

MATH 400: MATH AND KNITTING IN CROCHET

ANDY FISHER AND RACHEL VASAN

ABSTRACT. For our first MATH 400 presentation, we aimed to elaborate on how mathematics connects to knitting and crochet. This paper is organized into six parts. In part 1, we discuss our topic selection, specifically the relevance of our topic to our course goals. In part 2, we provide an introduction to knitting, crochet, and the basic mathematical principles that relate to these crafts. In part 3, we discuss the connections between crochet and fractals; in part 4, we discuss the connections between crochet and hyperbolic space. Part 5 considers the connections between knitting and topology; part 6 considers machine knitting, knitting as a form of coding, and the relationship between knitting and symmetries. Part 7 talks about further connections between knitting, crochet, and math, including Dr. Hinke Osinga's contributions to the field of mathematical knitting and crochet, and the mathematical principles related to knitted and crocheted clothes. We conclude in part 8 with a reflection on our experiences creating this project.

I. MATHEMATICAL CONNECTION

1.1. Topic Selection. When selecting our topic, we both wanted to find something that we had a pre-established interest in. Both of us crochet and knit, with Rachel being more partial to knitting, and Andy more partial to crocheting. Although we had a fair amount of experience with these crafts before creating our presentation, neither of us were familiar with the many connections between crochet, knitting, and mathematics. This topic fits with the goals of our course, as it shows how mathematics can connect to artistic fields. Our main goals in creating this presentation were to introduce knitting and crochet as inherently mathematical arts, and show how mathematics can have roots in all aspects of life – even our hobbies.

II. INTRODUCTION

2.1. What is Knitting? Knitting is a way of manufacturing a fabric that is made by creating a line of loops and then pulling another line of thread through the existing loops. This means that

the yarn goes along the row as shown in Figure 2.1.1. This fabric can be manufactured either by hand with a set of two needles and works along the fabric one stitch at a time or with a machine that uses a line of hooks to manipulate all of the stitches at the same time. We refer to the stitches currently being worked as active or live stitches.

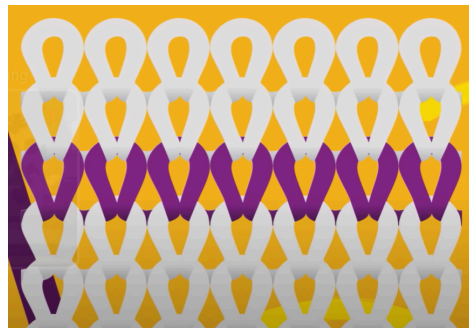


Figure 2.1.1[2]

There are a wide variety of fabrics that can be made by knitting. For the sake of simplicity during the presentation and this paper we will only discuss the fabric created through stocking knit stitch. It is important to note that the final fabric created by this form of knitting has a right and wrong side, which means that the two sides of the fabric look different. Most of the time the fabric that is used is the side that has the “v” pattern on it.

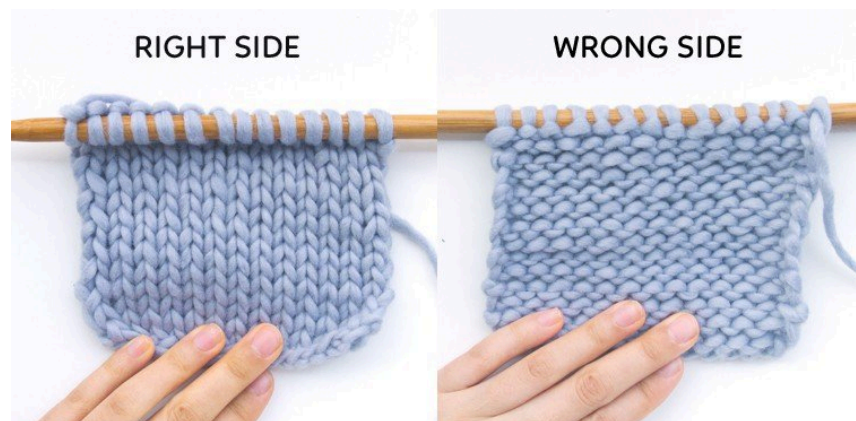


Figure 2.1.2 [13]

2.2. What is Crochet; How is it Different from Knitting? Unlike knitting, crochet uses a single hook – as opposed to two needles – to weave yarn into fabric; crochet cannot be replicated by a

machine. There is more variation in the stitches you can create in crochet as compared to knitting. The most common crochet stitches are chain, single crochet, half double crochet, and double crochet stitches. Slightly less common are the triple crochet and double triple crochet stitches. The primary visual difference between all of these stitches is their height; the relative height difference between double triple (DTR), triple (TR), double (DC), half double (HDC), and single (SC) crochet stitches can be seen in Figure 2.2.1.

