Mathematical Model for Athletic Training

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Two Types of Athletes

Genetically Talented

Hard-working

vs.

How do we manage athletic training?

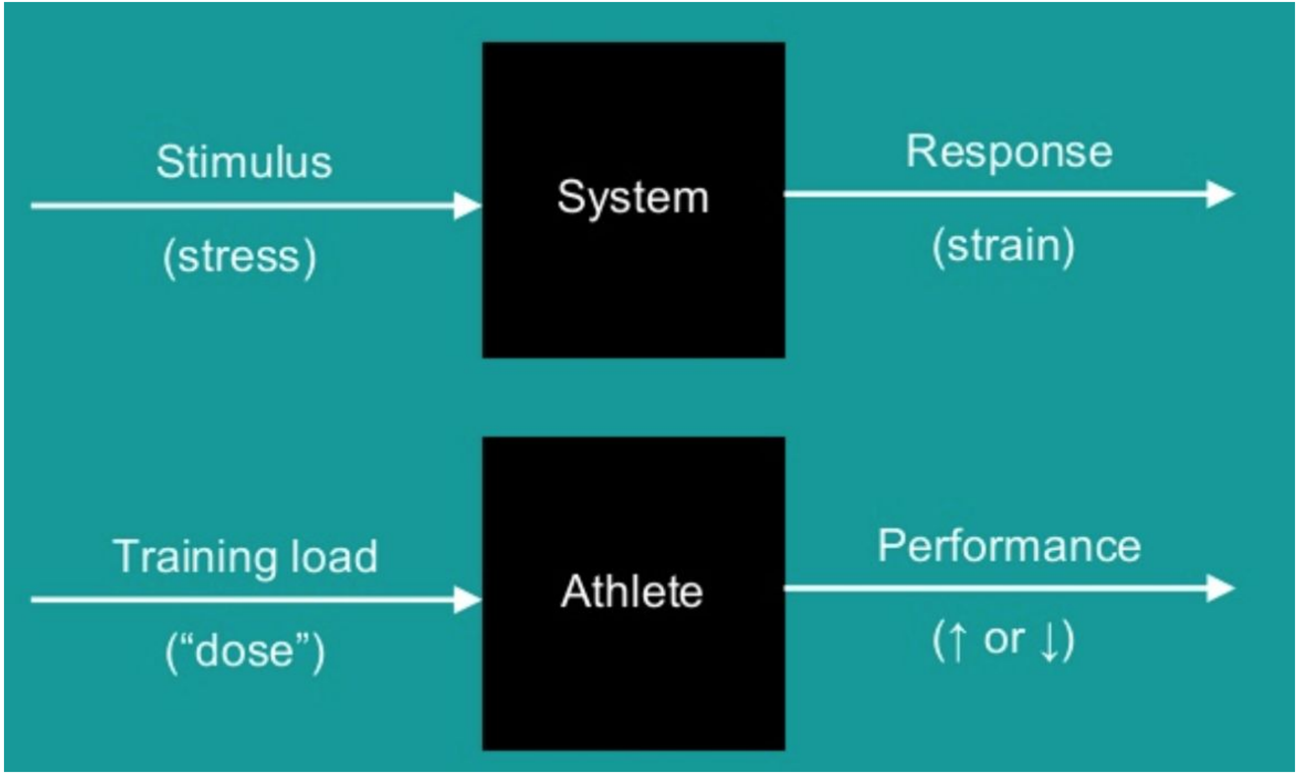
Common Methods

- Tradition (experience)
- Empiricism
- Application of basic training principles

More Quantitative Approaches

- Training Stress Balance (TSB) Model
- The Busso Model
- **The Banister Model**

Impulse-Response Model



The Banister Model

- Proposed by Banister in 1975
- Most experimentally validated
- Uses an exponential decay to model the effects of training stress
- Shows that a steady state of training will improve performance
- Quantify intensity based on heart rate

