Math 400: A Mathematical Approach to Walkability in Urban Networks

Grace Lupold

Abstract

Urban spaces are complex networks that often cannot be described solely by traditional Euclidean Geometry. Thus, non-traditional methods such as the urban web can be used to analyze their complexity. An urban web is a network of nodes and connections that describe a city. The nodes are where human activity occurs, they are places like homes, schools, and businesses, and the connections are how people get from place to place. However, there is not just one level of connection, as there exists a hierarchy of ways to get from node to node, with the most abundant and prioritized level being cars. Due to the post-World War II economic boom, American life has become car-dependent, harming the environment, economy, and psychology of residents. This paper will show the importance of urban spaces becoming less car-focused by analyzing the current usage of public transportation and pedestrian travel in American cities. In addition, the paper will look at specific examples of cities worldwide to display spatial modeling techniques, infrastructure changes, and the psychological effects of walkability.

1. The Urban Web

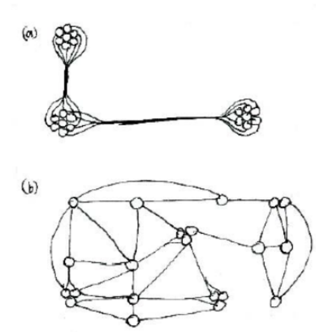
In urban design and development, cities can be understood and analyzed through various lenses. While basic Euclidean geometry is commonly used, considering cities as complex systems proves valuable in describing their irregularities. In my previous paper, I explored how fractal geometry can be applied to urban analysis. However, one can use many other non-traditional analysis methods to describe a city, one being the urban web. The urban web is a network, based on graph theory, that is composed of three components: nodes, connections, and hierarchy. [11]

Nodes, the first component, represent the locations of human activity. They are distinct from architectural sites, as an empty building without human activity will be isolated from the urban web. A node can be anything from a home, place of work, or even a hot dog stand on the side of the street. [11]

The second component of the urban web is the most crucial: the connections, which consist of the pathways between nodes. For a network to function, it needs to be well-connected by an idea known as the theory of multiple connectivity. From a geometrical perspective, only one unique straight line connects two points, but allowing for curved paths creates infinitely many possibilities. Thus, the connections between nodes in the web need to be curved and irregular to create multiple connections and a stable system. If a city is minimally connected by straight lines, one failed connection would collapse the entire network, a concept that is exemplified in Figure 1. [11]

Although its simplicity and order are visually appealing, the top image in Figure 1 lacks stability because of a singular path dependency. In contrast, the image below appears more chaotic but facilitates a stable system with multiple connectivity. In context, the former represents a simplistic, zoned city that lacks alternative routes. The latter displays a well-connected urban space where no single road is overly crowded or important. This argues against block-based cities, where straight lines hinder multiple connectivity unless the block size is small enough or diagonal roads create the necessary linkage. [11]

FIGURE 1 [11]



The hierarchical structure constitutes the final aspect of the urban web, and it reflects the multidimensional nature of the network. Because the way one connects to a city is different depending on what mode of transportation is used (road and pedestrian paths are often different), the urban web must have layers of connections. While all of these paths are vital, they cannot be the same (consider what would happen if pedestrian paths lay on the side of highways). However, they should intersect to accommodate the ability to change transportation methods and get to a destination easily (such as taking a car to a metro and then walking). When developers built cities in America, they prioritized car-centric connectivity, neglecting pedestrian, public transportation, and bike paths, making the entire urban network weak. [3] [11]

2. Networks Built for Cars

The dependency on cars in American life stems from the post-World War II economic boom, characterized by an automobile boom during the suburban sprawl. Upon being created to appeal to the American values of independence and privacy, suburbs were built at the expense of walkability and community amenities. For example, many suburbs lack sidewalks and pedestrian access to restaurants, schools, and grocery stores, which forces residents to rely on cars. However, integrating pedestrian-level connections into suburban spaces could alleviate many of the problems people associate with suburban living, enhancing walkability, cohesion, and community. [3] [5] [13]

