Mathematics in Gomoku

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Outline

- Basic Rules
- Tournament Opening Rules
- Graph Theory and Ramsey game

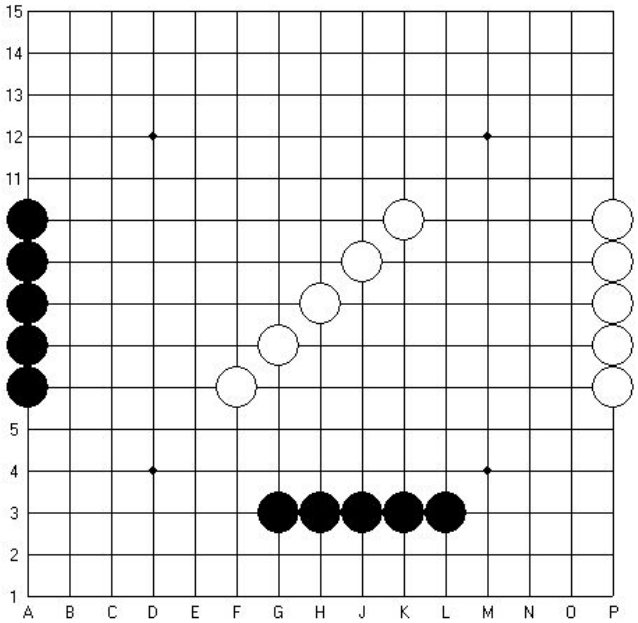
Rules

Players alternate turns placing a stone of their color on an empty intersection. Black plays first. The winner is the first player to form an unbroken chain of five stones horizontally, vertically, or diagonally.

m,n,k-game:

An m,n,k-game is an abstract board game in which two players take turns in placing a stone of their color on an m-by-n board, the winner being the player who first gets k stones of their own color in a row, horizontally, vertically, or diagonally. Thus, tic-tac-toe is the 3,3,3-game and gomoku is the 15,15,5-game. An m,n,k-game is also called a k-in-a-row game on an m-by-n board.

Gomoku (five in a row)



Have a try?

Winning Strategy

The first-player has a winning strategy for any m-n-k game.

We can prove this using the argument of strategy stealing

Proof

The strategy stealing argument assumes that the second player actually has a winning strategy. The first player makes an arbitrary move. After that, the player pretends that she is the second player and adopts the second player's winning strategy. She can do this as long as the strategy doesn't call for placing a stone on the 'arbitrary' position that is already occupied. But this extra stone can only help her, she can again play an arbitrary move and continue as before with the second player's winning strategy. This ensures that the first player wins, contradicting the assumption that the second player has a winning strategy.

Renju

Renju (Japanese: 連珠) is a professional variant of gomoku.

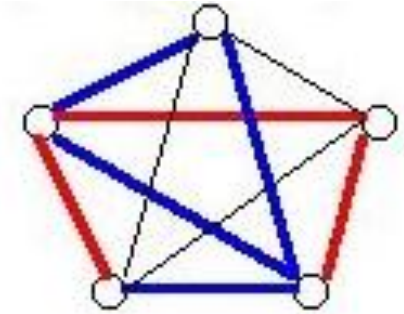
Forbidden moves:

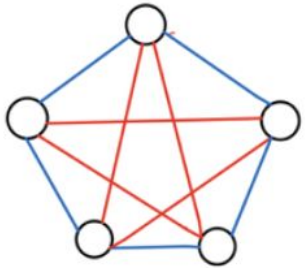
There are certain moves that Black is not allowed to make:

- Double three – Black cannot place a stone that builds two separate lines with three black stones in unbroken rows (i.e. rows not blocked by white stones).
- Double four – Black cannot place a stone that builds two separate lines with four black stones in a row.
- Overline – six or more black stones in a row.

Ramsey Game

Start with a complete graph -- that is, a set of points of all which are connected by lines. Take turns coloring one of the edges. The winner is the first person to color edges which form a triangle.





6-vertices Simple Ramsey Game

Ramsey Theorem **(Frank Plumpton Ramsey)**：Ramsey's theorem states that there exists a least positive integer R(r, s) for which every blue-red edge colouring of the complete graph on R(r, s) vertices contains a blue clique on r vertices or a red clique on s vertices. (Here R(r, s) signifies an integer that depends on both r and s.)

A clique is a subset of vertices such that every two distinct vertices in the clique are adjacent.

There must be a winner since R(3, 3) = 6 and any 2-coloring of K6 yields a triangle. (proved by F.P.Ramsey)

The player who moves first has a winning strategy.

- Simple Ramsey Game
- Tic Tac Toe
- Gomoku
- GO(Weiqi)

References

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Questions?