

# THE MATHEMATICAL IMAGE PROCESSING OF MASTERPIECES

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## ARTISTIC APPLICATION OF MATHEMATICAL IMAGE PROCESSING

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### Art Restoration & Conservation

True conservation calls for the preservation of the object to stop material decay and delay further restoration as long as possible.

### Mathematical Image Processing

The digital processing of an image to correct for visual imperfections in the least visually distracting way.

# CAUSES OF DAMAGE TO ART

- Wear & Tear
- Degradation
- Censorship
- Iconoclasm
- Previous conservation attempts

# CAUSES OF DEGRADATION

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Figure 1: Effect of dirty varnish layer

- Paper deterioration can be caused by photomechanical reactions, acidic hydrolysis and oxidation.
- Paint can crack and become brittle due to humidity levels, if no primer underneath, it can seep in between the paper fibers, which allows light to go through and the painting becomes “translucent”.
- Warping in the frame of a painting also makes the handling of it more difficult and dangerous.

# CAUSES OF DAMAGE TO ART

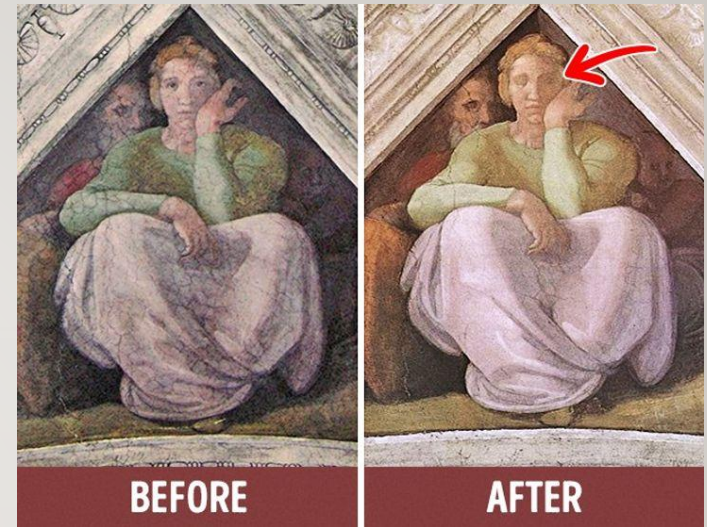
- Wear & Tear
- Degradation over time
- Censorship
- Iconoclasm
  - the action of attacking or assertively rejecting cherished beliefs and institutions or established values and practices.
- Previous conservation attempts

# PAST METHODS OF ART RESTORATION & CONSERVATION

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Same as today

- Methods:
  - Surface dirt removal
  - Varnish removal
  - Consolidation
  - Structural treatments
  - Inpainting
- Potential Problems:
  - Overpainting
  - Unfaithful to origin



# MODERN TECHNIQUES

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The Restoration of  
St. Francis by  
Baumgartner Restoration

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# MULTIDISCIPLINARY MODERN TECHNIQUES

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# BASICS TECHNIQUES

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Main objective of art restoration and conservation is reversibility.

- Digital Recoloration
- Image inpainting – filling in damaged areas in an image defined on a rectangular domain by transferring the information available in the intact areas of the image to the damaged areas in the image.

# EXAMPLE BASED RECOLORATION APPROACH- DIGITAL RESTORATION OF OLD PAINTINGS (ARORA, KUMAR, AND KALRA)

How to match the color space of two paintings to give clean appearance to the old painting using a statistics-based method.

- First, calculate the mean of pixel data along all three axes R, G and B for both the old and sample cleaned painting, denoted as  $(\bar{R}_{old}, \bar{G}_{old}, \bar{B}_{old})$  and  $(\bar{R}_{clean}, \bar{G}_{clean}, \bar{B}_{clean})$  respectively.
- Then calculate the covariance matrices for both the paintings  $Cov_{old}$  and  $Cov_{clean}$ .

$$Cov = \begin{pmatrix} cov(R,R) & cov(R,G) & cov(R,B) \\ cov(R,G) & cov(G,G) & cov(G,B) \\ cov(R,B) & cov(G,B) & cov(B,B) \end{pmatrix}$$

- Now, decompose the covariance matrix using [singular value decomposition](#) to get U and S.

$$Cov = U * S * V^T$$

$$Cov = U * S * V^T$$

- U and V are unitary matrices and are composed of eigenvectors of covariance matrix.
- S is a diagonal matrix of eigenvalues of Cov,  $S = \text{diag}(\lambda^R, \lambda^G, \lambda^B)$ .

