Unwrapping History with Mathematics

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Math-400

1 Invisible Library

The study of history is greatly limited by the information and materials available from each time period. A contributing factor to these limitations is the wear and tear of time on objects, particularly scrolls and books. There is a large portion of knowledge about history that is hidden by the damage from time. Accessing the information that is hidden in this invisible library could greatly improve the scholarship in history. However, physical and archaeological techniques have proven fruitless in revealing this knowledge. In figure 1.1, you can see the effects of a 1883-4CE attempt at physically unwrapping a scroll found in Herculaneum that was damaged by the Vesuvius eruption in 79CE. Figure 1.1 shows one of the many pieces of the scroll that were created in this process and highlights the damage that can be inflicted in attempting physical unwrapping of these scrolls. Physically unwrapping this scroll did not prove to be an effective tool at studying it and gaining information and it was not until a 1990's breakthrough with infrared light that text on these scrolls was readable [1]. Hence, studies of the invisible library have turned to digital methods of unlocking the invisible library.

Figure 1.1: Herculaneum Scroll, damaged by Vesuvius eruption, c. 79CE, further damaged by physical unwrapping in 1883-4CE. Photo by Henrik Knudsen [1]

2 The Volume Cartographer

Dr. Brent Seales, a professor of computer science at the University of Kentucky, has dedicated the past two decades to improving the study of the invisible library utilizing digital methods. He has pioneered this study and created a software package entitled The Volume Cartographer that can digitally unwrap scrolls with just a volumetric scan of the object.

Additionally, Dr. Seales has improved the field of archaeology with digital methods in many other ways during this research. In 1999, Dr. Seales began working on the digitalization of burned manuscripts at the British Library, creating the only digital copy of the manuscript *Beowulf* [1]. Dr. Seales furthered this work in 2000 by developing a method to digitally flatten the warped pages of these manuscripts to improve readability [5]. Additionally, Dr. Seales began to work on a software to digital unwrap scrolls and achieved a successful unwrapping of an Egyptian prototype scroll in 2003 [5]. Inspired by the success of these endeavours, Dr. Seales began working on using medical CT scans of archaeological objects to get a better understanding of their internal geometry [1]. In 2006, Dr. Seales utilized these methods to scan a 15th century bookbinding and confirm the theory that there was a fragment of Ecclesiastes hidden inside of it without damaging the bookbinding [1][5].

Working with the Institut de France, Dr. Seales and his collaborators created a complete digital rendering of the internal structures of a Herculaneum scroll in 2009 [5], furthering the development of the pipeline to be able to unwrap these scrolls digitally. In 2015, Dr. Seales and his collaborators successfully unwrapped and read a scroll from En-Gedi, that was revealed to be the book of Leviticus and the oldest Hebrew bible found other than the Dead Sea Scrolls [5]. The process used to achieve this result will be further explored in the next section. In 2018, Dr. Seales and his collaborators recreated the physically unwrapped Herculaneum scroll digitally and utilized hyperspectral 2D images to reveal the text hidden in the scroll [5]. In 2019, Dr. Seales and his collaborators utilized machine learning to reveal carbon ink from micro-CT scans, a task previously thought to be impossible [5].

Overall, the goal of Dr. Seales research is to take the techniques of mathematics and computer science and apply them to the field of archaeology. Through this interdisciplinary approach, Dr. Seales has begun the process of uncovering the secrets of history hidden in the digital library. Ultimately, the goal is for archaeologists and historians to be able to use this technology to improve their own research and understanding.

3 Unwrapping the En-Gedi Scroll

In 1970, during an excavation of a synagogue in a large, ancient Jewish community that was destroyed by a 600CE fire, the Holy Ark was found to contain multiple rolls of parchment that were heavily damaged by the fire [6]. Each of the scrolls (e.g. figure 3.1) were quite fragile and thus were preserved without further study by the Israel Antiquities Authority (IAA) until Dr. Brent Seales proposed testing his new virtual unwrapping method on them [6]. The volume cartographer computer

Figure 3.1: an En-Gedi scroll, damaged by fire in 600CE

framework allowed for the scroll to be fully unwrapped and read digitally without need for physically damaging the scroll [6]. Virtual unwrapping is a process that is divided into five parts: volumetric scan, segmentation, texturing, flattening, and merging and visualization [6].

