The Mathematics of Sports Betting

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Abstract

Sports betting is one of the fastest growing industries in the United State. The purpose of this paper is to give an introduction to sports betting. It begins by giving a brief overview of it's significance in the modern day, then gives examples of popular types of bets including moneyline, totals, and spreads, then finally exposes the reader to the mathematics behind betting, discussing odds and "the juice".

1 Introduction

1.1 The Impetus

Over the past 4 years I've watched a lot of sports. Starting around 2022 I noticed that I was getting a numerous amount of advertisements from various companies claiming that they would give me hundreds of dollars if I simply signed up for their service and spent $5 predicting the outcome of a game. Naturally, I assumed this was some sort of scam as it was seemed to good to be true. So I continued to ignore the advertisements for years, but I kept hearing from friends who were having success and enhancing their viewing experience of the games. Over winter break, in December of 2023, I was sitting at dinner with one of my best friends. Once every few minutes he'd pull out his phone and check the same app. I'd been in this same situation with him a lot recently, so I finally asked him if people could genuinely make money off of it. He looked me in the eye and said "Yes". You see the trick is gambling companies lure you in with a little bit of free money in hopes that they can psychologically exploit you into spending more than they give you, as long as you don't get addicted it is practically free money. That evening I downloaded DraftKings Sportsbook, and my journey began.

Up until about two weeks ago I had been betting pretty regularly on sports. It's become a regular topic of conversation within my friend group and the source of many emotions across the past few months. I wanted to bring it to the attention of the class for as I predicted (and verified during the course of my presentation) no one in the class had ever placed a bet on sports or at a casino before. As one of the oldest and most commonly used form of mathematics, I believe that even if you don't participate in gambling, everyone should understand the basics.

1.2 A Nationwide Pastime

In 2023, the National Collegiate Athletic Association (NCAA) conducted a survey to discover the prevalence of sports betting in collegiate life. Their findings revealed that 58% of students had engage in at least one sports betting activity. Further narrowing the demographic, 67% of all students surveyed who live on campus had participated in sports betting at least once (2). And, contrary to popular belief, the results of the survey found that both a
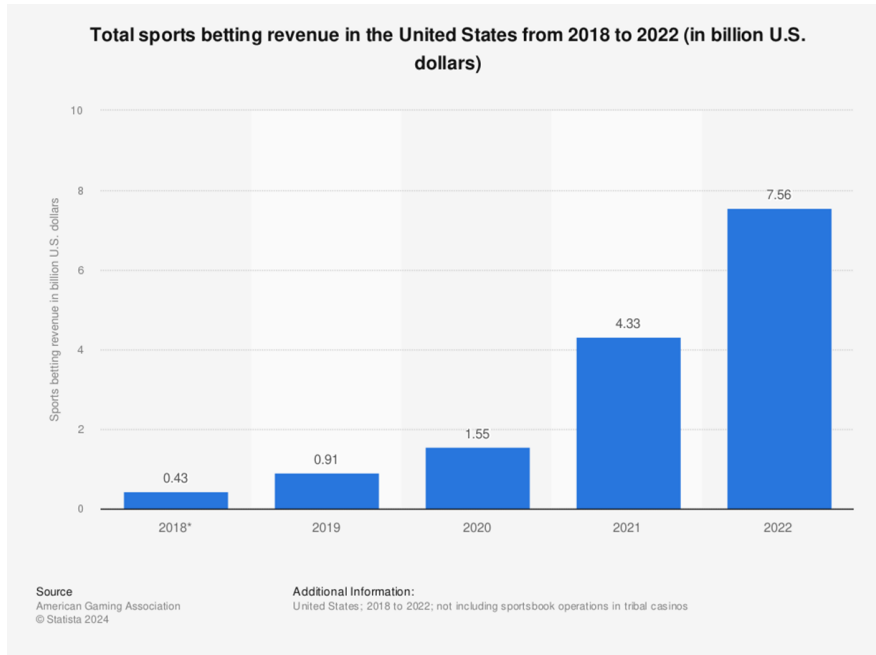
Figure 1: A bar graph showing the growth of sports betting from 2018-2022.

majority of men and women in college have participated in sports betting, emphasizing the universal draw of the activity. Obviously, sports betting is prevalent among young adults, but it has also emerged as a nationally recognized industry.

Sports betting became possible outside of the state of Nevada in May 2018 when the Supreme Court struck down the Amateur Sports Protection Act. Since then, 38 states have legalized sports betting, including Virginia (3). Over 300 billion dollars have been bet on the outcome of sporting events since legalization. In Virginia alone, 5.5 billion dollars were placed on sports bets in 2023. Figure 1 shows the exponential growth of revenue within the industry since it's legalization in 2018.

$$Revenue = Handle - Payout \tag{1}$$

The handle refers to the total number of dollars bet while the payout refers to the total amount of money the sportsbooks have to payout to winners. This difference being in the billions tells us that sportsbooks have an effective method of guaranteeing a profit, which we will examine further on.

