The Minimum Toll Booth Problem

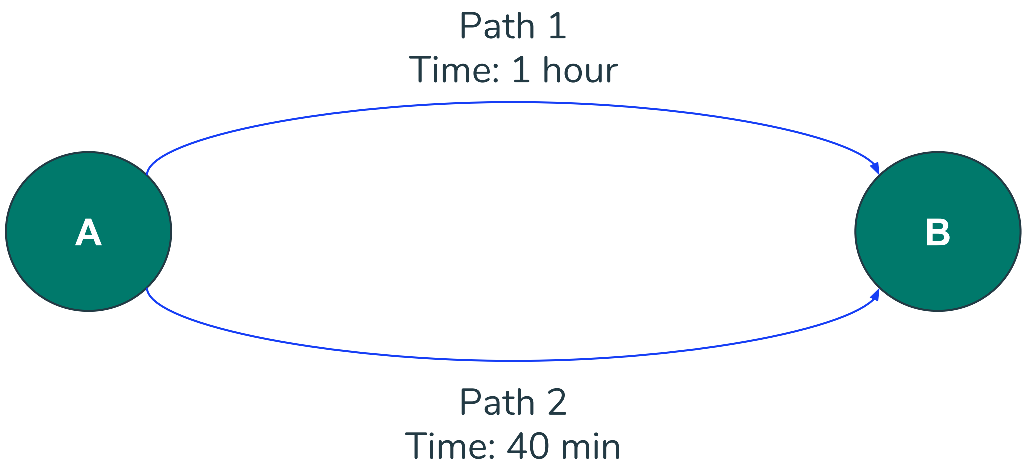
Sarah Hardy

Based mainly off research by Lihui Bai (Valparaiso University), Matthew T. Stamps (UC, Davis), R. Corban Harwood (George Fox University), Christopher J. Kollmann (Concordia Seminary)

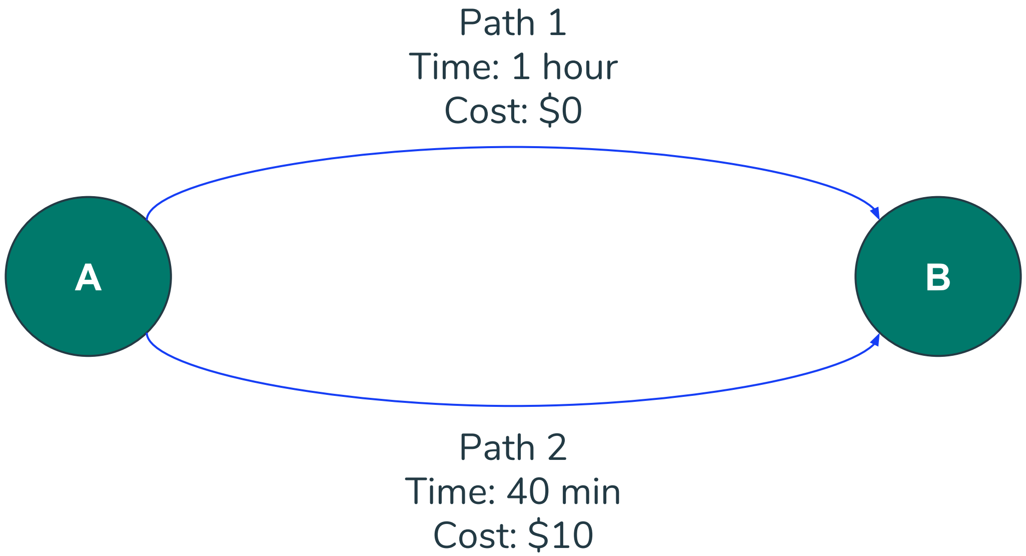
Overview

I. Introduction to Traffic Controls and Management
II. What is the MINTB?
    A. Defining the problem
    B. Illustrative Example
III. Genetic Algorithm (and other methods) for MINTB
IV. Improvements

Which path would you rather take?



What if there was a toll?



Traffic Control and Management

- Traffic congestion cost the US:
  - $126 billion in 2013
    - Estimated increase to $186 billion by 2030
  - 4.2 billion hours of travel delay in 2007
  - 2.9 billion gallons of wasted fuel in 2005

- Theory of traffic management via use of tolls: place tolls on congested roads to make routes less appealing and divert demand (and therefore reduce congestion)

- Traffic Management aims to:
  - Reduce travel time for the individual traveler
  - Reduce overall travel time for all travelers

What is the Minimum Toll Booth Problem (MINTB)?