Volumetric Scan

The unwrapping process begins with creating a digital model of the object by taking a volumetric scan. Since each artifact has different limitations due to scale, material, and location, a scan type is not specified for the digital unwrapping framework. Additionally, not specifying a type of scan allows researchers to take advantage of new technologies in volumetric scanning [6]. Some examples of volumetric scanning include micro-CT and the handheld Artec Space Spider 3-D scanner [1]. It is in this step where damage to the artifact can be incurred even if non-invasive methods are used, due to possible required transport and handling in the scanning process [6].

For the En-Gedi scroll, a micro-CT scan was chosen. In order to reveal the text, despite not knowing how the ink will appear, calibration scans were used to create a protocol to create visible intensity variation. Since no text is visible from the exterior of the scroll, the ink used is still unknown, though the scans revealed that the ink is denser than the other material, likely meaning it contains metal of some kind [6].

Segmentation

Once a volumetric scan has been taken, the next step is creating a digital model of the scroll itself with focus on areas where text is presumed to appear. The digital model will be in the form of a mesh, or a collection of 3d points that are combined in faces.

Though the faces can be any polygon, a triangular mesh is the best choice due to the unique representation of a triangle and other helpful geometric properties, such as ray intersection, shape dynamics, texturing, and rendering. Additionally, the mesh can vary in resolution as needed depending on the number of faces used [6].

Segmentation is the process of developing a digital model of the scroll by locating the layers in which the text is presumed to appear. Due to the damage that the scroll unwent, the layers are no longer simply folded or rolled in a predictable pattern, making it difficult to identify the layers and follow along them to create a model. Additionally, the animal skin material of the En-Gedi presents its own challenges in terms of density. All in all there are many locations within the scroll where the distinction of layers are ambiguous [6].

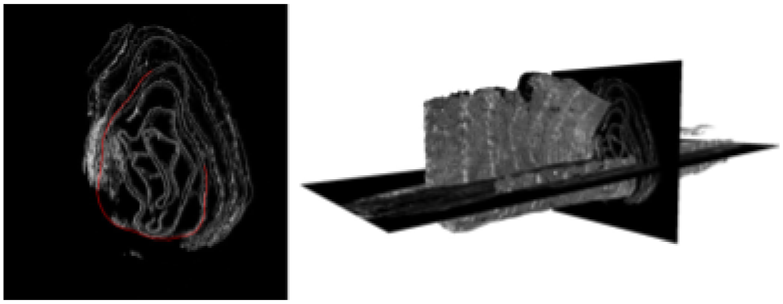
Figure 3.2: for segmentation, the scroll is split into smaller sections to improve accuracy

In order to improve the overall result and decrease the effects of errors on the entire mesh, the scan is split into smaller sections for the segmentation process, one such section is shown in figure 3.2. Segmentation begins with locating a coherent section of the animal skin and using a region growing algorithm to localize the layer of the scroll from there. Local estimates of the differential geometry of the scroll are found using second-order symmetric tensor and associated feature saliency measures and these are used to drive the algorithm. A second-order symmetric tensor is an invariant algebraic object that describes a relationship between objects in a vector space. Once the coherent section is located, the seed points propagate through the scrolls as a chain which traces out a layer of the scroll [6].

In order for this process to be successful, accurate localization is required, as this is the key to revealing the details and text within the scroll. In figure 3.3, the effects of accurate localization are shown, as the accurately localized path more closely follows the division in layers, allowing for more detail to be revealed [6].