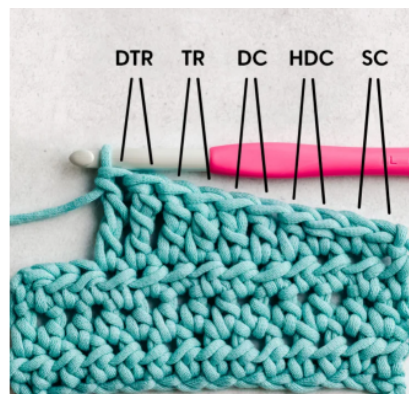


Figure 2.2.1 [1]

In contrast to knitting, in crochet, only one stitch is active at a time; in a practical sense, this means that a crocheter cannot drop finished stitches in a given row without first unraveling each stitch preceding it. In Figure 2.2.1, for example, the leftmost DTR crochet stitch is the only active stitch in its row; if we wanted to drop, say, an HDC stitch, we would first need to unravel the DTR, TR, and DC stitches that precede it. The nature of active stitches in crochet gives the crocheter more freedom to work their stitches in various directions.

As in knitting, crochet can be worked flat or in rounds. However, in contrast to knit stitches, crochet stitches are looped from top to bottom *and* side to side (Figure 2.2.2). This difference causes crocheted fabric and knitted fabric to have distinct appearances, and crocheted fabric to be, in general, larger than knitted fabric.

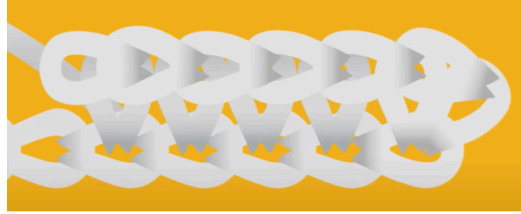


Figure 2.2.2 [2]

2.3. Basic Mathematics in Knitting and Crochet. There are a lot of ways that mathematics relates to the basic concepts involved in knitting and crochet. One of the first and most basic is referred to as yarn usage; this is the amount of yarn needed for a given project. Before someone starts a project, they first have to determine the yarn usage for that project and buy the corresponding amount. One of the notable differences between knitting and crochet is that crochet uses significantly more yarn. Hence, yarn usage is generally greater for crochet projects. The amount of yarn used for a knitting or crochet project also depends on the size of the knitting needles, or crochet hook, used. Hook and needle sizes are based on the diameter of the hook/needle in millimeters; using larger needles, or a larger hook, will mean that the circumference the yarn wraps around to create stitches is greater. Hence, the amount of yarn used is greater for projects that use large needles, or a large hook.

The next mathematical concept that exists in knitting and crochet is the differences in the size of the finished project. Even with the same size hook/needle, crochet fabric made with a single crochet stitch will be larger than knitted fabric made with the same number of stocking knit stitches. There have been a number of informal experiments conducted to see how much time crochet and knitting take; in general, most people find that for knit and crochet swatches with the same number of stitches, the knit fabric takes longer to create. This is interesting given that the knit swatch will be smaller.

Other basic mathematic principles involved in knitting and crocheting include counting stitches, increasing and decreasing, and scaling patterns. Counting stitches is the most basic way to create a pattern or a set of instructions for a knitting or crochet project, that someone else can follow. We count stitches to make sure that we are increasing and decreasing at the rate that will generate the desired shape. Increasing and decreasing allows us to make non-rectangular fabric; we increase by knitting or crocheting twice into a single stitch and decrease by knitting or crocheting two stitches together. Next, to discuss scaling patterns. When patterns for clothing are made, they are designed to fit one person. To make a pattern that will fit various body types, the original pattern will need to be scaled. This means that the proportions of the design are mostly maintained while the size of the final project is changed. This can be thought of as a geometric transformation. [14] [15]

