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Mathematics in Video Games

Abstract

With the rise in modern technology has come the rise of video games as a pastime as gaming systems have continued to advance and grow. With the recent new introduction of Virtual Reality in the gaming industry, this boom looks like it will only continue to grow in the next few decades. And with the current state of the world, video games are looking as appealing as ever. But have you ever wondered what goes on behind the scenes in order to create the games you know and love? Ever wondered just how exactly you are still hard stuck in Bronze in competitive Overwatch or why your shield keeps breaking way too quickly in Breath of the Wild? Stay tuned to find out the answers to your questions as I delve into worlds created entirely by numbers and coding.

Introduction

In the world of casual gaming, there's a lot to appreciate about games. But for those unaccustomed to gaming terminology, it can be a lot to digest without knowing what terms refer to what. Like with any subject, video games all tend to follow similar patterns and use similar terms to describe aspects of their games. Jargon can be hard to follow, so I will try to introduce all necessary terms here so you can refer back to them as needed.

This paper will dive into two specific examples from two different games to illustrate just a few ways math is used in the development and playing of games. Video games have a lot of different categories and genres but as I will be only focusing on two games for this paper, I will only introduce those that are necessary for the games I've chosen. The two games in question are Overwatch and Breath of the Wild. I will explore more about each game individually in their separate sections but briefly here I will discuss their differences in structure and content. Overwatch is a multiplayer shooter game that pits two teams of players against each other for matches. This means that each person on your team and on the opposing team is another player (a person) and not AI. These games are short (usually peaking at around 20 minutes) and you play multiple games generally during your play time if you play for an hour or so. There is a common objective for your team and the other team will fight you in order to keep you from obtaining that objective. Think of it kind of like basketball, or football, just all online and with different rules. Breath of the Wild on the other hand, is a single-player open world style game that pits you against enemies that are AI and perhaps the hardest enemy of all time; mother nature. Basically, it is you the player put out in a large world where you have to survive and explore all by yourself. Both games are rather enjoyable for different reasons (at least I believe) and they function very differently from each other. Overwatch is popular for its competitive

nature, while Breath of the Wild is known for the vastness of the world and the sort of thrill that comes with exploring something new. In this way, the structure and math behind these games varies drastically and I have chosen two small areas to highlight the math in each game and how it is utilized differently.

Overwatch

To start things off, let's take a look at Overwatch. As mentioned above, Overwatch has some analogies to some more traditional sports such as football. To continue this analogy, we can talk about the different characters in Overwatch and their different functions. There are currently 32 different playable characters in the game that each have their own unique set of abilities and they all can be placed into three different categories; support, tank, or DPS. Support characters are the easiest to explain as they are merely the healers of a team; for any damage the opposing team deals on their own team, support players can heal them back up to full health. Tanks are a bit more complex, but they are generally the characters with the most health (and thus the hardest to kill) and they generally have a shield of sorts for their team to shelter behind. DPS, which stands for damage per second, are the characters that deal out the most damage to the other team and are generally the most popular category among players. Like in football, each character fills in a needed spot on the team and certain team compositions work better than others (another area that affects the statistics later on). This game is centered on teamwork and competition and these are the areas I will delve further into.

As with other sports, there needs to be a way to rank teams against each other and unlike with other sports, there also needs to be a matchmaking system that creates these teams as there are no set teams in the Overwatch competitive world. I would like to point out here that this paper is not discussing the world of Pro-Overwatch teams as those teams function on a different system and do in fact play with set team rosters. This paper will instead focus on just the competitive Overwatch game mode. So back to the early question; how do we rank individual players and create evenly-matched teams to play against each other? To put it shortly, Overwatch borrows a well-known ranking system from chess and makes a few tweaks.