In essence, urban planners must prioritize a human scale when designing cities. A human scale is defined as everything people interact with that is meant to be experienced on foot. While it may sound strange to argue that the world is not built for humans, much of the current infrastructure lacks a human scale. Think about the large gaps in between buildings to accommodate multiple lanes of traffic, billboard signs on the sides of highways, and skyscrapers built to benefit a skyline. In each instance, pedestrians are forced to fit into a city that is not built for them, making the urban network fail. [2] [4]

Moreover, car-dependent cities pose significant mobility, safety, and environmental challenges. Thus, urban webs must transition away from car dependency. While walkability is ideal, it is impractical to navigate most urban spaces solely on foot. However, with public transportation, the next tier of the urban web, one can be connected without reliance on individual vehicles. [8]

3. Public Transportation

The US has been known to have challenges with all forms of its public transportation systems. This stems from the suburban sprawl, where urban developers neglected the importance of transportation networks. Instead of building rail infrastructure through cheap undeveloped land first (and then building a city around that rail line afterward), the US waited until suburbs were established to create metro stations and new rail lines. Because they waited, the metro lines today fail to connect enough cities and often still rely on cars to get to the stations. Even where public transportation exists, systems are often expensive and irregular, making it infeasible for day-to-day use. [3] [5]

While some may argue that the failure of public transportation is inevitable because of the structure of American life (the dependence on suburbs, lack of space for rail lines, and size of cities), other countries have succeeded. After WWII, other European countries experienced automobile booms like the US, yet, they prioritized transit. Especially because many of the historic cities could not accommodate a car-centered lifestyle (narrow streets, no space for parking, etc.), city life was never focused on automobiles. In the chart pictured below (Figure 2), one can see the consistently higher numbers of riders on public transportation systems in European cities. Many Europeans can use public transportation not just because it is available, but because it is reliable and affordable, making it practical for day-to-day use. [3] [5] [9]

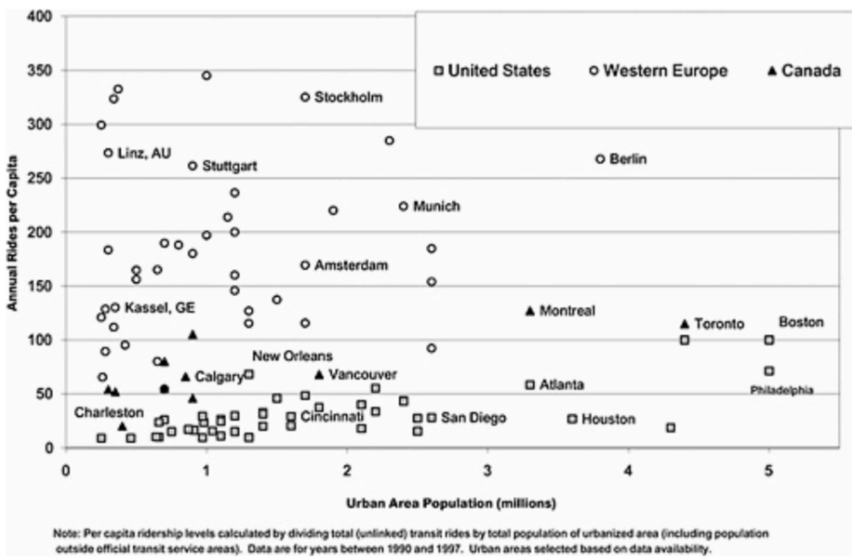


FIGURE 2: Public Transportation Use by City [9]

Even in Canada, where suburban living is common, well-established public transportation systems succeed. Toronto, for example, successfully established a large enough bus network so that nearly all residents, including those in the suburbs, are within a 15-minute walk of a 24-hour bus route. These buses serve as a feeder to bigger metro stations that can quickly connect people to the city center. Surprisingly, many of the suburban stations get more riders a day than stations in New York City. Creating new, cheap, reliable bus routes does not require any changes to the physical infrastructure of a city, and is a great way for the US to begin shifting its focus away from car-centric life. But even with new public transportation systems, an urban web is not successful without an emphasis on the vital component of connectivity: walkability. [3] [9]