Objectives:

1. Encourage travelers (driven only by personal travel costs) to choose routes that ***benefit all travelers*** and allow for the ***efficient use of transportation systems***
2. Determine tolls that require the ***least number of toll booths***

Other methods of congestion control:

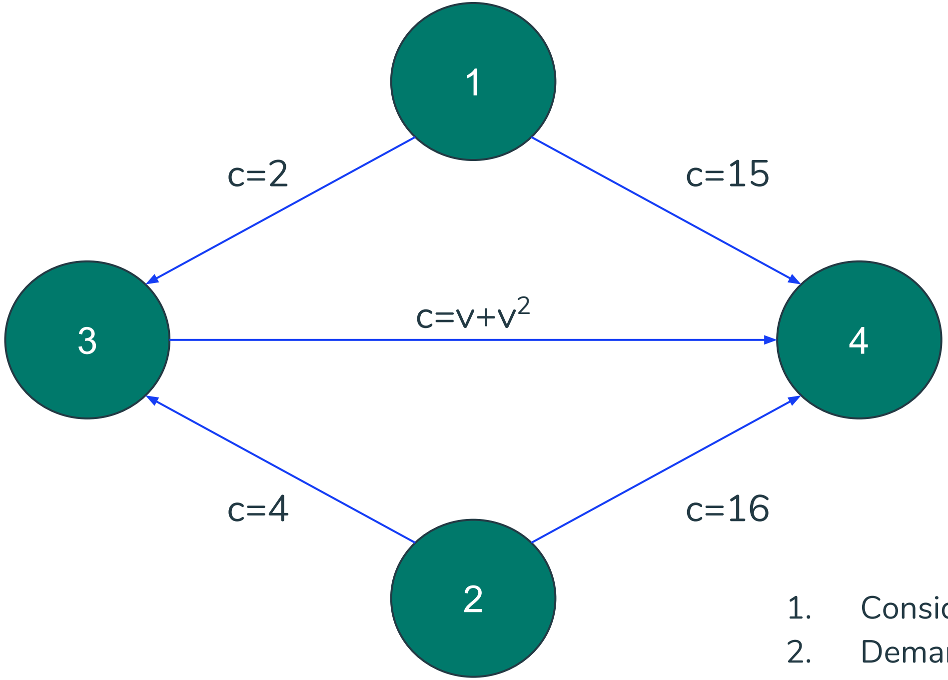
- Marginal Social Cost Pricing (MSCP)
- Minimize Total Toll Revenue (MINREV)
- Minimize the Maximum Toll on a Network (MINMAX)

Defining the MINTB

- Represent the transportation network as a set of arcs (roads) and nodes (intersections)

  - O-D pair where o is the origin node and d is the destination node
- D_k = demand of the O-D pair k = # of travelers from origin to destination
- x^k = flow vector for the O-D pair k
- v = aggregate flow vector
- s(v) = travel cost vector for a given flow v

4 Node, 5 Arc Network with given costs



1. Consider O-D pairs 1→4 and 2→4
2. Demand on each pair is 2 cars

User Equilibrium (UE)

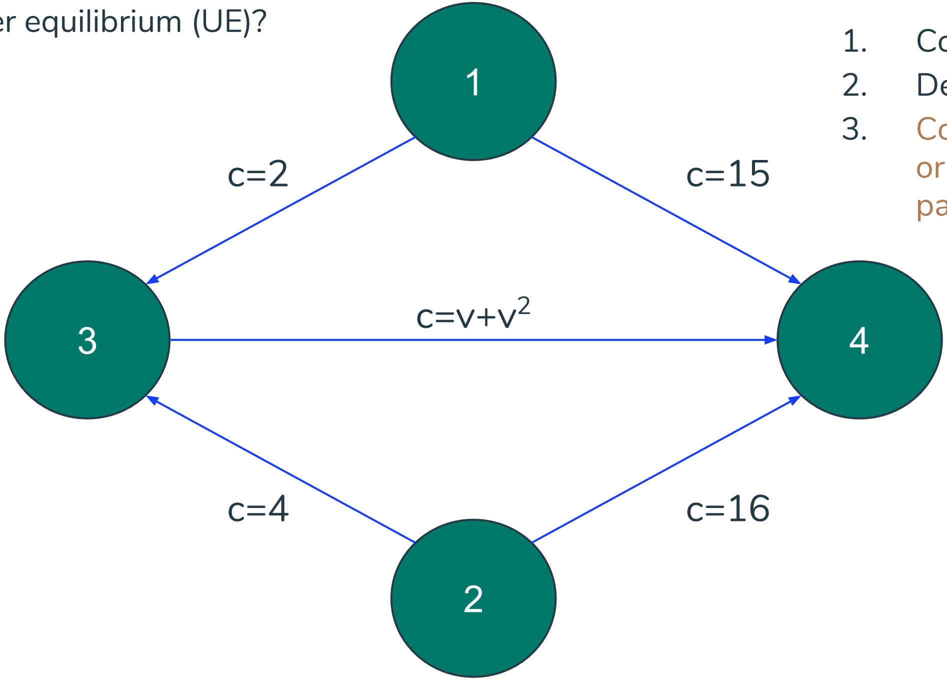
Definition: describes the behavior of users (travelers) when every traveler chooses the least costly path available