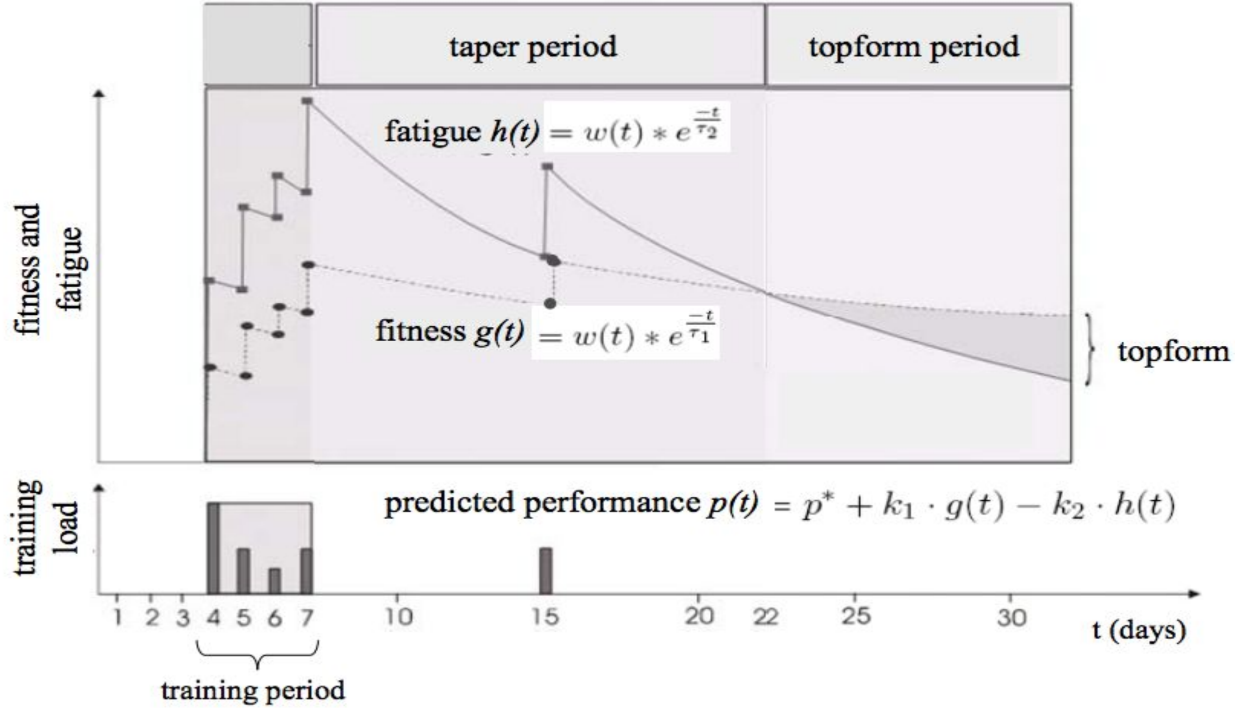
The Banister Model

$$p_t = p_0 + k_a \sum_{s=0}^{t-1} e^{-(t-s)/\tau_a} w_s - k_f \sum_{s=0}^{t-1} e^{-(t-s)/\tau_f} w_s$$

- *ka* and *kf* = fitness and fatigue magnitude factor, respectively
- *τa* and *τf* = fitness and fatigue decay time constant, respectively
- *wt* = the known training load per week/day from the first day of training to the week/day preceding the performance

* The original paper used values of ka=1.0, kf=1.8~2.0, *τa*=49~50, *τf*=11

Graphical Illustration



Flaws of the Banister Model

- The parameters are specific to each person and training system, so it takes a lot of steps just to start using the model to predict
- All trainings are summed into one, not considering individualization caused by variability in training types
- Ignores the effect of overtraining and assumed that any workload would result in improved performance of some degree (direct linear relationship)

Examining the Model

- People have used R^2 as a measure of validity of this model
- Only means how much variability is explained
- Not necessarily accurate predictions

Starting with…

Selected characteristics of the subjects training over the one-year period studied.

S	G	Age (years)	Height (cm)	Weight (kg)	Training (km)	Event	Best Perf	Number of Perf's	CV of Perf (%)
1	F	24	168	61	1402	100 Free	00:55:65	11	2.3
2	F	21	173	62	1856	200 Butt	02:10:8	12	1.6
3	F	26	179	59	1677	200 Free	01:59:86	21	2.2
4	M	27	185	84	1751	200 Medl	02:01:83	12	1.9
5	M	23	181	81	1340	100 Breast	01:03:51	14	1.8
6	F	26	168	50	1477	200 Back	02:15:00	18	2.4
7	M	20	186	80	1815	100 Free	00:51:5	12	2.6
8	F	19	167	52	1916	200 Free	02:03:51	11	1.6
9	M	23	188	84	1843	400 Free	03:53:42	13	1.5
Mean		23.2	177.2	68.1	1675			13.2	2.0
s.d.		2.8	8.4	14.0	215			2.4	0.4

Examining the Model

Due to the nature of the measures of swimming performances, we express the performance at time t as follows:

$$p_t = \frac{\min_t(P_t)}{P_t} * 100.$$

Examining the Model

From Matlab:

τa = 41.4 days

τf = 12.4 days

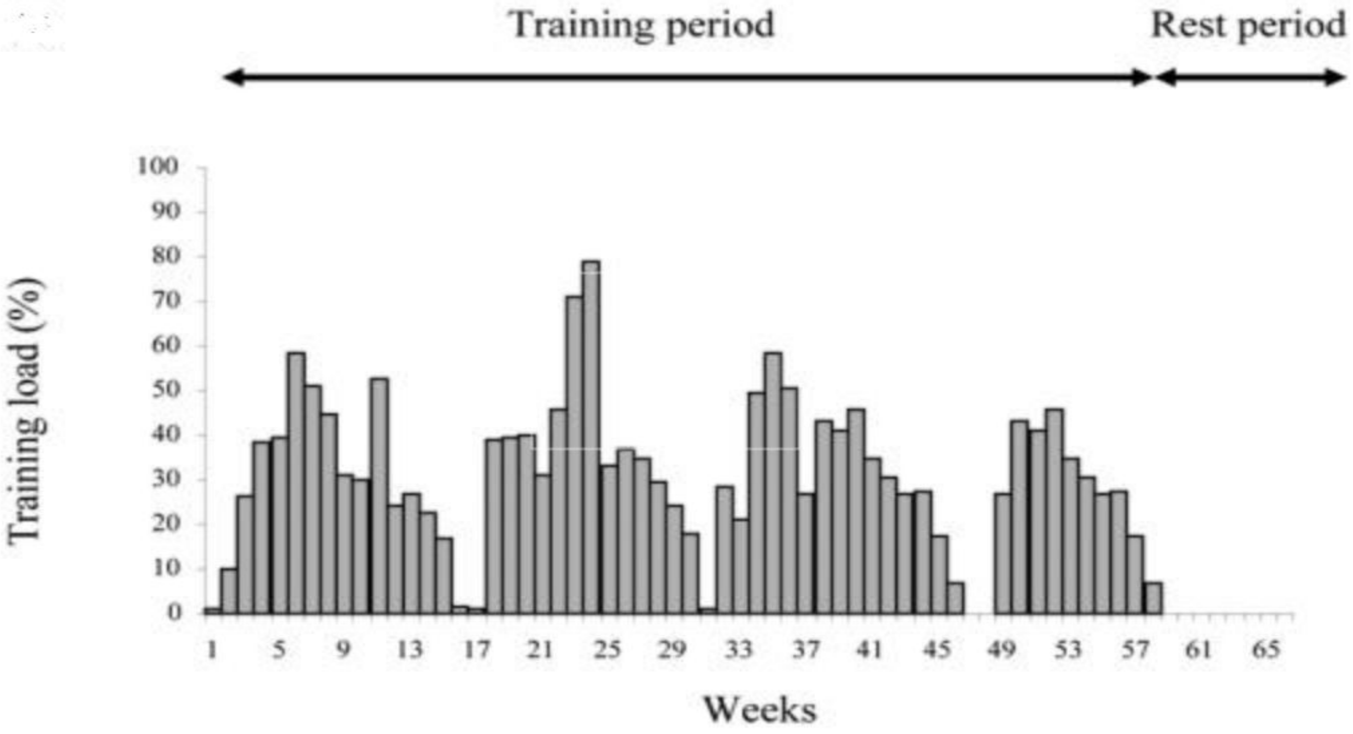
ka = 0.128 a.u.

kf = 0.055 a.u.

Quantify Training Load

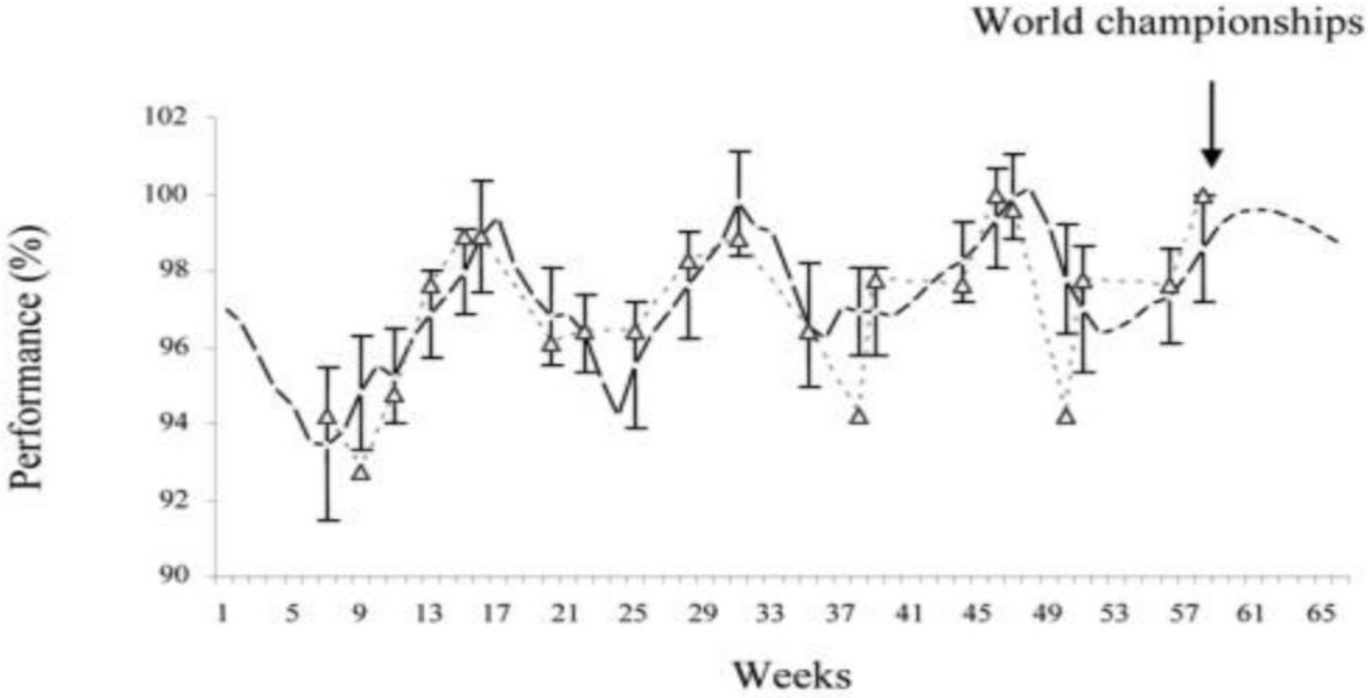
- Blood lactate concentration
- 7 intensity levels
- I3 => onset of blood lactate accumulation