4. Walkable Cities

The pedestrian connections between nodes in a city are the most important part of the urban web. In one study conducted in Great Britain, they focused on the indirect effects heavy road traffic had on residents. Participants were surveyed about the busiest road near their homes, focusing on factors such as the number of lanes, the presence of sidewalks, and speed limits. The survey then asked for details about accessibility to important destinations, like grocery stores. It considered factors such as walking distance, whether they have to cross the road to get to it, and whether they choose to walk or drive there. Their findings indicated that destinations on the other side of major roads reduced the probability of a resident making a trip there. This trip suppression was found to reduce physical activity, social interaction, accessibility, and mental health. Beyond the findings of this survey, walkable cities improve air quality, lower the risk for obesity, heart disease, and diabetes, facilitate civic engagement, are cheaper than owning a car, and stimulate economic growth. [1] [13]

Thus, there is no question that urban spaces need to shift away from cars. To achieve this, cities need to prioritize the human scale. There are many ways to make the shift, including turning road areas into pedestrian zones, creating isolated bike and walking paths, removing homogeneous urban regions (or zoned areas), connecting pedestrian paths to public transportation systems, and implementing superblocks. Superblocks were an experiment completed in Barcelona in 2016, where they took a 3x3 section of city blocks, closed off the interior roads to through traffic, and made the area a pedestrian zone. This initiative effectively reduced traffic, air pollution, and noise, all while increasing access to parks and recreational areas at minimal cost. All they had to do was restrict vehicle traffic. [2] [4] [8] [10]

FIGURE 3 [10]

However, the viability of superblocks for other cities hinges on existing infrastructure. A city that supports superblocks is one with high density, active street life, and good transportation. Residents must be able to conduct their daily lives without a need for cars. According to one article, cities such as Mexico City, Paris, and Tokyo are ideal candidates for this system, while Atlanta and London present challenges. See Figure 3 for the visualization of where superblocks could fit in major cities. [10]

5. Measuring Walkability

The next section of my research focused on how walkability can be measured, as it is important to have a standardized system to assess and compare pedestrian-friendliness across different communities. There are two general categories when it comes to quantitatively measuring walkability. The first quantifies it retrospectively, and examples include the Pedestrian Environment Review System (PERS) and Walk Score. PERS focuses on the quality of sidewalks, public transportation, and public spaces and rates an area with a number from -3 to 3 (with 3 being incredibly pedestrian friendly and -3 the opposite), but it does not take into account accessibility to amenities. Conversely, Walk Score is a numerical rating between 0 and 100 based solely on how many amenities are available to pedestrians. It is a points-based system that looks at the distance to each amenity in a given category. If a location is within a 5-minute walking distance (.25 miles), it gets maximum points, and a decay function reduces the points for more distant amenities until something is a 30-minute walk away (in which case no points are awarded). [6] [14] [15]

Upon looking at the walk scores of notable cities in the US, New York, Boston, and Williamsburg have high scores of 88, 83, and 88 respectively, indicating great walkability. However, out of 130 US and Canadian cities with populations exceeding 200,000, the average walk score was only 48. In my hometown of Ellicott City, Maryland, the walk score is only 23 (a low enough score that almost all errands require a car). By using systems like PERS and Walk Score, urban planners, and new residents can work to make cities less car-centered. [14] [16]

The second form of measuring walkability predicts pedestrian traffic, with examples such as Spatial Syntax and Urban Network Analysis (UNA). UNA is the more commonly used, as it is a toolbox that can be installed in ArcGIS to build models of pedestrian travel. To utilize UNA, users input an urban web themselves to analyze, designating nodes as either origins or destinations and specifying all connections present. Each edge and node can also have a weight aspect to it, which represents the number of people that use a building, the capacity of the road, or any other aspects the researcher would like to focus on. UNA then employs one of the five measurement indices to estimate pedestrian flow. [12]