Elo Ranking System

The Elo Ranking system is best known for its use in professional chess as it was developed for this use by the Hungarian-American physicist Arpad Elo. While still used in chess today, this system has also been adopted by other video games as well as American football and basketball to rank teams and individual players. Players are assigned a rating that is a numerical value with the higher ranking player expected to win over a player of lower rank. In chess, for example, the ratings are on a scale from 0 to over 2400 with the current highest ranked player sitting at 2882. The expected score will be the difference between the rankings of two players which can also be considered the likelihood of winning for a certain player. This system does not directly measure a players skill but instead infers it based on the starting rank and outcomes of the game and the skill of the opponents played. This system was incredibly useful when first

developed as it simplified the predictions for game outcomes significantly, but now has rather decreased usefulness as computers are able to use much more data inputs to calculate results and such a simplified system has less need. However, it is clearly still used today as competitive Overwatch utilizes the numerical ranking system to determine players skill level.

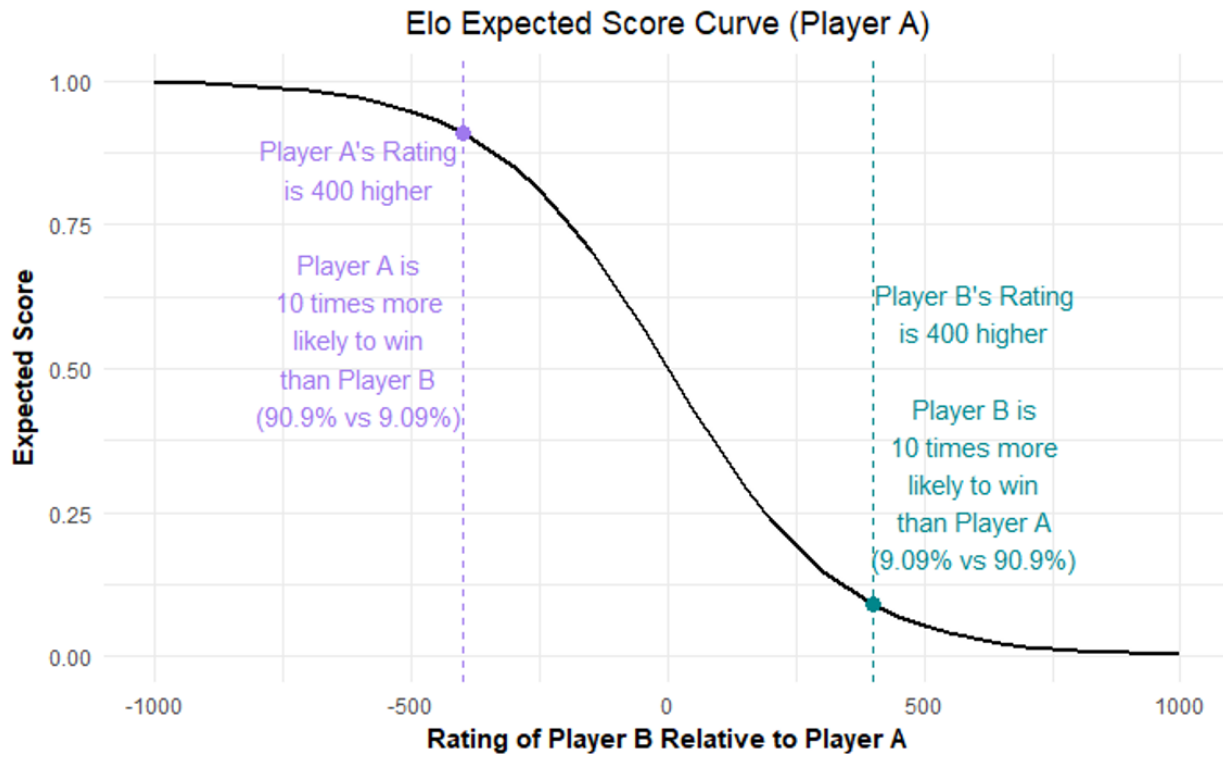
As mentioned above, the expected score is a key aspect in this ranking system. To illustrate this system, let's take a look at the ranks of two players: Player A and Player B. The expected score is calculated by taking the probability of winning added with one-half the probability of drawing or tying. So if player A has ranking R_a and Player B has ranking R_b then Player A's expected score (E_a) can be calculated through the following formula:

$$E_a = \frac{1}{1 + 10^{\frac{R_b - R_a}{400}}}$$

Upon closer examination, this formula looks very close to the Logistic Function as seen below:

$$f(x) = \frac{L}{1 + e^{-k(x - x_0)}}$$

To fit the Elo Expected Score formula to that of the Logistic Function, we first set L = 1 which will bound the probability of winning at 100%. Then, we can set x_0 as R_a which then ensures that when Player B's rating is equal to Player A's ranking, then the expected score will be 50%. Then, we set k = 1/400 and can take out the negative sign before it to flip the graph and change the base from e to 10 without further disruption. Then, we have the Elo Expected Score formula as above and we can now graph it rather easily from what we know about Logistic functions.



In this graph, the rating of Player B relative to Player A sits on the x-axis (R_b - R_a). When this is zero, Player B's ranking is equal to Player A's. On the y-axis, we have the Expected Score on a scale from 0 to 1 as related to percentages (0 = 0%, 1 = 100%). The curve then represents the Elo Expected Score of Player A based on these factors. The closer the player's rankings are to each other, the more this curve looks linear and is even easier to plot.

So how does Overwatch use this calculated Expected Score to rank players? This system is used once a player has an established rank already, but how does a player get a rank in the first place? This is an area that is a little more unclear and I do not have a clear answer here as much of it is hidden from the general public. But once a player's rank is established, the system then uses this rank to establish a team of six similar ranked players which it places against a similarly ranked team by calculating the expected scores for each player and getting as close to 50% as possible. So for Overwatch, first a player plays 10 matches at the beginning of each season of competitive play called Placement matches. After these games, using algorithms that are mostly still secret, a player is ranked from 0 to 5000 with each rank corresponding to a name from bronze to grandmaster. Then the matchmaking algorithm uses the expected score from above to create evenly skilled matches and then the game gets played. Once each game is finished, the ranks of each player are adjusted depending on the outcome (increasing rank for a win, decreasing for a loss, and no change for a draw) and the next match can start! Now this is obviously an oversimplification of a very complex system that has many variables impacting the outcome, but is a first step to understanding just how math can be utilized in a game's ranking system

The Elo Ranking System is a great basis to build a more complex model of ranking from and the developers behind Overwatch have effectively implemented it to organize their competitive gaming scene. The math in this section is really just a tip of the iceberg for all the calculations that go on in a highly complex game such as Overwatch, but it is a useful mental exercise to help being to shed light on the interesting things going on behind the scenes in video games.

Breath of the Wild

While Overwatch seems like the more complex of the two games, at least in understanding the play style and dealing with multiple players at a single time, Breath of the Wild's complexity should not be underestimated. Whereas Overwatch contains itself in small areas or maps that the players can explore, Breath of the Wild manages to create almost an entire world for the character to travel. The sheer size of this game is breathtaking and I would almost suggest buying it for the game design alone. It's a gorgeous and ambitious game that manages to do a lot with honestly very little apparent structure. In creating such a realistic world, the calculations behind every little detail, from how fast an apple falls once a tree is cut, to how much damage the main character Link takes per second when on fire, analyzing all the instances of math in this game is near impossible for anyone who was not a developer. To narrow the scope, I will be focusing on a simple example of calculations in the game that involves shield mechanics.

But first, to briefly introduce the game itself. As mentioned previously, this game is an open world game that has a huge map to explore. Besides an actual plot and storyline that I will not go over, some of the more interesting aspects of this game include a rather unusual feature about the weapons and gear obtained throughout. Starting with nothing but quite literally the shirt on his back, the main character Link must gain weapons and materials from the world around him, opening up a huge variety of weapons to be discovered. In total, there are around 126 different weapons in the game, some rare enough that they aren't encountered until the end of the game. With a limited amount of space to carry these weapons, you would think all these different weapons wouldn't truly matter once you discovered your favorites, but that's where Breath of the Wild throws in a twist. For the sake of realism, the weapons, shields, and bows that Link collects all break down after a certain amount of usage. It's an interesting gaming mechanic that was met with, well, mixed reviews to say it succinctly as it allows for a wide variety of weapons to be used throughout the game, but can also be quite annoying when your best weapon breaks in the middle of a fight. This unique mechanism is actually the place this paper will continue to explore with the next example of some rather simple math for a simple problem; when will my shield break?