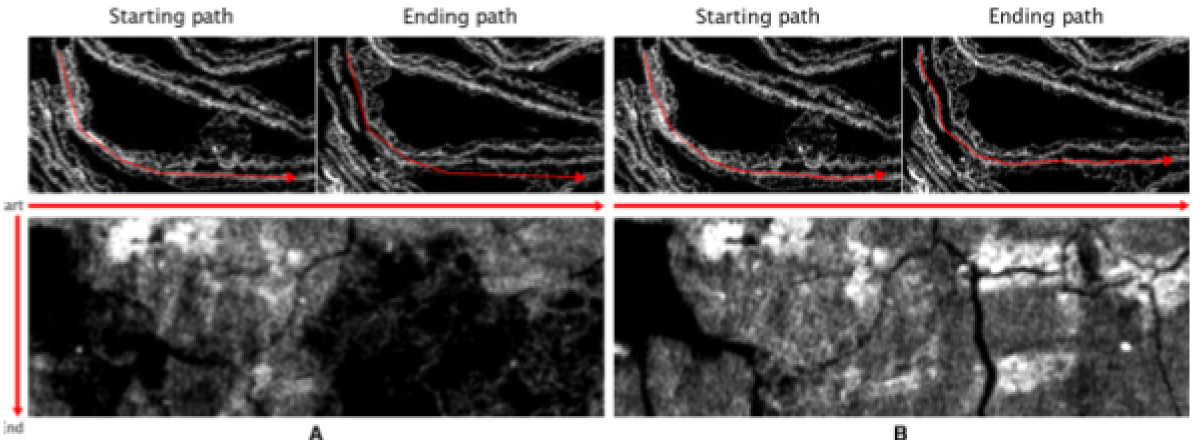

Figure 3.3. (A) Texture generated when the surface is only partially localized. (B) Texture generated when surface is accurately localized.

Texturing

Texturing is the beginning of revealing the text on the digital model, through assigning intensity to each of the points of the mesh. Due to the use of a triangular mesh, this can be achieved by assigning each point intensity based on the intensity information obtained from scanning. From the micro-CT scan, information on density is given and used to determine the intensity of each point, where less dense points appear darker and more dense points appear brighter. In the En-Gedi scroll, the ink was shown to be more dense than the animal skin, so it appears brighter [6].

Ideally, a direct approach to texturing where intensity is assigned based on 3D location in relation to the scan would produce the best texture, but this requires both the scanned volume and localized surface mesh to be perfect. However, as a result of the errors in segmentation, a filtering method to reduce noise is necessary. In figure 3.4, the difference in final textures between the direct approach to texturing and an approach that includes a filter for noise is shown. For the En-Gedi scroll, a neighborhood-based directional filtering method was used. Each texture intensity is determined by both its 3D location and a filter based on parameters, such as the point's surface normal direction, the shape and extent of the local texturing neighborhood, and the type of filter applied to the neighborhood. In two-sided manuscripts such as books, the direction of the points surface is incredibly important to accurately texture [6].

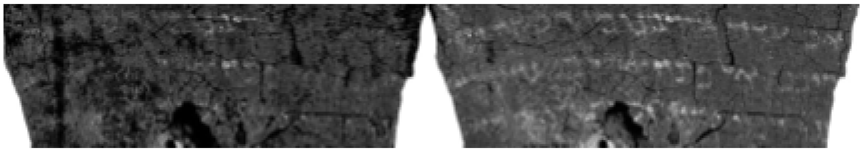

Figure 3.4: a texture generated without directional texturing (left) v.s. a texture generate with directional texturing

Flattening

Flattening is the process of converting the 3D mesh of the scroll into a 2D image to improve readability. Due to the original geometry of the scroll and the process of segmentation, the mesh contains areas of high curvature that make reading the scroll in its original format quite difficult. There are many methods of flattening, including purely geometrical methods such as Angle-Based flattening and Least Least Squares Conformal Mapping, as well as physics based flattening. In figure 3.5, the differences in each

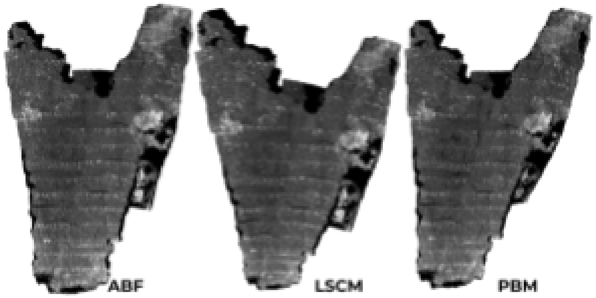

Figure 3.5: A section of the En-Gedi scroll flattened with Angle-Based Flattening (ABF), Least Squares Conformal Mapping (LSCM), and Physics Based Modeling (PBM)

of these flattening methods is shown through their applications to a section of the En-Gedi scroll. As each method intends on preserving different aspects from the 3D mesh in conversion, the geometry of the final flattened images can look rather different. Each method has their own individual strengths and weaknesses that need to be taken into account when choosing which one to use. Despite being highly effective, the purely geometrical methods lack a way to include more specific details on the scroll, so the En-Gedi scroll was flattened using a physics based method [6].