The first index, reach, is defined by the equation $\Sigma_{j\in G-\{i\};d[i,j]\leq r} W[j]$. The r represents the maximum radius allowed, d[i,j] is the shortest path between nodes i and j, and W[j] is the weight of the destination node. This equation sums over all of the destination node weights to find the reach of node i. In simpler terms, the reach counts how many other buildings are reachable based on a given search radius. The next index the user can choose is the gravity index, defined by the equation $\Sigma_{j\in G-\{i\};d[i,j]\leq r} \frac{W[j]}{e^{\beta\cdot d[i,j]}}$, where all variables are defined the same, and beta controls the effect of distance decay. Beta is chosen based off of how strongly the distance between two nodes affects the result. This is a modification of reach that takes into account the spatial impedance needed to reach the destination. The next index, betweenness, is an estimation of how many times a node lies on the shortest path between two other nodes. It is defined by $\Sigma_{j\in G-\{i\};d[i,j]\leq r} \frac{n_{jk}[i]}{n_{jk}} \cdot W[j]$, where n_{jk} is the number of shortest paths from building j to building k, and $n_{jk}[i]$ is the subset of these paths through i. This index helps analyze whether certain locations are more likely to have passersby. The fourth index is closeness, which is the inverse of the total distance from a node to all other reachable nodes. It indicates how close each location is to all surrounding locations. The equation is defined as $\frac{1}{\Sigma_{j\in G-\{i\},\, d[i,j]\leq r}(d[i,j]\cdot W[j])}$. Finally, straightness measures how closely the shortest network resembles a Euclidean distance, or how "straight it is." It is described by the equation $\Sigma_{j\epsilon G-\{i\};d[i,j]\leq r}\frac{\delta[i,j]}{d[i,j]}\cdot W[j]$, where $\delta[i,\ j]$ is the shortest distance (not paying attention to roads) between the two buildings. Based on what a researcher wants to measure, they can choose the appropriate index to model pedestrian flow through an urban web. [12]

In a study conducted in Guangzhou, China, researchers utilized UNA to analyze pedestrian traffic patterns. They chose to designate residential buildings as origins, with the number of residents being the weight, and public spaces as destinations, with either area or visitor count as the weight. For the study, they selected an index called patronage-betweenness to form the model, which is a combination of betweenness and a probability value that measures the probability that a person starting at point i will end at point j. Since the city is small and dense, many residents choose to walk to get around, and thus the results will help researchers understand what pedestrian traffic is like. Their output is displayed in Figures 4 and 5, showcasing paths taken from residential buildings to public activity areas in the first, and to commercial facilities in the second. Analysis of these graphs reveals that high pedestrian flow rates primarily occur on public municipal roads, leading to congestion, especially at intersection points. Additionally, the researchers were able to see which areas had the highest pedestrian traffic to better understand and resolve any pedestrian congestion issues. [15]

FIGURE 4: From Residential to Public Activity [15]

FIGURE 5: From Residential to Commercial

6. WalkUPS

With so much increasing research and emphasis on walkability, cities are now being constructed with pedestrians in mind. Over the past twenty years, the number of Walkable Urban Places (WalkUPs) in the Washington DC area has doubled, rising from 24% to about 48% of all the income property in the area. In the report, they found that there were six main types of WalkUPs, categorized based on their characteristics. In urban environments, there is downtown, downtown adjacent, and urban commercial. Downtown areas, such as downtown DC, are predominantly office space, downtown adjacent communities still have substantial office space but devote about ¼ of the space to residential areas, and urban commercial are locations that focus less on offices, featuring residential and retail spaces. We find the remaining three categories in suburban areas with suburban town centers, strip commercial redevelopments, and greenfield areas. Suburban town centers are focused on residential and retail space and often are centers of a community (like Old Town Alexandria), strip commercial redevelopment has a retail focus often being built around urbanized regional malls, and finally greenfield is the most balanced division consisting of outdoor spaces and parks. The distributions of these categories are seen below, illustrating the diverse ways in which walkability can be incorporated into urban spaces. [7]