Shield Breakage

Well, maybe not so simply. The math is only addition and multiplication, but a table is needed as well as a few different variables. Some factors that I will not be including but are still

important to consider as it adds to the overall complexity of the system are any buffs (bonuses that make Link more powerful) that Link receives from food or his armor as well as any combos or critical hits the enemies weapon does. So lets paint the scene; Link, our character, has a shield and is blocking hits from an enemy that is attacking him. The shield he holds carries the following stats: G = the shields guard stat (how good of a shield it is) and D = the shields durability (how strong it is). The enemy and their weapon's attack stats will be represented by A and P will represent the amount of durability points that are taken away from the shield with each hit. H stands for the number of hits a shield has taken . Then, we have two rules to follow:

1. If $A \leq G + 10$ (weaker enemy), then the shield will break when H = D (when the number of hits to the shield equals to its durability)
2. If $A > G + 10$ (stronger enemy), then we need a table to figure out what P equals and thus how many hits the shield can take before it breaks.
    a. We calculate P = A - G -10 and get the following table:

A-G-10	P
1-5	2
6-10	3
11-15	4
16-20	5
21-25	6
26-30	7
31-35	8
36-40	9
41-45	10
46-50	11
51-55	12
56-60	13

b. Then, the shield will break when $H \geq D/P$

And thus, we know when our shield breaks! But only against one enemy, when in reality, the player will probably face multiple enemies of varied statistics and weapons with one shield. But no fear! We can just use the above to calculate singular hits from different enemies that the shield has taken and keep track of its current health after different encounters. This is all a very complex way to say the harder an enemy hits a shield, the more damage it will take and the sooner it will break, a very logical conclusion. Yet someone had to come up with this algorithm and code it into the game in order for it to take place as one would expect. From this example, we can see just a small taste of the construction that takes place beneath the surface of video game worlds and how numbers act as the backbones of the worlds in unique ways. Breath of the Wild in particular is ripe for deconstruction as it creates a whole world simply from numbers and lines of code taught to run a certain way. The amount of variables that feed into every small decision is mind-blowing and the overall work behind these video games isstaggering. Hopefully through this paper you have gotten just a small taste of what lies beneath these games sitting on your shelf, or better yet, has encouraged you to try them out yourself (thought I will warn you playing them involves a lot less math for better or worse.)

Conclusion

Perhaps one of the most interesting things about video games is the fact that they are entirely fabricated worlds. Everything from the color of an apple on a tree in Breath of the Wild to how fast a certain character's gun fires in Overwatch is all determined by lines of code. This paper has focused mostly on uncovering a few of these hidden lines of code and logic in the game to bring light to the often unappreciated elements of math in these games. Some of these systems hidden in games are not published or advertised and it can take a lot of work to track down exactly how they work and what formulas are used to calculate them. However, even if the casual viewer may not see them, these numbers and math are always there because the developers themselves used formulas to create the worlds in these games. Where this gets interesting is when we try to view the world around us in similar ways. Mathematics is constantly trying to describe how the world around us functions in terms of numbers and equations but we don't actually know if they exist inherently and we are discovering them, or if we are merely creating things to best match what we see. We do not know if there is a developer and code behind all of our world or if we are just fabricating these equations from something that functions not on math but on something else entirely. Video games are fascinating to look at as they attempt to replicate similar worlds to the one we live in using numbers, equations, and coding. It's fun to uncover the math behind the games in a sort of treasure hunt for the end result. What's more fascinating is trying this in the real world where the treasure may not exist at the end and we can only hope to get closer and closer to it as time goes on. And with those final thoughts, I invite you, the reader, to try two things; (1) find math in something unusual around you, and (2) try and figure out why it is there. Good luck!

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