Goal: cost of each utilized path for each O-D pair is less than or equal to the cost of every non-utilized path for that pair

Issue: each traveler chooses based on their own interests but these may not be in the best interest of the network as a whole

Let's go back to our example:

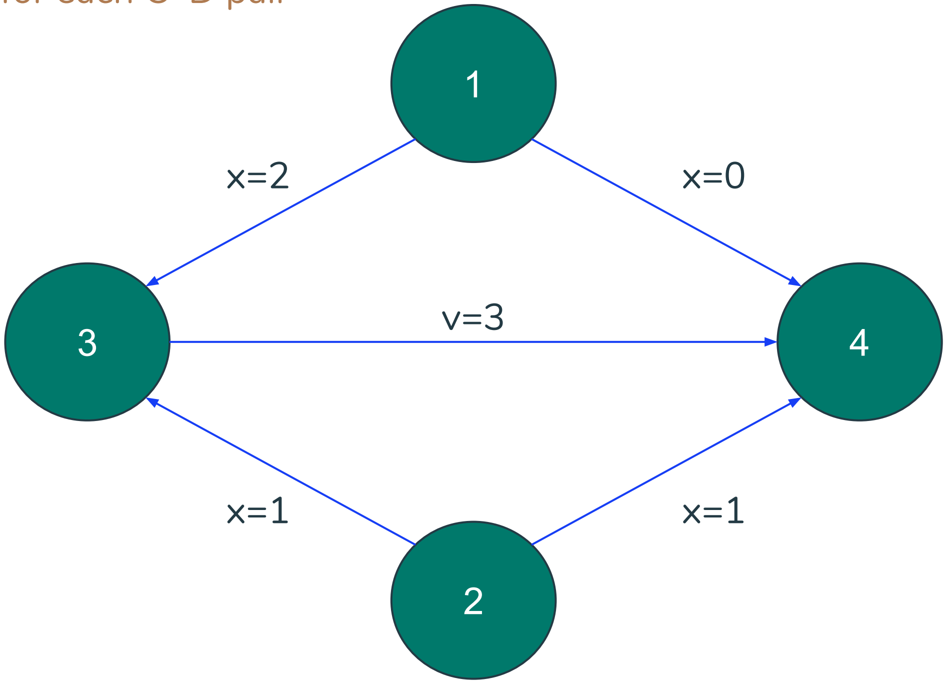
What is the flow (v) and total travel cost if the network is in user equilibrium (UE)?



1. Consider O-D pairs 1→4 and 2→4
2. Demand on each pair is 2 cars
3. Cost of each utilized path is less than or equal to the cost of non-utilized paths for each O-D pair

User Equilibrium (UE) Solution

Cost of each utilized path is less than or equal to the cost of non-utilized paths for each O-D pair



Total travel cost = 60

System Optimal (SO)

