Electoral College and Gerrymandering:

A Mathematical Exploration of the Issues of Representation in the United States

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Math-400

In the year 2020, the United States of America is having both a presidential election and a Census collection. As a result of these two momentous political events, questions on fairness of representation in the United States are brought to the forefront. Through a mathematical study on apportionment of House of Representatives seats, issues of the gerrymandering and the electoral college are brought into new light.

1 **Apportionment of House of Representatives Seats**

The United States Federal legislative branch consists of two houses, the Senate and the House of Representatives. As determined by the constitution, representation for states in each of these houses are determined differently. In the Senate, there is equal representation for each of the states, with two senators elected from each state, resulting in 100 senators overall. In the House of Representatives, there is proportional representation for each of the states, determined by the population of each state. Though originally, the total number of representatives was meant to increase according to the overall population of the country, it was capped to 435 representatives in 1929 [14]. Each ten years, the number of representatives per state is recalculated according to the populations as counted in the Census [10].

According to the Constitution, each state must have at least one representative. The US Congress is responsible for determining the method of apportionment and then calculating the number of representatives per state according to this method [10].

Numerous apportionment methods have been used throughout the history of the US Congress.

Historical methods use the calculation of

$$\frac{state\ population}{\frac{total\ population}{number\ of\ house\ seats}}$$

but vary depending on the rounding method. According to the Hamilton Vinton method, each state is assigned the rounded down above calculation number of seats and the surplus seats are assigned to the states with the highest fractional component of the above value. The Webster method rounds the above calculation up if the fractional component is greater than or equal to .5 and rounds down if it is less than .5. Finally the Jefferson method rounds down the above calculation for each state and assigns each surplus seat to the state with the largest critical divisor, that is the state population divided by the current number of seats plus one [5].

The current method used to apportion House of Representatives seats was developed by Huntington in 1921 in order to establish a mathematical basis and reasoning for the process of apportionment. The overall goal of this method, the Huntington-Hill Method is to reduce the disparity between the number of people represented by each representative across states. It accomplishes this by maintaining a mathematically determined priority list and assigning the next representative to the state with highest priority. Priority calculated by dividing the population of each state by the geometric mean of its current and next seats, $priority = \frac{P}{\sqrt{n(n-1)}}$, where P is the state population and is the state's current number of representatives plus one [3].

While the Huntington-Hill method does attempt to reduce the disparities across states in terms of representation per population, these disparities still exist due to the wide range of state populations. As of the 2010 Census, Wyoming has the smallest population with 568,300 people, while California has the largest population with 37,341,989 people. Currently, Wyoming has 1 representative while California has 53 representatives in the House. This results in the representative for Wyoming representing only 568,300 people, while the representatives for California represent 704,566 people each. The largest disparity exists between the state of Rhode Island with 527,624 people per representative and Montana with 994,416 people per representative [9]. It is possible that increasing the number of representatives could improve these disparities or another method could be developed.

2 **Redistricting and Gerrymandering**

Once the number of representatives per state is determined by Congress, each state's governor is provided with the number of representatives and the current Census population for their state. From there, it is up to the state to determine the districts for each representative to represent [10]. Many states give the power of redistricting to the state legislature, while others give this power to commissions that are either political or independent [6]. Each state must abide by the criteria set forth by the federal government when creating their districts. The federal government requires all districts to be equal in population and prohibits discrimination by race. States are then free to adopt their own criteria and practices to ensure fairness in creating these districts.

Many states have adopted the some of the same basic criteria for redistricting, focusing on ensuring compactness, contiguity, preservation of counties and other political subdivisions, preservation of communities of interest (i.e. communities that share a common political interest), preservation of cores of prior districts to ensure continuity of representation, and avoid pairing incumbents. Some states have added additional criteria, such as a prohibition on favoring or disfavoring an incumbent, candidate or party and on using partisan data or ensuring competitiveness in a district with equal representation of both major parties or ensuring proportionality, that is matching the make-up of the district with that of the state. However, very few states have adopted all of this criteria and many simply have adopted a few of these principles, while ignoring the rest [8].