1.3 How to Play

So what does sports betting actually entail? Simply put, it is the act of gambling on specific outcomes within sports events. For example, if the New York Yankees are playing the Boston Red Socks in a Major League Baseball game, you could visit a casino or download a sports betting app on your phone and put money on one of the teams to win the game, say the Yankees. If the Yankees were to win, you would win the amount of money you bet plus a

multiplier of your initial bet. If the Yankees were to lose, the sportsbook would keep your bet.

There are more types of bets beyond the simple "Who will win?" question. Popular bets include player props (e.g. Steph Curry will score more than 20 points tonight), total points (e.g. Barcelona vs Real Madrid will have less than 4 goals scored in the game), and futures (e.g. The New England Patriots will win the Super Bowl in 2025). Different bets can also be combined into one big bet called a parlay. An example parlay would be the Boston Celtics will beat the Miami Heat AND the Kansas City Chiefs will beat the Buffalo Bills AND Travis Kelce will score one touchdown. You would only get paid if all of the statements come true, but the payout will be much larger.

1.3.1 Reading Odds

Accompanying each statement is something called it's odds. For example, a statement like Steph Curry will score at least 20 points tonight might have odds of +100. These odds represent the multiplier you will get on your stake should the statement come true. To best interpret the practical meaning of odds, we'll take a look at the most commonly used odds systems.

British Odds (Fractional Odds). British odds are expressed as a ratio.

$$\frac{DollarsMade}{DollarsBet} \tag{2}$$

So if the odds are $\frac{3}{4}$ I would make 3 dollars for every 4 dollars I bet. An easy formula to express the total payout of placing a bet on a statement given its British odds is:

$$TotalPayout = Stake \cdot \frac{N}{D} + Stake \tag{3}$$

The "stake" is another way of saying the amount of money bet on the statement.

An example using British odds would be, the statement Liverpool will beat Luton Town has odds $\frac{1}{4}$. If I bet \$5 on Liverpool to win the match, the payout could be calculated as follows:

$$\begin{aligned} TotalPayout &= 5 \cdot \frac{1}{4} + 5 \\ &= \frac{5}{4} + 5 \\ &= 5.25 \end{aligned}$$

So you would only win \$0.25 from this bet, were it to win. This indicates that Liverpool are considered heavy favorites. In fact, we can easily see that the larger the ratio is, the less likely it is for the statement to come true.

European Odds (Decimal Odds). Another odds system that is relatively intuitive is the European Odds system. These odds are expressed as a decimal greater than 1. A simple formula to calculate the total payout of a bet using these odds can be seen below.

$$TotalPayout = Stake \cdot D \tag{4}$$

D is a decimal greater than 1.

An example using European odds would be, Napoli will beat Barcelona has 2.95 odds. If I bet $10 on Napoli, the payout could be calculated as follows:

$$TotalPayout = 10 \cdot 2.95$$
$$= 29.50$$

So if Napoli wins, you would profit $19.50. Note how the if $D \leq 1$, you would not make any money if your bet won (you would in fact lose money when $D \neq 1$). If $D > 2$, then the statement is considered unlikely to occur. If $1 < D < 2$, the statement is considered likely to occur. Again, the less likely the statement is to occur, the higher the payout.

American Odds   That brings us to the American odds system, which is a bit more complex. This system is almost exclusively used in the United States. American odds feature two types of odds: plus odds and minus odds. To calculate the total payout using American odds, we need two equations, one for each type of odd. We can write plus odds as +x (e.g. +100, +250, +310) and minus odds as -x (e.g. -100, -150, -300). To calculate total payout using American odds, consider the following formulas:

For plus odds:

$$TotalPayout = Stake \cdot \frac{x}{100} + Stake \qquad (5)$$

$x \geq 100$

For minus odds:

$$TotalPayout = Stake \cdot \frac{100}{x} + Stake \qquad (6)$$

$x \geq 100$

An intuitive was to think about the different types of odds is that plus odds are the amount that you would win if you bet $100 on the statement, and minus odds are the dollar amount that you would have to bet to win $100. We can also see in the above formulas that for plus odds, as x grows the payout gets bigger, meaning that if a statement has large plus odds, then it is considered less likely to occur. On the other hand, for minus odds as x grows the payout gets smaller, meaning that a statement with high minus odds is considered likely to occur. We can also see given the formulas that for plus odds, the multiplier will always be greater than or equal to 1, while the opposite is true for minus odds. So we can conclude that all statements with plus odds are considered unlikely to occur and statements with minus odds are considered likely to occur (except for +100 and -100 which both imply a 50% probability of occurrence).