Example of Subject 3



Before we see the result…

Example of Subject 3



Parameter Observations and Predictions

S	P_0	CI p_0	k_a	CI k_a	k_f	CI k_f	τ_a	CI τ_a	τ_f	CI τ_f	R^2	MIW
1	0.92	0.89, 0.95	0.002	−0.040, 0.044	0.016	−0.018, 0.051	40	8, 71	5	−6, 16	0.69*	3.17
2	0.95	0.92, 0.98	0.106	0.022, 0.189	0.129	0.071, 0.185	13	−6, 33	11	7, 15	0.84†	1.99
3	0.97	0.93, 1.01	0.039	0.006, 0.071	0.048	0.016, 0.081	33	8, 57	27	8, 46	0.65*	2.69
4	0.97	0.95, 0.98	0.050	−0.029, 0.068	0.068	0.048, 0.088	27	14, 40	20	15, 25	0.97§	0.80
5	0.98	0.95, 1.01	0.003	−0.040, 0.046	0.022	−0.012, 0.057	41	11, 70	9	−1, 19	0.78†	1.80
6	0.90	0.84, 0.94	0.009	−0.014, 0.034	0.016	−0.009, 0.041	45	18, 71	18	4, 32	0.61§	2.04
7	0.90	0.78, 1.02	0.018	−0.012, 0.046	0.028	−0.003, 0.057	57	39, 75	31	17, 46	0.95§	1.55
8	0.95	0.92, 0.97	0.083	0.019, 0.148	0.112	0.048, 0.176	23	11, 34	16	9, 24	0.92§	1.58
9	0.93	0.90, 0.96	0.010	−0.024, 0.046	0.012	−0.016, 0.042	65	50, 81	38	3, 64	0.73*	2.44
Mean	0.94	0.90, 0.98	0.036	−0.012, 0.077	0.050	0.014, 0.086	38	17, 59	19	6, 32	0.79	1.89
s.d.	0.03		0.038		0.044		16		11		0.13	0.49

Conclusion of Test

- Does not show difference among groups of swimmers that have different targeted trainings
- Peak performance in the rest period after 3-4 weeks of zero training which is not consistent with other proved ideas
- Does cover a good amount of sample variability but the intervals are a bit too large that the information becomes less useful

Modified Model by Busso

- The Busso Model was built on Banister's model
- Considered training monotony
- kf is changed into a function of training and is put into summation

$$k_2^i = k_3 \sum_{j=1}^{i} w(j) e^{-(i-j)/\tau_3}$$

$$\mathrm{p}(t) = \mathrm{p}(0) + k_1 \sum_{i=1}^{t-1} w(i) e^{-(t-i)/\tau_1} - \sum_{i=1}^{t-1} k_2^i w(i) e^{-(t-i)/\tau_2}$$

Other Suggestions

- Limit maximum fitness level
- Consistent, universal intensity classification to ensure stability of prediction
- Model fitness/fatigue parameter changes to be able to use functions to implement more frequent updates without numerous measurements
- Modify constants that are highly correlated

Other Non-traditional Approaches

- Linear mixed model
- Neural network

Questions?

Source

- http://ovidsp.tx.ovid.com/sp-3.31.1b/ovidweb.cgi?&S=IIDKFPMBLMDDMEPGNCEKKAFBMIIFAA00&Complete+Reference=S.sh.22%7c1%7c1
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