The Golden Ratio: Mathematics in Nature and Art

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The Golden Ratio is a proportion that has come to represent beauty and perfection in mathematics, art, and nature. In this paper I seek to define the Golden Ratio and explore the ratio's history and its connection with the Fibonacci Sequence. Further, I intend to present some techniques for using the Golden Ratio and its derivatives in art and photography. Last, I will reflect on classmate feedback and discussion to introduce areas for further study and improvement for future presentations.

Introduction

The Golden Ratio, denoted by the Greek numeral Phi (ϕ), is an irrational number that represents a proportion that has been lauded as the most perfect and beautiful throughout history. Using a line segment as an example, the Golden Ratio would be attained when, $\frac{a}{b} = \frac{a+b}{a}$ for parts $a > b$ (Pierce).

Figure 1: Golden Ratio Line Segment

Using this line segment representation and the quadratic formula, ϕ can be derived as shown below (Pierce):

$$\frac{a+b}{a} = \frac{a}{b} = \phi$$
$$\frac{a+b}{a} = 1 + \frac{b}{a} = 1 + \frac{1}{\phi}$$
$$1 + \frac{1}{\phi} = \phi$$
$$\phi^2 - \phi - 1 = 0$$

Thus by the quadratic formula:

$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$$

History

The Golden Ratio can be found everywhere if only one decides to look. It is in nature - in the distribution of sunflower seeds, the cochlea of your inner ear, and in the galaxies. It is in art - as a modern day photography technique and in the works of old master artists such as Da Vinci and Michelangelo. And it is in architecture - as seen in the Parthenon and Great Pyramids of Giza (Ilić). To some, this seems like information bias, the more we look for it the more it is found, but to others it is proof that the Golden Ratio is as old as the Earth itself.

One of the oldest and most prominent occurrences of the Golden Ratio is the Great Pyramid of Kufo. The Great Pyramid is part of a collection of three pyramids in Egypt built for the Pharaoh Kufu in 2694 BC. It is the last remaining of the Seven Wonders of the Ancient World and covers and impressive 13 acres. Despite its size and age, the Great Pyramid was built with a staggering amount of precision. The maximum difference in side measurements is only 4.4 cm with the space between individuals blocks no more that 1/50th of an inch (Bartlett 299). This attention

to detail in construction implies a conscious effort and a concentrated plan. Is it possible that the Ancient Egyptians were adhering to the Golden Ratio in their search to make a perfect resting place for Pharaoh Kufo? At first this seems like an easy enough question to answer - just measure and calculate. Due to the age of the Pyramid, however, this is not as simple of a task as it seems. In his article "The Design of the Great Pyramid of Khufu," Christopher Bartlett uses historic data and projection techniques to relate the hypotenuse of triangular cross sections of the Pyramid with base width.

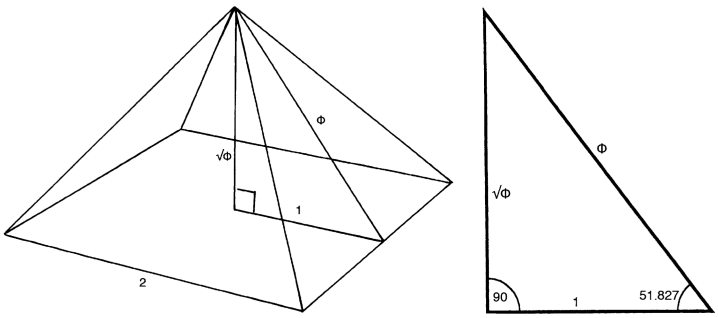

Figure 2: Great Pyramid Cross Section (Bartlett 302)

As an example of his calculations, a side height of 146.515 m and base width of 230.363 m were calculated using data from Cole (1925), Dorner (1981), and Lehner (1997). The hypotenuse was then calculated and the differing lengths compared as shown below:

$$\sqrt{146.515^2 + 115.182^2} = \sqrt{34,733.538^2} = 186.396m$$
$$\frac{186.396}{115.182^2} = 1.618$$

This calculation shows a proportion matching that of the golden ratio to five decimal places (Bartlett 301). There is no real way to know what the Ancient Egyptians were thinking, but the Great Pyramid was certainly built with a precision that implies a greater purpose - whether or not it was mathematically derived.