For an example, let's say that the statement is the Detroit Pistons will beat the Los Angeles Lakers, and the odds for this statement are +350. Say we want to place a 10 dollar bet on the Pistons.

We can immediately tell that this statement is unlikely to occur because of the plus odd associated with it. Using our formula for plus odds, we can calculate:

$$TotalPayout = 10 \cdot \frac{350}{100} + 10$$
$$= 35 + 10$$
$$= 45$$

So if the Pistons were to win, we would profit $35. We will use the American style odds for the remainder of this paper.

1.3.2 Moneyline

We will now move away from odds and review the 3 most common bets that every sportsbook is sure to carry. Sportsbooks all have a similar interface and for every sports game they will have a table like the one in Figure 2.

Figure 2: An example sports betting interface

Let's begin by focusing on the "Money" column on the far right of the table. We can see that "Money" for the Miami Heat is +280 and "Money" for the Boston Celtics is -350. So we have the odds, but the question is what is the statement they correspond to? "Money" is short for moneyline. A moneyline bet is just a prediction of who will win the game. So the statement would be: The Miami Heat will beat the Boston Celtics, and the odds for the statement would be +280. Alternatively, you could bet on the statement: The Boston Heat will beat the Miami Heat, and the odds for the statement are -350. So just by looking at the money column, we can easily tell who is predicted to win the game. Since the Celtics moneyline odds are minus, we can infer that they are favored to win.

1.3.3 Over/Under

Moving to the middle column of Figure 2, we have the odds odd for the total points scored, or more commonly: the over/under. This bet is quite simple as well. Looking at the tile in the first row, we see that the odds are -110 and there is an "O 210.5". The statement for this tile is as follows: The total number of points scored in this game will be over 210.5, and the odds are -110. Similarly, the bet corresponding to the tile below would be: The total number of points scored in this game will be under 210.5, and the odds are -110. Note that the over/under tiles don't necessarily depend on an an individual team, rather on the game as a whole.

1.3.4 Spread

Finally, we have the spread bets, which tend to be the most difficult to understand. Looking at the tile in the first column of Figure 1, we see that the Miami Heat are +7.5 with -105 odds. The statement for this tile is: The Miami Heat will win OR lose by less than 7.5 points, with the odds being -105. Alternatively, for the Celtics spread tile the statement would be: The Boston Celtics will win by at least 7.5 point, with -115 odds.

For example, if the final score of the game was MIA: 110 BOS: 115, then the Miami spread bet would win and the Boston spread bet would lose. In this case, it could be said that even though Miami lost, they "covered the spread".

2 The Mathematics

2.1 Probability vs Odds

We've been referring to the likelihood of our bets occurring in terms of odds, but what exactly are odds? The formulas for probability and odds and the following example highlight the key difference:

$$P = \frac{\#of successes}{\#number of outcomes} \tag{7}$$

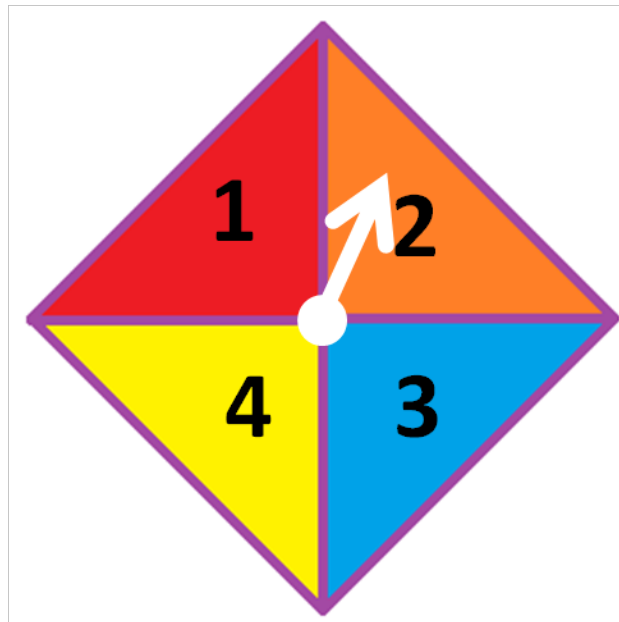$$O = \frac{\#of successes}{\#number of failures} \tag{8}$$