III. CROCHET AND FRACTALS

3.1. A Brief Introduction to Fractals. The Fractal Foundation defines fractals as “infinitely complex patterns that are self-similar across different scales”. Put a bit more simply, a fractal is a never-ending pattern, and if you were to zoom in on one continuously, the image you see would essentially stay the same. Although the concept of a fractal is rooted in geometry, we see fractals in other mathematical disciplines; the Mandelbrot Set, and the Cantor Set, for example, are fractals. We also see examples of fractals in nature; natural forms of crystals, and plants such as broccoli, exhibit fractal properties. [3][4]

3.2. Fractal Crochet. Fractal properties may be utilized to develop crochet patterns, or alternatively, crochet may be used to model fractals. Both of these processes are referred to as “fractal crochet”. Figure 3.2.1 shows a crochet pattern that utilizes fractal properties. To develop

this pattern, the crocheter used a repeated set of stitches throughout their project, resulting in many iterations of the same pattern. [5]



Figure 3.2.1 [5]

As mentioned previously, crochet may also be used to model fractals. To better understand this, we will consider a crochet model of the Sierpinski Triangle. Figure 3.2.2 displays a computer-generated image of this geometric fractal; Figure 3.2.3 displays its crochet model. To create this model, the crocheter first sketches the basic shape they want to create and repeat; this shape is displayed in Figure 3.2.4. The crocheter then traces out a path that goes over each line in this shape. Next, they decide how long they want each side of the smallest triangle to be; in this case, the crocheter chose a length of five stitches per side. Next, they crochet five chain stitches, placing a marker in the final stitch. They repeat this process until the path they are crocheting along connects with itself; at this point, the crocheter places their hook into the previously marked connecting stitch, and works a chain stitch into it. Putting this process into simpler terms, the crocheter creates one row – or, one straight line – and attaches this row to itself at previously marked stitches – or, points. [6]

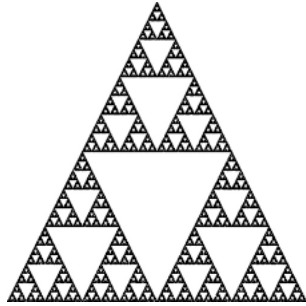


Figure 3.2.2 [6]



Figure 3.2.3 [6]

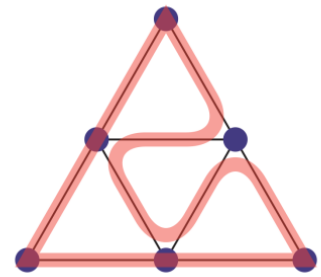


Figure 3.2.4 [6]

IV. CROCHET AND HYPERBOLIC SPACE

4.1. **A Brief Introduction to Hyperbolic Space.** Hyperbolic space is non-Euclidean space. That is, hyperbolic space does not satisfy Euclid's Parallel Postulate, which states: given a line, l , and a point P which does not lie on l , there is a unique line – say t – such that t passes through P and is parallel to l . For such a line l and point P in *hyperbolic* space, there exist at least two lines passing through P and parallel to l . Every point on a hyperbolic plane is a saddle point, and hyperbolic planes have negative Gaussian curvature; Figure 4.1.1 displays an example of a hyperbolic plane. [7][8]

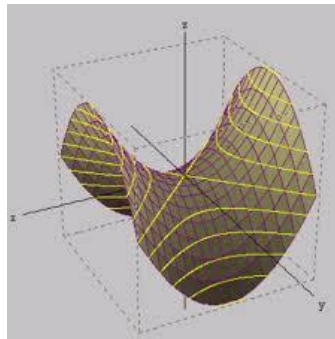


Figure 4.1.1 [7]

4.2. **Crochet and Curvature.** It is possible to crochet surfaces with negative Gaussian curvature; this is referred to as “hyperbolic crochet”. Similarly, it is possible to crochet surfaces with zero

curvature or positive curvature. We will examine how surfaces with zero, positive, and negative curvature are crocheted by considering three examples.

(1) Our example of a crocheted surface with zero curvature is a flat circle (see Figure 4.2.1).