Since many states rely on individuals with vested interest in how the districts are drawn, this opens up these districts to potential gerrymandering. Gerrymandering is the deliberate manipulation of district lines to favor one particular group. Gerrymandering is typically achieved in two ways, cracking or packing the votes of a particular group. Cracking involves splitting the members of this group into multiple districts, so that they do not have a majority in any district. Packing involves putting most of the members of this group into one district so that they consist of a minority in all of the rest of the remaining districts. Gerrymandering does not necessarily have to be negative, as these methods can be used to ensure that a district is competitive and gives equal power to each group [1].

Both partisan and racial gerrymandering have been ruled unconstitutional by the Supreme Court. Despite having struck down districts for racial gerrymandering, the Supreme Court has yet to strike down any districts for partisan gerrymandering, as they have yet to find a suitable test for proving it has occurred. One measure that has been used is the efficiency gap, that is a measure of the wasted votes. Wasted votes are votes for the losing candidate or votes above the 50% threshold for the winning candidate, which measures the methods by which gerrymandering is accomplished. Though this measurement does not show intent, as it is possible for gerrymandering to occur accidentally [4]. Looking at the new criteria of proportionality of districts, it is possible that due to the makeup of the state for the efficiency gap to show that gerrymandering has occurred. In a random generation of millions of reasonable district maps for the state of Maryland, it was found that nearly all of these maps favored the Democratic party [4].

Mathematicians and political scientists have proposed a list of criteria that need to be met to be able to claim that intentional partisan gerrymandering has occurred. These criteria are:

1. evidence of partisan bias, likely indicted by the efficiency gap measure
2. indications that the bias has and will endure for the whole decade, measured by looking at census projections
3. the existence of at least one replacement plan that does not show the current bias
4. proof by simulations that the plan is an extreme outlier to prove intentionality
5. evidence of prior knowledge that the map was more biased than necessary [4].

Mathematicians have also proposed another way to avoid intentional gerrymandering in redistricting and that is to use the shortest splitline algorithm to create congressional districts. The shortest splitline algorithm recursively splits the population into districts of equal population by drawing the shortest possible line to create these splits. The algorithm is as follows:

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“ShortestSplitLine( State, N ){
    If N=1 then output entire state as the district;
     A = floor(N/2);
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B = ceiling(N/2);

find shortest splitline resulting in A:B pop ratio;

Use it to split the state into the two HemiStates SA and SB;

ShortestSplitLine(SB, B);

ShortestSplitLine(SA, A);}" [7].

Though this algorithm solves the issues of intentional gerrymandering, it does open up the door to unintentional gerrymandering as it does not take into account the geographical makeup of the state.

3 **Electoral College**

The USA does not directly elect their president and vice president, but instead uses a system known as the electoral college to do so. The electoral college gives each state electors corresponding to the number of house and senate seats that they have. Additionally, the District of Columbia gets three electoral votes, despite not having voting members in Congress. Most states use a winner-take-all system, where whoever gets a plurality of the votes in the state earns all of the electoral votes for that state. Only Maine and Nebraska use a district system, where their two electors corresponding with the Senate seats vote for the candidate with the plurality and the electors corresponding with the House of Representatives seats vote for the candidate who gets a plurality in that district. However, some states do not require the electors to vote for this candidate and it is up to the elector to decide who to vote for [13].

As a result of this system and the mathematical issues of apportionment as previously discussed, the electoral college allows for the candidate who lost the national popular vote to win the presidency. This has occurred four times in US History, in 1876, 1888, 2000, and 2016 [2], meaning there is an error rate of 4/58=7%. Taking into account, the states with the least number of people per electoral college vote, Wyoming being the most represented with 192,579 people per electoral college vote, and the winner-take-all system for most states, it is possible to win the US presidency with only 22% of the popular vote [9].

4 **Conclusion**

Issues in the disparity between states in the number of people per representative for each state compound with gerrymandering and the electoral college to create a variety of issues in terms of the fairness of the government and representation in the US government. Further use of mathematics in exploration of solving these issues and identifying the issues as they occur can only improve the representation and democracy in the US. Mathematics has shown promise in improving many of these areas, particularly in gerrymandering, and could improve issues within apportionment and the electoral college as well.

5 **Works Cited**

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