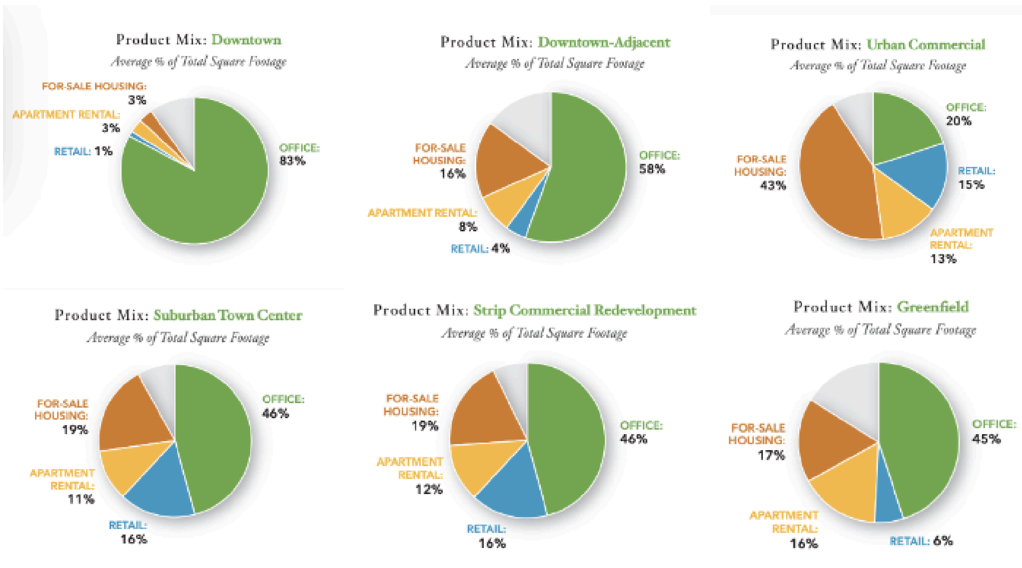


FIGURE 6: Distribution of WalkUP Usage [7]

From this study, they found that Walk Score explains 67% of the increase in economic performance, that about 77% of WalkUPs have access to rail transit, and about 34% of jobs in the DC area are located in these WalkUPs. The study was from a real estate perspective, but it displays how walkable cities can vary in appearance, how they benefit the economy, and their connections to public transportation systems. [7]

7. Conclusion

In conclusion, cities are complex and irregular systems that should be analyzed as such. Utilizing tools like the urban web allows one to understand the utilization and connectivity of different urban spaces. However, the conventional urban development of American cities prioritizes a car-centric infrastructure, neglecting the importance of public transportation and walkability. By implementing cost-effective solutions such as superblocks or expanded bus routes, cities can improve the well-being, safety, and economic performance of their area. Embracing the chaos and complexity of urban spaces as urban webs is thus an essential tool to create any successful city.

8. Reflection

In reflection on my presentation, I was pleased with the response I got. This topic was a continuation of my earlier talk on fractals, and the idea of walkability was recommended to me by a lot of my classmates. I enjoyed the discussion after the presentation about whether walkability is feasible, and I especially think since we live in such a walkable community right now, it is easy to relate to the benefits. If I were to present again, I would spend more time discussing and showing the benefits of walkability to Williamsburg itself, as sometimes the big-picture ideas do not keep the audience as engaged. I also found my presentation was connected to others in the class, especially those who touched on graph theory and discussions of the importance of infinity. I feel like in combination with all the other presentations, this topic offered a new way to understand old math ideas. In terms of future research, this topic could easily lead to an investigation of traffic patterns and how urban planners decide where to build roads and pedestrian paths when building cities. Overall, I had a lot of fun completing this research project and I hope that others in the class enjoyed learning about something new.

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