This surface is achieved by crocheting in the round, from the innermost to the outermost row, and increasing the number of stitches per row at the proper rate. But how is this “proper rate” determined? First, we take one crochet stitch to be our unit of measurement. This means that our first row will be a circle of radius 1; the circumference of this circle must of course be 2π . Because the number of stitches used per row must be an integer, we approximate π at 3; then 2π is approximately 6, and our first row should have 6 stitches. Our second row will be a circle of radius 2; the circumference of this circle must be 4π . Again approximating π at 3, we determine that our second row should have 12 stitches. If we continue on in this fashion, linearly increasing the number of stitches per row by a factor of 6 – or “ 2π ” – the end result will be a flat circle. [9]



Figure 4.2.1 [9]

(2) Our example of a crocheted surface with positive curvature is a sphere (see Figure 4.2.3).

As with a flat circle, this surface is achieved by crocheting in the round. The number of stitches per row must be increased at a linear rate, until the radius of the sphere is reached; at this point, the number of stitches per row must be *decreased* at the same rate up to the final row (if instead, we continue to increase at our linear rate, the resulting

surface would not be a sphere, but rather a paraboloid). When crocheting a sphere, the number of stitches per row must be increased at a slower linear rate than that of a circle; this causes the crocheted fabric to distort upwards instead of lie flat. For a more rigorous mathematical explanation of the precise rate of increase in stitches per row of an approximately ideal sphere, see the [following website](#). [9][10]



Figure 4.2.2 [9]

(3) Our example of a crocheted surface with negative curvature is a hyperbolic pseudosphere (see Figure 4.2.3). This surface is achieved by crocheting in the round and increasing the number of stitches per row at an *exponential* rate. In this case, because the number of stitches in a given row is so much greater than the number of stitches in the preceding row, the crocheted fabric is forced to curve up and down. We can think of this as a more dramatic, less uniform example of the distortion seen in example (2). [9]



Figure 4.2.3 [9]

4.3. Hyperbolic Crochet. Using crochet to model hyperbolic space, or alternatively, using properties of hyperbolic space to create crochet patterns and art, is called hyperbolic crochet. In both cases, the crocheter works in the round, increasing the number of stitches per row at an

exponential rate. Varying the rate of exponential increase per row will yield different end products; a crochet piece for which the number of stitches per row is increased at a very high exponential rate will curl in on itself more tightly than a piece for which a slower exponential rate is utilized. To visualize this difference, consider the piece shown in Figure 4.3.1, which utilizes a slower exponential rate than the piece shown in Figure 4.2.3.



Figure 4.3.1 [11]

The variation that can be achieved in hyperbolic crochet sees beautiful applications in art; for example, artists such as Christine and Margaret Wertheim have utilized this variation to create realistic crochet coral reefs, as seen in Figure 4.3.2. This work of art includes components that mimic hyperbolic space as well as components with positive curvature; see if you can spot a few! [12]



Figure 4.3.2 [12]

V. KNITTING AND TOPOLOGY

5.1. **A Brief Introduction to Topology.** Topology is the field of mathematics that studies spaces but considers all surfaces the same that are invariant under continuous deformation. In other words, topology studies surfaces but regards all surfaces that can be made by continuous change of shape as the same surface. We define a surface as a set of connected discs. [31] [32] [33]

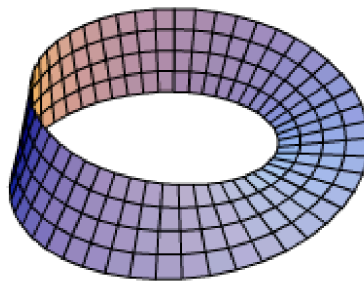


Figure 5.1.1 [32]

5.2. **Knitting Topological Surfaces.** In order to create a knitted pattern that mirrors a topological surface, you first have to calculate the difference between the height and width of a knitted stitch which is $\frac{2}{3}$ stitches * rows. Then you represent the discs of the surface as rectangles and connect them either by accounting for it in the pattern and knitting multiple discs together or perhaps more simply by sewing them together. Because of how formulaic this method is it is possible for these patterns to be computer generated. This technique works well for simply connected surfaces however it will fail for not simply connected surfaces. For these surfaces, the best way to knit them is to create an approximated surface that is simply connected. If one wants to add more information onto the surface you are able to add a metric which allows the product to be a proportional representation of the surface. [31] [32]

VI. KNITTING AND TECHNOLOGY

6.1. A Brief Introduction to Machine Knitting. Before people started using man-made fibers to make clothing the way that they made stretchy clothing was to use knitted fabric. Knitted fabric continues to be very common in a lot of modern clothing from t-shirts to activewear. To make the volume of clothes that was necessary and at the speed that was needed to be made knitted fabrics needed to be able to be made by machine.

