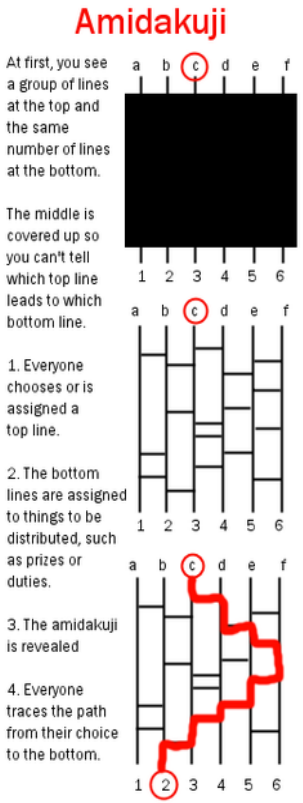
Factorization of permutation

Chi-Kwong Li
Department of Mathematics, The College of William and Mary;
Institute for Quantum Computing, University of Waterloo

Based on the paper:
Zejun Huang, Chi-Kwong Li, Sharon H. Li, Nung-Sing Sze,

Amidakuji/Ghost Leg Drawing



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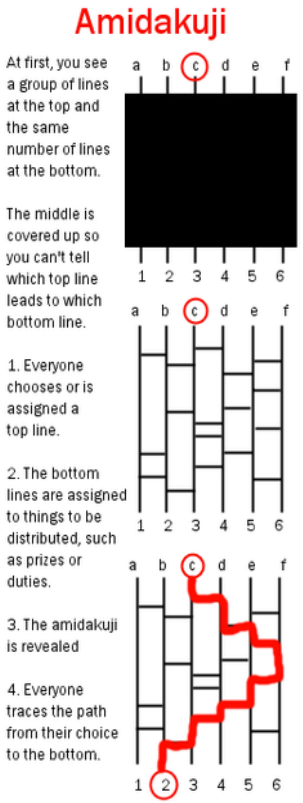

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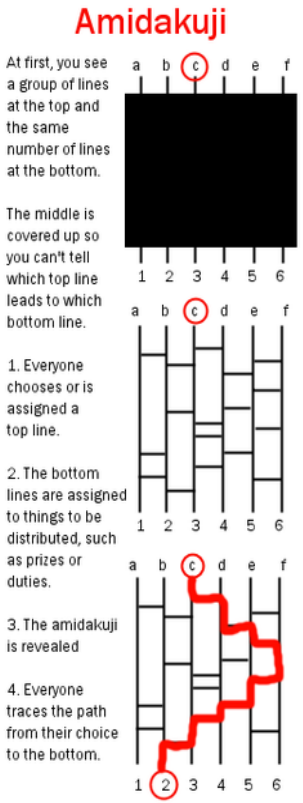

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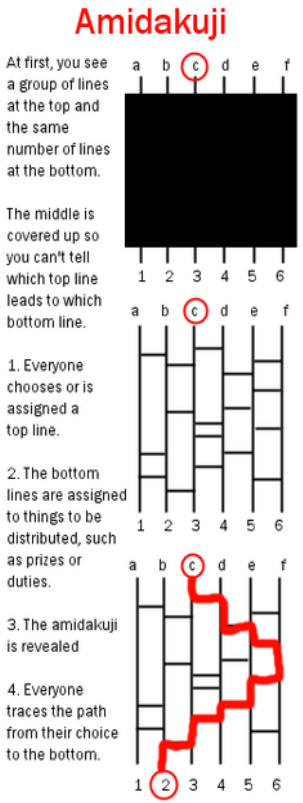

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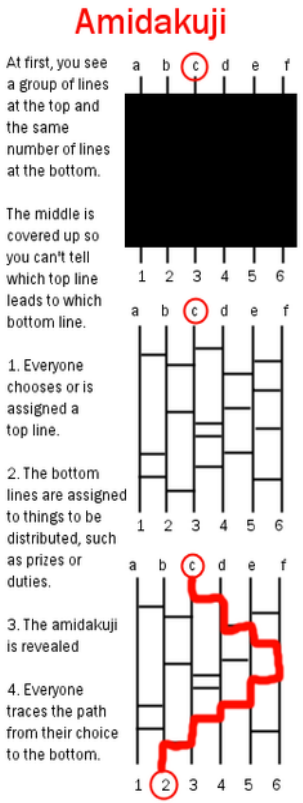

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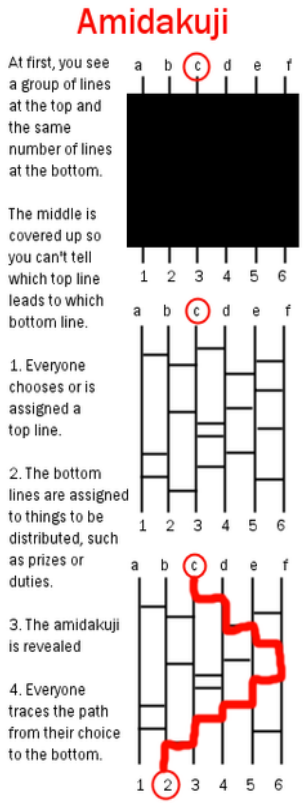

