$

$(1+1+1+1)! = 24$ (factorial allowed)

Fun fact:

$(0!+0!+0!+0!)! = 24$ We do not need any card!

Five cards?



Questions?

References

1. Facts about 24 the math game - 4nums.com, solves 24 the math game. (n.d.). Retrieved April 14, 2022, from https://www.4nums.com/24_game/facts/
2. "Rules of Card Games: Twenty-Four". www.pagat.com. Retrieved 2022-04-09