Today the Golden Ratio is represented by the Greek numeral Phi (ϕ) chosen in honor of one of the great Greek sculptures of the 400s BCE, Phidias (Blacker et al.). Phidias led design and supervised the construction of the Parthenon in Athens, Greece. Now a famous landmark, the Parthenon was originally intended as a temple for Athena the Greek goddess of wisdom ("Parthenon"). In the spacing of the columns and overall height and width of the Parthenon, the Golden Ratio can still be seen.

Figure 3: The Parthenon

(*"Parthenon, Athens" by szeke is licensed under CC BY-NC-SA 2.0*)

Whether or not the Golden Ratio is as old as the Earth itself, its influence can certainly be traced back through history and nature.

Connection with Fibonacci

The Fibonacci sequence, named after the Italian Mathematician Leonardo Fibonacci, is a sequence in which the next term can be defined based on the previous terms. Any starting numbers are acceptable, but the classic example begins with $0, 1$ producing the following sequence:

$$0, 1, 1, 2, 3, 5, 8, 13, 21, \cdots$$

In function notation this would be:

$$f_{n+1} = f_n + f_{n-1} \text{ where } f_0 = 0 \text{ and } f_1 = 1 \text{ for } n \in N$$

When each term of the sequence is divided by the one previous, it produces a new sequence which tends towards ϕ (Pierce). This connection with the Golden Ratio is important for further study of the ratio, especially as an artistic technique.

Techniques in Art

The Golden Ratio is believed by many to be an indicator of beauty and perfection. As such, it can be used as a tool for creating effective compositions in art. Two methods for using the Golden Ratio in art are the Phi Grid and the Golden Spiral.

The Golden Spiral is constructed using the Fibonacci sequence. As this sequence can be used to define a sequence that converges to ϕ, the fibonacci sequence can be used to assemble rectangles that are approximately proportioned according to the Golden Ratio. Further, we can connect squares with side lengths equal to each of the terms of the fibonacci sequence and add arcs to create the Golden Spiral as shown below:

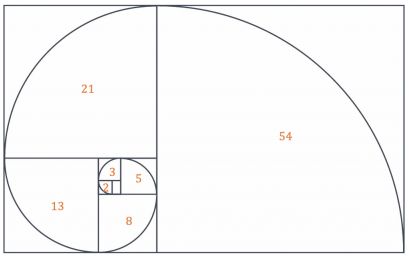

Figure 4: Golden Spiral

When used in art, the most complicated part of the composition is placed in the tightest segment of the Golden Spiral and the rest of the composition roughly lines up with the rectangles

or follows the shape of the spiral. (Grigonis)

The Phi Grid is made of rectangles that follow the Golden Ratio. As opposed to the classic compositional Rule of Thirds, the Phi Grid has a narrow section through the middle intended to focus the viewers eye on the center of the composition.

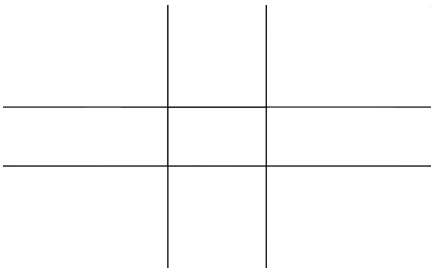

Figure 5: Phi Grid

For examples of the Golden Spiral and Phi Grid in practice, one need not look further than the master artist Leonardo Da Vinci.