In the physics based method, the mesh is treated as a mass-spring model, where each vertex is a mass and the edges of each face are springs. The mesh is then relaxed into a plane, thereby unwrapping the scroll. Using this method allows for a large number of configurations, as the forces on each point and face can be manipulated, thus allowing the both the geometric properties of the mesh and the physical properties of the scroll to be factored in [6].

Any flattening method distorts the final image, since it is impossible to perfectly convert a 3D object to a 2D image. Despite the scroll initially being a planar object, the damage that it underwent while wrapped up forever changed its shape. Therefore, it is important to study the effects of the flattening and distortion on the digital model, as this could contribute to the readability of the text.

In order to create a visualization of the distortion, distortion maps were generated based on a number of pre-existing distortion metrics to which colors were mapped then applied over the flattened and texture image of the mesh. Each distortion metric measures a different kind of distortion, thus each ones provides new information on what was lost in the transformation from 3D to 2D

The isometric error was described through the per-triangle texture stretch metrics L^2 and L^∞ as defined by Sander et al. L^2 described the average case stretch scenario, while L^∞ described the worst case stretch scenario over the set of faces [4].

$$S_s = \frac{q_1(t_2-t_3) + q_2(t_3-t_1) + q_3(t_1-t_2)}{2A}, \quad S_t = \frac{q_1(s_2-s_3) + q_2(s_3-s_1) + q_3(s_1-s_2)}{2A}$$

$$a = S_s \bullet S_s, \quad b = S_s \bullet S_t, \quad c = S_t \bullet S_t$$

$$L^2(T) = \frac{\sqrt{a+c}}{2}$$

$$L^\infty(T) = \sqrt{\tfrac{1}{2}(a + c + \sqrt{(a-c)^2 + 4b^2})}$$

where $\{q_1, q_2, q_3\}$ was the three dimensional triangular face and $\{p_1(s_1, t_1), p_2(s_2, t_2), p_3(s_3, t_3)\}$ was the flattened face.

Angular distortion was measured using the distortion metric introduced by the ABF flattening method [3]. This metric was described by the relation between the flattened angles and the optimal angle as determined by the angles in the three dimensional space.

$$F(\alpha) = \sum_{i=1}^{T} \sum_{j=1}^{3} w_j^i (\alpha_j^i - \phi_j^i)^2$$

where i iterated through each triangular face in set T, j iterated through each angles in the face, w_j^i was a per-angle weight, and α_j^i and ϕ_j^i were the flattened and optimal angles respectively.

Blender is an open source three dimensional modeling and rendering application that contains metrics for area and angular error [3]. The area error metric was a normalized comparison of the area of the three dimensional triangular face and the flattened triangular face.

$$\alpha = \frac{A(T)}{A(M)}\,,\ \beta = \frac{A(t)}{A(m)}$$

$$E(T) = \{1 - \frac{\beta}{\alpha},\ \mathit{if}\ \alpha \geq \beta$$

$$\{1 - \frac{\alpha}{\beta},\ \mathit{if}\ \beta > \alpha$$

where A(x) was the area function, T and M represented the three dimensional triangular face and mesh, and t and m represented the corresponding flattened triangular face and mesh.

The color mapping was created through comparison of the individual faces to the rest of the faces. For each data point, the z-score was calculated and then was compared to the rest of the data to place it along a color spectrum. The location was then applied to a color along the RGB spectrum. This color mapping method, shown in figure 3.6 produced an understandable scale where red represented high distortion, green represented average distortion.