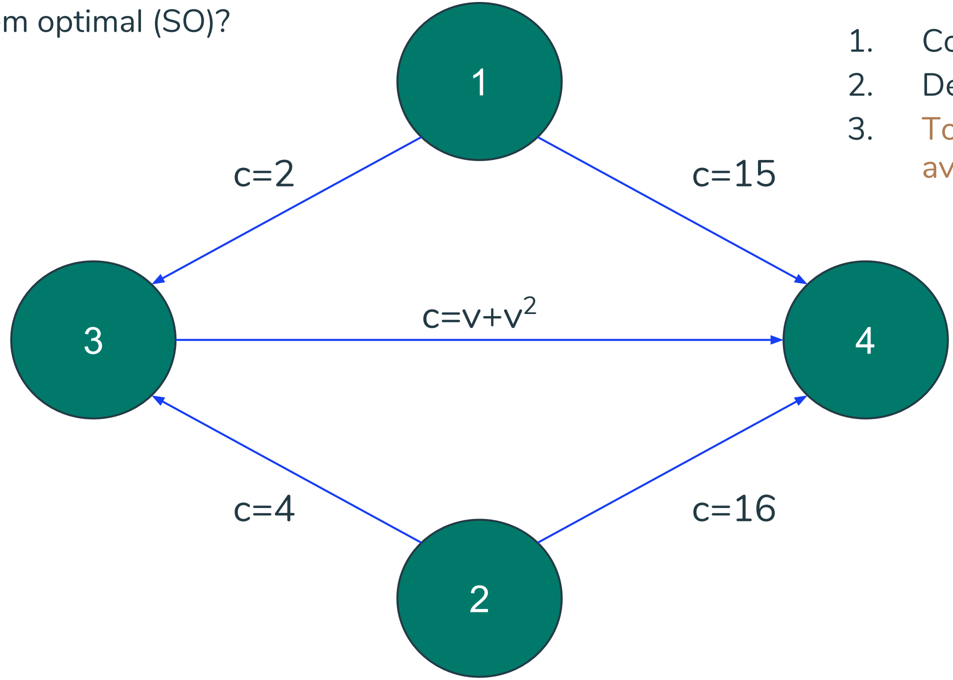
Definition: minimizes total travel cost when the network is working as efficiently as possible

Goal: average cost per user is minimized for each O-D pair

Issue: some travelers may pay higher costs than they would in UE

Let's go back to our example:

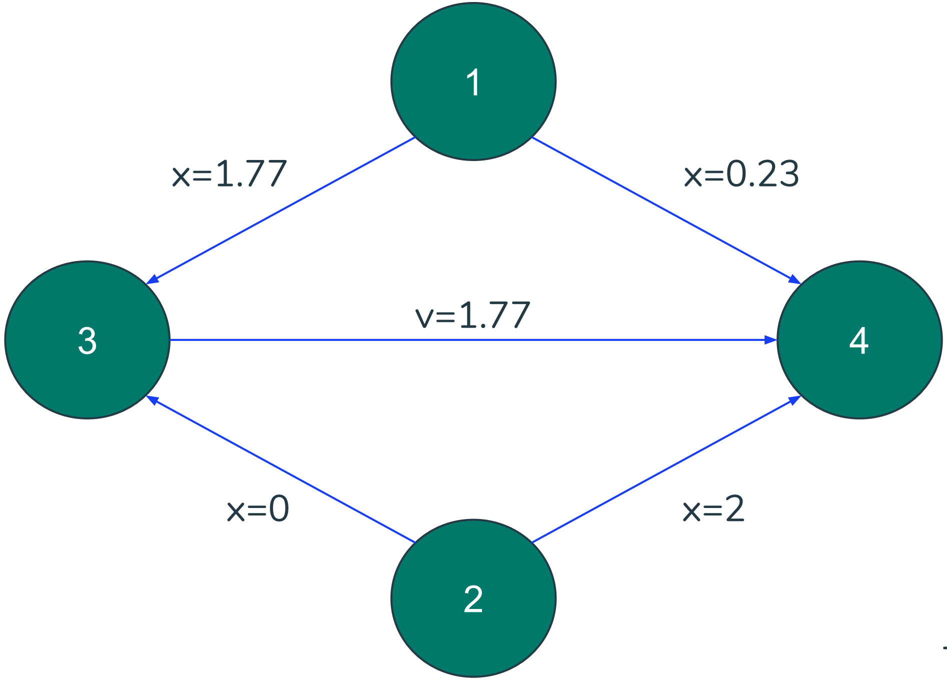
What is the flow (v) and total travel cost if the network is system optimal (SO)?