#### Rotation matrices

$$R_{old} = U_{old}, R_{clean} = U_{clean}^{-1}$$

#### Translation matrices

$$T_{old} = \begin{pmatrix} 1 & 0 & 0 & \bar{R}_{old} \\ 0 & 1 & 0 & \bar{G}_{old} \\ 0 & 0 & 1 & \bar{B}_{old} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$T_{clean} = \begin{pmatrix} 1 & 0 & 0 & \bar{R}_{clean} \\ 0 & 1 & 0 & \bar{G}_{clean} \\ 0 & 0 & 1 & \bar{B}_{clean} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

#### Scaling matrices

$$S_{old} = \begin{pmatrix} \lambda_{old}^R & 0 & 0 & 0 \\ 0 & \lambda_{old}^G & 0 & 0 \\ 0 & 0 & \lambda_{old}^B & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$


$$S_{clean} = \begin{pmatrix} s_{clean}^R & 0 & 0 & 0 \\ 0 & s_{clean}^G & 0 & 0 \\ 0 & 0 & s_{clean}^B & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where  $s_{clean}^R = 1/\sqrt{\lambda_{clean}^R}$ ,  $s_{clean}^G = 1/\sqrt{\lambda_{clean}^G}$ ,  
 $s_{clean}^B = 1/\sqrt{\lambda_{clean}^B}$

- The final transformation will have the following form:

$$I = T_{clean} \cdot R_{clean} \cdot S_{clean} \cdot S_{old} \cdot R_{old} \cdot T_{old} \cdot I_{old}$$

where  $I = (R, G, B, 1)^T$  and  $I_{tgt} = (R_{tgt}, G_{tgt}, B_{tgt}, 1)^T$



# THE KULLBACK–LEIBLER DIVERGENCE AS A METRIC FOR SIMILARITY

# SIMILARITY METRIC

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- Use KL-divergence values to find out the relative entropy between two color distributions.
- In order to assess if the new sample painting has similar color distribution to the old painting, perform eigen-space transformation followed by KL-divergence calculation.

## DEFINITION OF THE KULLBACK–LEIBLER DIVERGENCE

- The Kullback-Leibler divergence is a non-symmetric measure of the difference between two probability distributions P and Q. KL measures the expected number of extra bits required to code samples from P when using a code optimized Q, rather than using a code based on P. With P being the observed distribution of data; Q approximating P.

$$D_{KL}(P||Q) = \sum_i P(i) \ln \frac{P(i)}{Q(i)}$$

# USING KL DIVERGENCE

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- Recap:
  - Calculate the mean of pixel data along the three axes and compute the covariance matrices between the three components in the color space for both the given old and sample painting.
  - Then,, decompose the covariance matrix using [SVD algorithm](#) and obtain the rotation and scaling matrices.
  - Corresponding eigenvalues and eigenvectors are obtained from the covariance matrix and put diagonally into S.
  - The point clusters of the image is translated to the origin by subtracting the mean obtained along the three reference axes. Eigenvectors are aligned with the reference axis by applying the rotation matrix on all the pixels of the image.
- The eigen-vectors of both the old and sample painting are now, aligned with the standard R, G and B axes. Now, find the KL-divergence values between these transformed distributions.
- Low values of KL-divergence justify the visual similarity of the distributions.

# GENERATING RESULTS

- Generate the results in  $l\alpha\beta$  color space to minimize correlation between channels for many natural scenes. There is little correlation between the axes in  $l\alpha\beta$  space, to apply different operations in different color channels without interacting with the other colors.
- Then, compute the mean and standard deviation for both the old and sample clean painting in  $l\alpha\beta$  space.
- Subtract the mean from the data points:

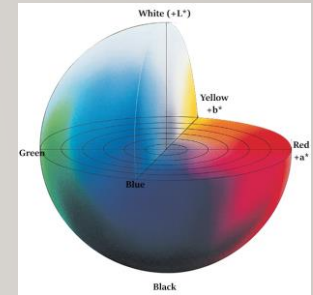
$$\bullet \quad l^* = l - \langle l \rangle, \alpha^* = \alpha - \langle \alpha \rangle, \beta^* = \beta - \langle \beta \rangle$$

- Then, scale the data points comprising the old image by factors determined by the respective standard deviations:

$$\bullet \quad l' = \frac{\sigma_t^l}{\sigma_s^l} l^*, \alpha' = \frac{\sigma_t^\alpha}{\sigma_s^\alpha} \alpha^*, \beta' = \frac{\sigma_t^\beta}{\sigma_s^\beta} \beta^*$$

where  $\sigma_t^l, \sigma_t^\alpha, \sigma_t^\beta$  are the standard deviations for the sample clean painting in  $l, \alpha, \beta$  dimensions respectively, and  $\sigma_s^l, \sigma_s^\alpha, \sigma_s^\beta$  are the standard deviations for the old painting in  $l, \alpha, \beta$  dimensions respectively.