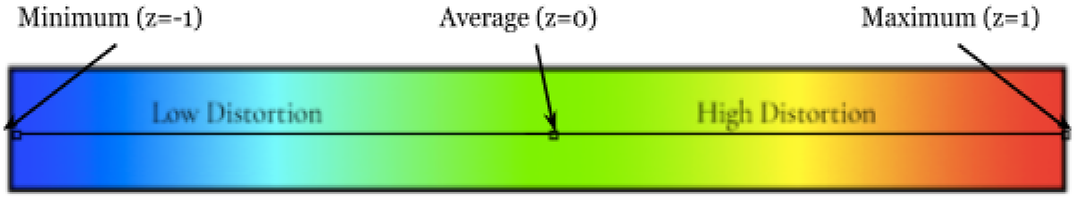

Figure 3.6: The color continuum shows distortion with a minimum z-score of -1 to a maximum z-score of 1, any z-scores less than the minimum or greater than the maximum are assigned the color of the corresponding end point.

Analysis of the distortion produced by each of the flattening methods reveal that they all perform relatively well with only minor differences in distortion [3]. In figure 3.7, the area error distortion maps are shown for each of the flattening methods. Notably, the physics based model shows the stretching in the areas highlighted in green, likely caused by the relaxation into a plane, which is very different from the two purely geometric methods which show more generalized areas of distortion. The best method of flattening may be a combination of methods, in which tracking the physical state of the mesh during LSCM or ABF, allows for physics based modelling to account for the mesh's physical properties [6].

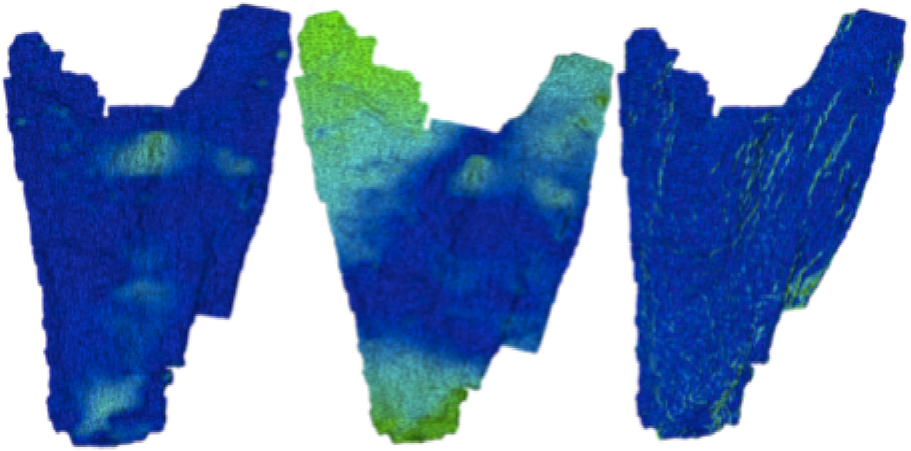

Figure 3.7: Area Error Maps: ABF (left), LSCM (middle), PBM (right)

Merging and Visualization

In order to improve accuracy during the process of unwrapping, the scroll was digitally split into smaller sections which must be merged together for final visualization. In the process of segmentation, the mesh was split into 140 smaller sections, which were then merged into 7 sections for texturing and flattening. A set of all transformations done to the mesh is maintained so that each pixel of the final image can be mapped back to the original voxel, which is a 3D pixel, allowing for analysis and verification of the final image [6]. In figure 3.8, the final unwrapped and textured image of the En-Gedi scroll is shown, in relation to the original wrapped form.