1. Consider O-D pairs 1→4 and 2→4
2. Demand on each pair is 2 cars
3. Total travel cost is minimal (minimum average cost per traveler)

System Optimal (SO) Solution

Total travel cost is minimal (minimum average cost per traveler)



Total travel cost = 47.67

Tolled User Equilibrium (TUE)

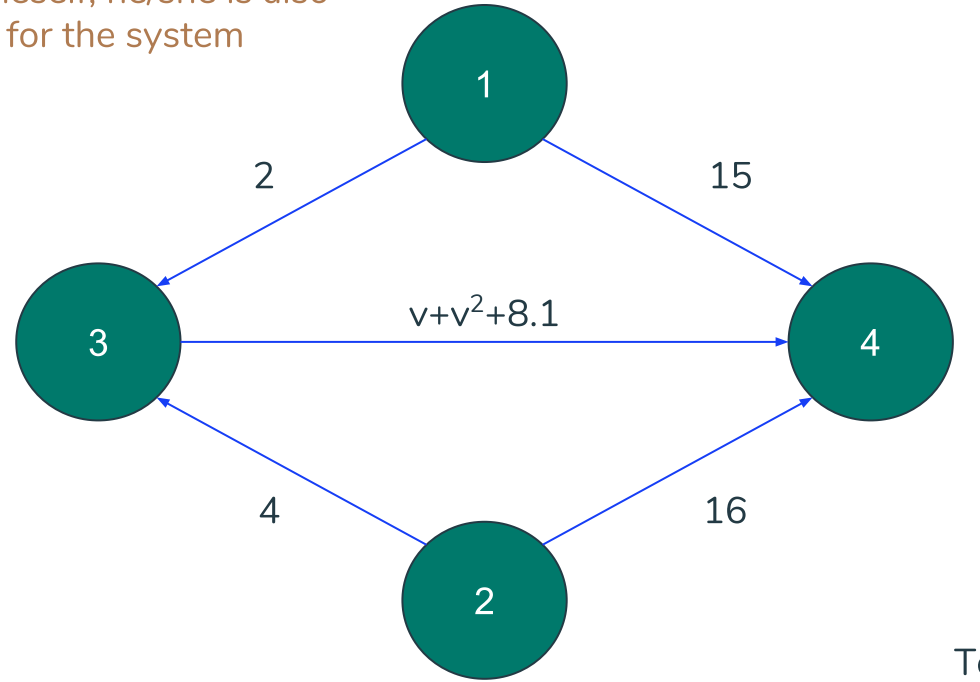
Definition: describes network behavior where tolls can be added to produce the system optimal flow

Goal: tolls allow the system to function efficiently so that when the user chooses the option best for herself, she also is choosing the best option for the network as a whole (should be selected to cause TUE to equal SO)

Issue: difficulty in determining toll cost so that TUE minimizes the total number of tolls (for the MINTB problem)

Tolled User Equilibrium (TUE)

Impose toll so when every traveler chooses what is best for oneself, he/she is also doing what's best for the system



Total travel cost = 62.01

Heuristic Algorithms to Solve MINTB

- Heuristic definition: involving or serving as an aid to learning, discovery, or problem-solving by experimental and especially trial-and-error methods
- Why do we need heuristic methods to solve this problem?
  - Mixed integer linear programming problem ($Ux^k = b_k$ where U is a node-arc incidence matrix for the network, x^k is the flow vector for O-D pair k, and b_k is the demand vector)
  - Extremely difficult to solve
- Methods:
  - Genetic Algorithm (which we will explore further)
  - Combinatorial Benders Cut
  - Dynamic Slope Scaling Method
  - Series Parallel Graphs
  - Bi-Level Programming Approach

Genetic Algorithm for MINTB (GAMINTB)

Idea: utilize idea of natural selection to filter through random vector solutions for the MINTB to find the optimal solution

Steps:

1. Initialization -- generates N initial “chromosomes” of length = # arcs (roads) with 0 (no toll booth) or 1 (toll booth) randomly assigned to each index
2. Evaluation -- rank chromosomes on feasibility and number of toll booths
3. Selection -- select “parent” chromosomes using weighted probability based off rank
4. Alteration -- crossover and mutation to “reproduce” new solution vectors
5. Termination -- repeat process for new “generation” for a desired number of “generations”

Improvements to the GAMINTB

1. Immigration: “breed” percentage of the new generation and randomly generate the rest
2. Increase the number of generations
3. Weighted rather than uniform probability (which they did test and proved to be more effective)
4. Larger networks (greater than 4 or 5 nodes)
5. Does not consider time-dependent road cost or tolls
6. Does not consider other factors beyond user cost and number of tolls (such as MSCP method does)

Questions?

Sources

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