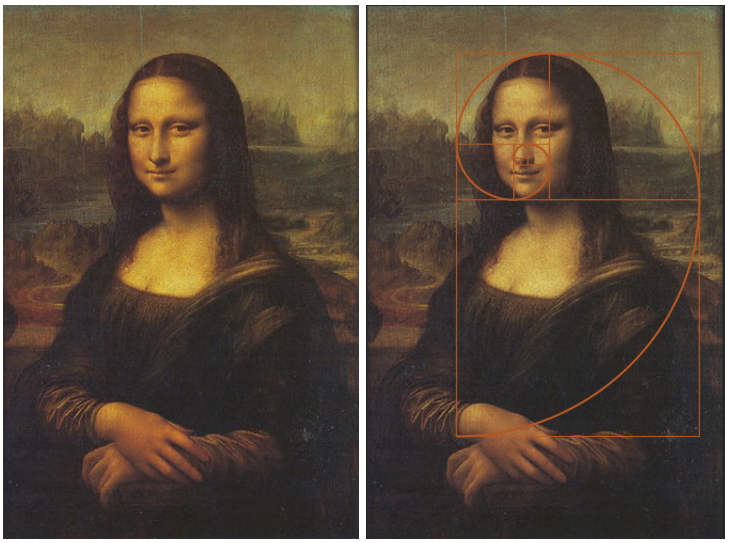

Figure 6: *Mona Lisa* Leonardo Da Vinci (Pickering)

(*"Mona Lisa" by Joaquín Martínez Rosado is licensed under CC BY 2.0*)

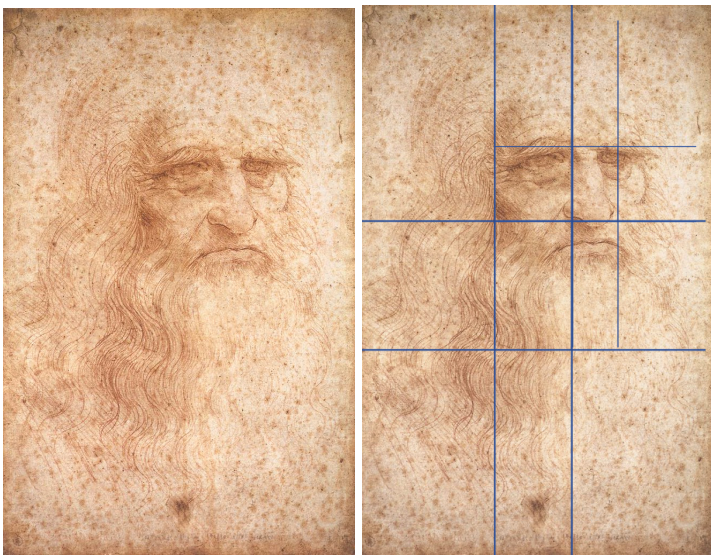

Figure 7: *Self Portrait* Leonardo Da Vinci

(*"Leonardo da Vinci, Self-portrait, c. 1512, red chalk on paper, 333 x 213 mm (Biblioteca Reale, Turin)" by Emme Debi is licensed under CC BY-NC 2.0*)

While this is certainly not exhaustive proof that using Golden Ratio is the best way to ensure an effective composition, these art pieces are good examples of how the idea of the Golden Ratio can be put into practice.

Reflection

Overall I think my presentation went smoothly and I am happy with the feedback I received from the class. One of the most interesting ideas brought up in the discussion board was whether the Golden Ratio's use in art was limited to European artists. The student wanted to know if other cultures had ratios that were seen as the most beautiful and thus used in art and architecture. I presented the Golden Ratio as a universal constant, but then only shared art pieces that I was most familiar with - those belonging to famous European artists. If the Golden Ratio is truly universal it would have to appear all over and so further study of other cultures would

be mandatory. I would also be interested in researching the Golden Ratio's relationship with neuroscience and psychology. What is happening in our brains when we see the Golden Ratio and why is it so pleasing to the eye? Is this socialization or a biological response? I think that the Golden Ratio is so interesting because there is always more to learn and more subjects to be explored using the ratio as a lens for examination.

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