Figure 3: 4 sided spinner

The probability of a the spinner landing on 1 would be the number the number of success (spinner lands on 1) over the total number of outcomes (spinner lands on 1,2,3,4). So the probability would be $\frac{1}{4}$. On the other hand, the odds of the spinner landing on 1 would be the total number of successes, over the total number of failures (spinner lands on 2,3,4). So the

odds would be $\frac{1}{3}$. We can convert this to American odds by multiplying by $\frac{100}{100}$, yielding $\frac{300}{100}$ which using our minus formula gives -300 odds. This, however, seems like a contradiction. It was previously said that statements with minus odds are considered likely to happen, but we know that the probability of the statement "The spinner will land on 1" is 25%. What we need to consider now is that when we read odds from a sportbook, they are written from the perspective of the sportsbook. Odds of -300 imply that the sportsbook has a 25% chance of winning the bet, meaning we lose 25% of the time and win 75% of the time. The bottom line is: minus odds mean long odds for the sportsbook, short odds for the bettor, while plus odds mean long odds for the bettor, and short odds for the sportsbook.

Traditionally, as humans we prefer to think in terms of probability rather than odds. One can convert from American odds to probability in the following manner, using the example of +350:

$$\begin{aligned} +350 &= \frac{350}{100} \qquad \qquad \text{(Convert to British odds)} \\ &= \frac{35}{10} \end{aligned}$$

Using our formula for odds, the number of successes is 35 and the number of failures is 10, giving us 45 total outcomes. Using our formula for probability, we get:

$$\begin{aligned} P &= \frac{35}{45} \\ &= 0.778 \\ &= 77.8\% \end{aligned}$$

2.2   The Juice

We saw earlier that the revenue of the sports betting industry is in the billions, so now we will examine how they actually make their money. We can do this by referring to the example given in Figure 2.

Consider the over/under column. Note that the odds of the total points scored being over 210.5 is the -110, the exact same as the odds for the total number of points being below 210.5. Since there is only two options, the game will either finish with less that 210.5 points or more than 210.5 points, the probability of the two scenarios should add up to 100%. However, using the method from above, we can derive that the implies probability of -110 is 52.4%. Meaning that the two probabilities add to 104.8%. This extra 4.8% is called the "juice". What it means is that in order to profit from your betting, you have to win over 52.4% of the time on bets that the sportsbook considers 50-50. Assuming the sportsbook has done their research correctly, this is very difficult over a long period of time, and unless you have have some sort of unaccounted for edge, it become impossible to sustain.

Looking at the example from the Miami vs. Boston game, say across all bets, $1,000,000 is bet on the over at -110 and $1,000,000 is bet on the under at -110. As we said, if the sportsbook has set the line correctly, they both have a 50% chance of occurring. No matter what, the sportsbook has to payout $1,000,000 \cdot \frac{100}{110} \approx \$909,091$ and the sportsbook keeps the other $1,000,000 from the losing bettors. Assuming they set the line right, meaning bettors are split 50-50, they will profit $90,909 or about 4.54% of the total stake.

2.3 Minimizing the Juice

So the question now becomes: Are there any strategies to help me be a successful bettor? There aren't any proven profitable strategies in gambling, otherwise the industry would have gone under long ago. Sometimes, the sportsbooks will do promotions that boost odds in favor of the bettor, just as a ploy to further suck them in.

However, there it is notable that the juice isn't standardized at 4.8%. Some sportsbooks market themselves as "reduced juice" sportsbooks, meaning that they have on average a lower winning commission. These sportsbooks are best for players who place a lot of bets and are looking to beat the 52.4% rule. Other sportsbooks have bigger juices, but give great new customer bonuses that attract novice players, so it's very important it to consider the type of bettor you want to be before committing to a particular bookie.

3 The Presentation

3.1 Reflection

Overall, I'm pretty happy with how my presentation went. I was really lucky that this entire sports betting phase of my life overlapped with this semester, as it really was the perfect topic for me. The feedback I received was very positive. People seemed to enjoy the easy to follow along aspect of the presentation, and I'm happy I got to introduce my classmates to something they had little to no experience with. I think it was super important to have a topic that I could connect to personally, I love sports, and I am passionate about this type of stuff. Plus gambling is just fun any money tends to keep people engaged. I was happy to go with a very simple mathematical topic to begin with just to get my feet wet, but I think I'm going to have to be a bit more technical with my next presentation.

3.2 Looking Forward

I'm still deciding what to do for my next presentation. As I said before, I want to do something that I find fun and exciting. Someone suggested just a broader survey of gambling, but I fear that I might be getting a bit tired of all of the probability and odds talk. That being said, there are some interesting papers about strategy in gambling that could be cool to explore, but at the end of the day, we know that you're never going to beat the casino so that's a bit depressing.

References

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