Hence, the knitting machine was invented by William Lee in 1589. This machine featured a row of hooks that held loops of yarn that could then be used as live stitches. This simulates the way that loops are held on a pair of needles. Then a moving carriage holding yarn slides along the bed of hooks as it slides over the hooks it places another length of yarn into these hooks. After the new yarn is in the hooks the old stitches are pulled over the new yarn creating new stitches. This creates a stocking knit stitch. (There exist two types of knitting machines one that knits a flat fabric and one that knits in a circle. For the sake of this presentation and paper we will only discuss the flatbed knitting machine that makes flat fabric.) This means that in one action we can knit across an entire row of stitches. [16] [17] [18] [19]

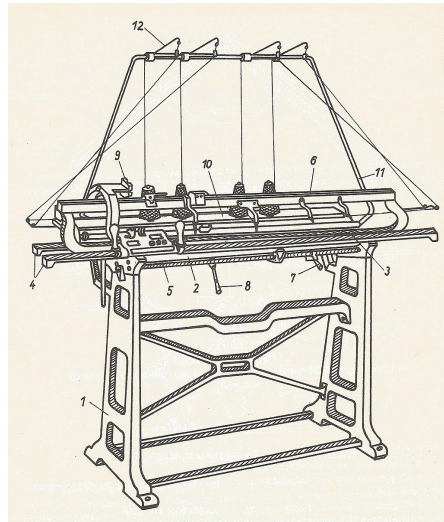


Figure 6.1.1 [20]

6.2. Machine Knitting and the Analytical Engine. The next thing to tackle was how to knit more complicated garments. The form of knitting shown below is called lace knitting. The lace is created by knitting some stitches while skipping others. The information regarding which stitches to stitch and which to skip was uploaded into the knitting machine using punch cards. These punch cards tell the machine which hooks to push forward which controls which stitches are knit in the next row. After the row is completed the punch card is advanced so the machine can read the next row. The way that the punch cards and the fabric correspond is shown below (Figure 6.2.1).

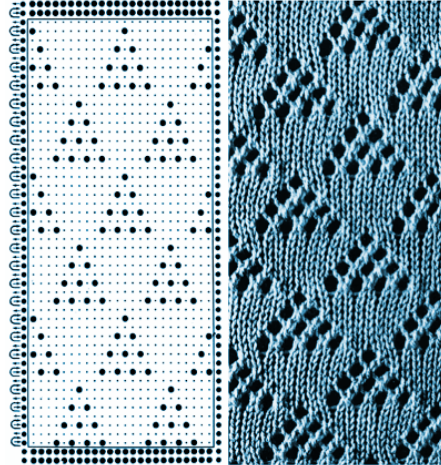


Figure 6.2.1 [22]

This way of inputting information was the first instance of punch cards being used to input information into a machine. This went on to inspire the later punch cards that were used to code machines. [19] [21]

6.3. Knitting as a Form of Coding. Not only was machine knitting an inspiration for computers but also hand knitting can be thought of as an algorithm. Some of the similarities between the two are the use of loops, patterns, symbols, and testing. We will first discuss loops; for repeated knitting patterns instead of writing the sequence of stitches repeatedly the pattern writer will write the stitches and then write the number of times the sequence of stitches should be repeated. Then for symbols, there is a set of somewhat standard abbreviations and symbols that correspond with certain stitches so that the patterns can be written more efficiently. Then once a pattern is written it has to be test knitted to ensure that the pattern can be replicated by others and that the pattern has not only been written correctly but written in a way that others can understand.