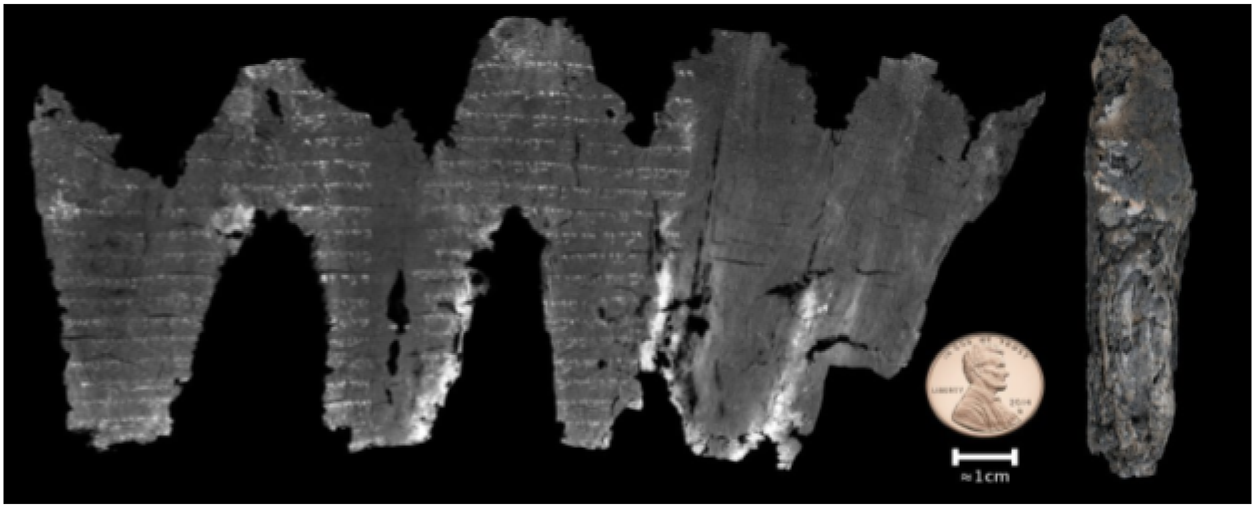

Figure 3.8: the final unwrapped visualization of the En-Gedi scroll

During the process of segmentation, texture merging is used to provide the user with feedback on the overall progress and quality of the segmentation thus far. Texture merging is "the alignment of texture images from small segmentations to generate a composite master view" [6]. However, this method of merging generates quite a lot of distortion, since each section of the mesh is flattened independently, and thus is not suitable for scholarship [6].

In order to produce the final image of the scroll, mesh merging is used. Mesh merging is done after segmentation and it connects each of the smaller pieces together in 3D to match up with the original geometry of the scroll. Since, this is a more time intensive process, it is not suitable for quick user feedback, however as the algorithm is improved, it could become feasible to use for feedback [6].

Additional visualizations that can be made available are the distortion mappings of the flattened mesh and a 3D print of the scroll.

4 Further Research and Applications

The ultimate goal of the Volume Cartographer is a software that is available for researchers to easily and quickly virtually unwrap scrolls, thereby improving the scholarship of these scrolls that are otherwise unreadable. Further research could be done into the use of similar methods for restoring damaged manuscripts or art. Additionally, this software is being improved and tested on a variety of ink and materials.

As mentioned previously, a major breakthrough was the reading of carbon ink via micro-CT scan. Prior to this breakthrough, it was thought that the micro-CT scan hid all information from the carbon ink. However, it was discovered that there is still a signature of the ink that can be revealed through machine learning, the process of which is shown in figure 4.1. This research allows for more text to be read through the methods discussed, including the Herculaneum scrolls, which use a similar ink to carbon ink [2].

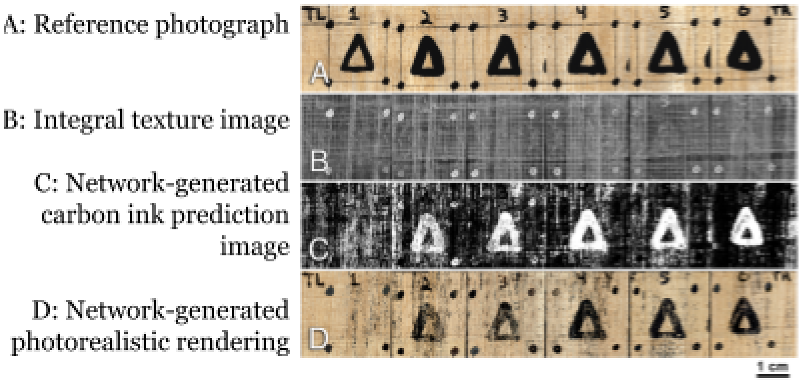

Figure 4.1: Results of the machine learning method to reveal carbon ink

Overall, this research highlights what can be accomplished when mathematical techniques are applied to fields and topics that are not commonly considered related. The use of mathematics in this research has resulted in new knowledge about history and archaeology. Additionally, this research establishes a connection between math, computer science, and archaeology that will only improve the scholarship of all three fields.

5 Works Cited

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