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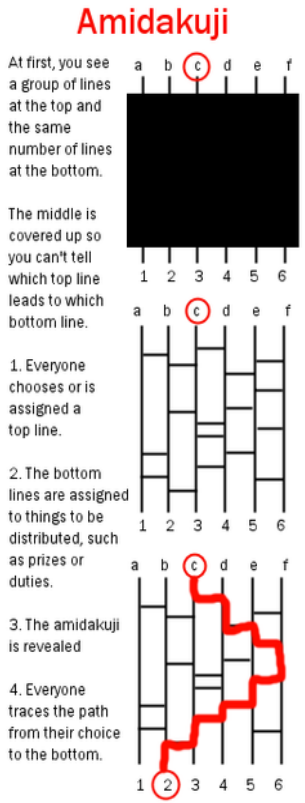

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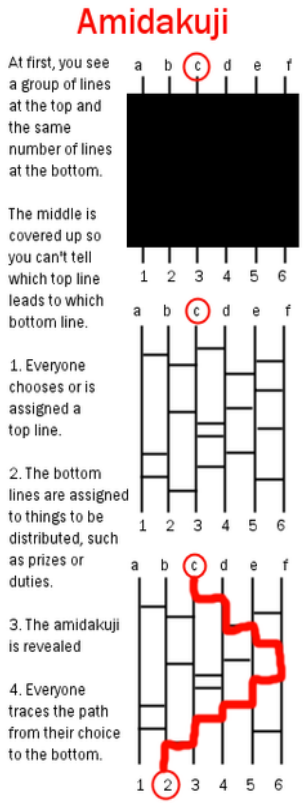

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- Why do we always get an one-one correspondence (bijection)?
- Can we get all possible job assignments?
- What is the minimum number of horizontal segments needed for a given job assignment?

Answer of Question 1

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- What if there is one horizontal line segment?
- An easy induction argument!

Bubble sort

- Regard the job assignment as a permutation (a seat assignment)

$$\sigma = [i_1, \ldots, i_n] = \begin{pmatrix} 1 & 2 & \cdots & n \\ i_1 & i_2 & \cdots & i_n \end{pmatrix}.$$

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Example For $\sigma = [5, 3, 1, 2, 4]$, total number of inversions is: $4 + 0 + 2 = 6$, and

$$\sigma \to [3, 5, 1, 2, 4] \to [3, 1, 5, 2, 4] \to [3, 1, 2, 5, 4]$$
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$$(n-1) + \cdots + 1 = n(n-1)/2 \text{ steps.}$$

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Example. $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 4 & 5 & 9 & 6 & 7 & 1 & 8 & 2 \end{pmatrix} = (1, 3, 5, 6, 7)(2, 4, 9)(8).$

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- Determine the optimal (smallest) $r^* = r^*(n, m)$ so that every $\sigma \in S_n$ is a product at most r^* transpositions in G_m.

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- To find r^* and the permutation which is most difficult to get restore, we use the breadth first search.

Partial results of the general problem

We have the following list for $r^*(n,m)$ for S_n and $(i, i+\ell)$ with $\ell \le m$,

$n\backslash m$	1	2	3	4	5	6	7	8	9	10	11
2	1										
3	3	2									
4	6	4	3								
5	10	5	5	4							
6	15	[7]	6	6	5						
7	21	[10]	8	7	7	6					
8	28	[14]	[10]	9	8	8	7				
9	36	[16]	[11]	10	10	9	9	8			
10	45	[19]	[14]	[12]	11	11	10	10	9		
11	55	[23]	[16]	[14]	13	12	12	11	11	10	
12	66	29^*	20^*	17^*	16^*	14	13	13	12	12	11

where the entries marked by brackets are obtained by computer programming.

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Theorem [Jerrum, 1985], [van Zuylen et. al, 2014]

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The number of steps is at most $[n^2/4]$ attained at the following permutation:

(1) $[k+1,\ldots,n,1,\ldots,k]$ if $n = 2k$ or $n = 2k+1$,

(2) $[k+2,\ldots,n,1,\ldots,k+1]$ or $[k+1,\ldots,n,1,\ldots,k]$ if $n = 2k+1$.

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Example $p = [6,5,1,2,4,3]$, $d = [5,3,-2,-2,-1,-3]$,

$$\tilde{d} = [-1,3,-2,-2,-1,3],\ \tilde{p} = [0,5,1,2,4,9],\ \iota(\tilde{d}) = 6,$$

Note For $[4,5,6,1,2,3]$, $d = [3,3,3,-3,-3,-3]$ and $\iota(d) = 9 = [6^2/4]$.

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1	2	6	11	18	25	35	45	58	71	???

- If we use L, S, L^{-1}, then the maximum steps needed are:

S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}
1	2	6	10	15	21	28	36	45	???

Conjecture We need at most $\binom{n}{2}$ steps, and the worst case is

$$[2, 1, n, n-1, \ldots, 3].$$

Related research

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- Quantum computing.
  It is of interest to decompose certain quantum gates into simpler quantum gates (CNOT gates).

Let me know if you have any ideas.

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Thank you for your attention!