All of these features lead me to think that knitting can be thought of as a coding language or algorithm. I think this is a really interesting idea because it offers another way to teach children or even adults logic and how to follow a set of instructions or think in a mathematical framework. [23] [24]

6.4. Knitting and Symmetries. Another aspect of math in knitting is the use of symmetries. Symmetries in knitting are often created using loops because it allows designers to create a smaller pattern and then repeat the pattern until you create the size that you need. What is interesting is that there are mathematicians who are creating knitted garments that include all types of symmetries. But these symmetries also exist in garments that were invented by people who had no knowledge of mathematics or the complexities of what they were creating. It is also very interesting to me to see how completely independent communities all had examples of symmetries. This feels like a strong argument that supports the idea that mathematics is discovered not invented. [25]



Figure 6.4.1 [25]

VII. MORE CONNECTIONS BETWEEN MATHEMATICS AND FIBER ARTS

7.1. Dr Hinke Osinga. In researching this topic, I kept coming across Dr. Hinke Osinga and so wanted to include someone who to the best of my research appears to be one of the first pioneers of this space. Dr. Hinke Osinga is a mathematician at the University of Bristol. She was working

on a project one day when she was asked, “Why don’t you crochet something useful?”. This inspired her realization that she could crochet the Lorenz manifold (Figure 7.1.1). Though she had been knitting since she was seven years old, it was this experience that made her realize how useful knitting and crochet are as tools for visualizing mathematical objects. Representing 3D mathematical objects on a flat page often requires removing or simplifying aspects of the object. This makes it difficult for people to fully understand the objects in their full or 3D form. By using knitting and crochet, we can help new mathematicians build intuition with a more accurate representation of these objects. [26]



Figure 7.1.1 [26]

7.2. Mathematics in Wearable Clothing. Not only are knitting and crochet powerful tools for understanding mathematics but math has also been inspiring knitting and crochet. For example, people have made are Mobius strip scarves (Figure 7.2.1) and Klein bottle hats. There is also an example of someone who made a blanket using squares whose size followed the Fibonacci sequence (Figure 7.2.2).



Figure 7.2.1 [27]



Figure 7.2.2 [28]

One artist even created a scarf in which the number of rows in each section is equal to the next prime number so the scarf follows the sequence of prime numbers as they appear in the real number line. The pattern also makes a joke poking fun at the fact that while there are an infinite number of prime numbers, meaning the scarf can be made infinitely long, as these numbers get farther apart the sections of the scarf will get increasingly longer. There was also an artist who

made a cowl that features a fractal through the colorwork pattern (Figure 7.2.3) and a book entitled *Making Mathematics with Needlework* by Sarah-Marie Belcastro and Carolyn Yackel.

Which explores further how to knit mathematical objects. [27] [28] [29] [30]



Figure 7.2.3 [30]

VIII. REFLECTION

8.1. **Andy's Takeaways.** My objectives for this presentation were to introduce crochet, explain how mathematics can be used to crochet, and describe how crochet can be used to model mathematical concepts. Fractal crochet and hyperbolic crochet were the most interesting connections I found between mathematics and crochet in my research. I think offering a brief explanation of fractals and hyperbolic space before explaining these connections aided the clarity of my presentation. However, in my next presentation, I may want to go more in-depth on fewer topics; the feedback Rachel and I received from our peers was generally positive, but some people noted that they wished we had discussed the math involved on a deeper or more technical level. Looking back, I think it might have been more engaging to take a deeper look at the mathematical connections seen in hyperbolic crochet, and shorten my discussion of fractal crochet. I also think it could have been interesting to briefly note how crochet might be utilized as a tool for math education.

8.2. **Rachel's Takeaways.** My objectives for this presentation were to expand the idea of where mathematics can be found. I understand that most people in the class probably have not knitted before so I wanted to make sure that I explained enough of the basics of knitting that people were able to understand how the concepts applied without explaining too much that they felt overwhelmed. If I were to do the presentation again I would have liked to spend more time discussing the topological applications of knitting and get into more of the technical math and proofs that exist. I think overall our feedback was positive which was good to hear but a few people asked for more emphasis on the technical aspects of the topics we discussed. I liked that we were able to incorporate a lot of visual examples because I think it made both the crafting and the mathematical concepts more approachable for people who did not have previous experience in the topics and I think it made the overall presentation more interesting. I think that these topics could be really interesting when discussing mathematics as something that is either discovered or invented. I would have liked to discuss more why we see examples of these mathematical concepts from people who have no understanding of mathematics.

REFERENCES

- [1] Schipper, F.M., *A Mathematical Study of Crochet*, fse.studenttheses.ub.rug.nl/27795/1/bMATH_2022_SchipperFM.pdf. Accessed 25 Mar. 2024.
- [2] “Why It’s Impossible to Build a Crochet Machine.” *YouTube*, Half as Interesting, 6 Mar. 2023, www.youtube.com/watch?v=ElmnSsCadK8.
- [3] “What Are Fractals?” *Fractal Foundation*, fractalfoundation.org/resources/what-are-fractals/. Accessed 25 Mar. 2024.
- [4] “What Is a Fractal? - The Ultimate Guide to Understanding Fractals.” *Iternal Technologies*, 24 Sept. 2020, internal.us/what-is-a-fractal/#:~:text=A%20Fractal%20is%20a%20type,similar%20to%20the%20whole%20image.
- [5] Mlnarik, Kelsey. “Fractal Crochet: An Introduction and Beautiful Examples.” *Crochet Concupiscence*, 20 Dec. 2022, www.crochetconcupiscence.com/stunning-examples-of-crochet-fractals/.
- [6] Stokes, Felix. “How to Crochet a Fractal.” *Chalkdust*, 23 Oct. 2019, chalkdustmagazine.com/blog/how-to-crochet-a-fractal/.
- [7] “Euclidean Space.” *Encyclopædia Britannica*, Encyclopædia Britannica, inc., www.britannica.com/science/Euclidean-space. Accessed 25 Mar. 2024.
- [8] “Hyperbolic Geometry.” *Wikipedia*, Wikimedia Foundation, 26 Jan. 2024, en.wikipedia.org/wiki/Hyperbolic_geometry.
- [9] Lambert, Anna. “The Wonders of Mathematical Crochet.” *Chalkdust*, 21 July 2020, chalkdustmagazine.com/blog/wonders-mathematical-crochet/.
- [10] “The Ideal Crochet Sphere.” *Ms Premise-Conclusion*, 18 July 2017, mspremiseconclusion.wordpress.com/2010/03/14/the-ideal-crochet-sphere/.
- [11] “Hyperbolic Space; Crochet Models.” *The Institute for Figuring // Online Exhibit: Hyperbolic Space*, theiff.org/oexhibits/oe1e.html. Accessed 25 Mar. 2024.
- [12] Wertheim, Margaret, and Christine Wertheim. “Art - Crochet Coral Reef.” *Margaret Wertheim*, www.margaretwertheim.com/crochet-coral-reef. Accessed 25 Mar. 2024.
- [13] Lauren, and Pat Dalton. “Stockinette Stitch Knitting for Beginners.” *Sheep and Stitch*, 21 Aug. 2020, sheepandstitch.com/library/stockinette-stitch-knitting-for-beginners/.
- [14] Zimmermann, Kat. “Knitting vs. Crochet: Speed & Yarn Use.” *Craftematics*, Craftematics, 28 Aug. 2023, www.craftematics.com/post/knitting-vs-crochet-speed-yarn-use.
- [15] Andrea, et al. “Does Crochet Use More Yarn than Knitting?” *Yarnandy*, 13 Jan. 2023, yarnandy.com/does-crochet-use-more-yarn-than-knitting/.
- [16] “Machine Knit Sweater for Beginners | Any Flatbed Machine | Pattern & Tutorial.” *YouTube*, YouTube, 14 Jan. 2023, www.youtube.com/watch?v=AwFmx2xWR7w&t=33s.
- [17] “Basic Flat Bed Knitting.” *YouTube*, YouTube, 2 Feb. 2016, www.youtube.com/watch?v=xYzK15yVjXw&t=316s.
- [18] Ann, Krista. “The Surprisingly Controversial History of the Knitting Machine.” *Interweave*, 22 Feb. 2024, www.interweave.com/article/knitting/history-of-knitting-machine/.
- [19] “Flat Bed Machine.” *Flat Bed Machine - an Overview | ScienceDirect Topics*, www.sciencedirect.com/topics/engineering/flat-bed-machine#:~:text=The%20typical%20flat%20machine%20has,a%20bi%2Ddirectional%20cam%20system. Accessed 25 Mar. 2024.