- After this transformation, the resulting data points have standard deviations that conform to the sample clean painting.
- Next, instead of adding the averages that was previously subtracted, add the averages computed for the sample clean painting.
- Finally, convert the result back to RGB color space.

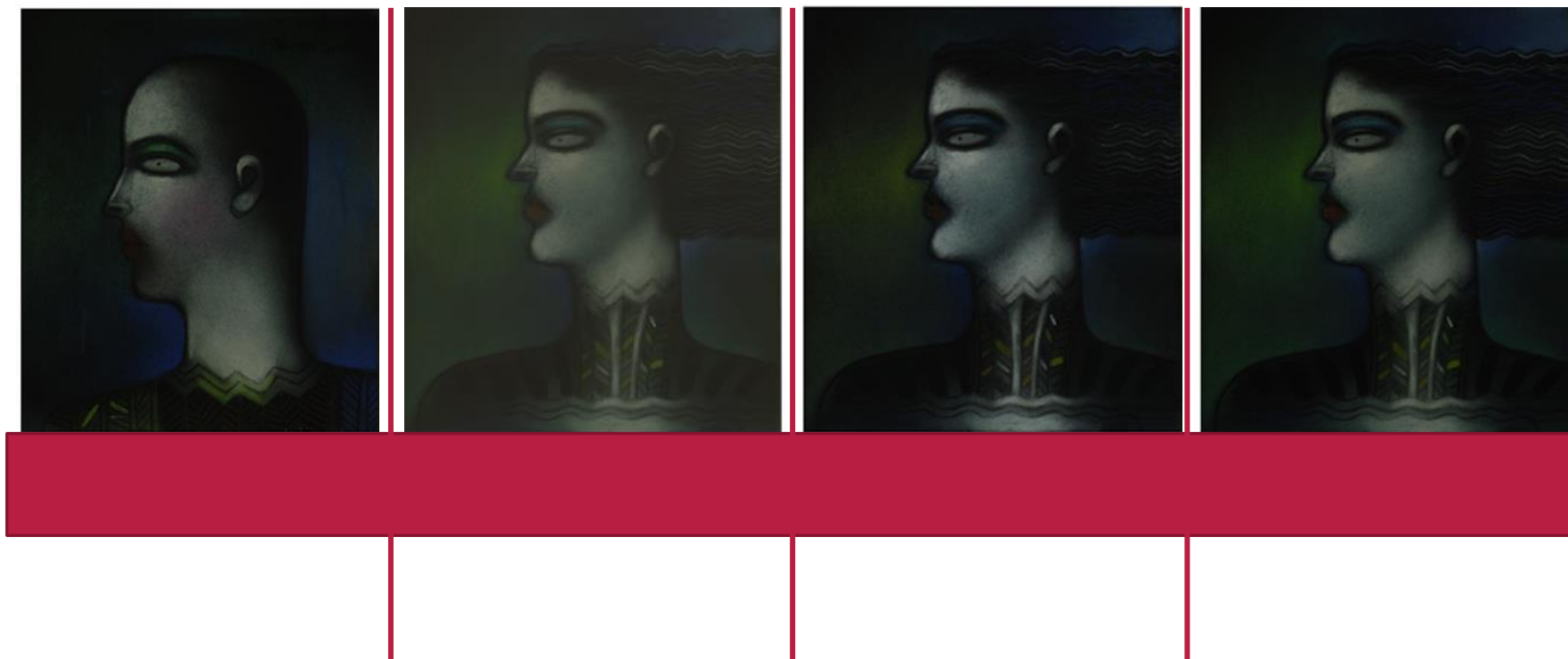




- a) Old
- b) Chemically cleaned
- c) Example
- d) Restored

Channels	Fig4(b)(KL-divergence)	Fig4(d)(KL-divergence)
Red channel	0.43	0.48
Green channel	1.90	1.91
Blue channel	0.37	0.94

Table 6: KL-divergence values for the manually cleaned painting and the restored painting with reference to old painting for Fig 4



# EXAMPLE-BASED RECOLORATION WITH DIFFERENT EXAMPLE PICTURES



(a) Old painting (b) Chemically cleaned painting



(c) Example painting (d) Result  
Figure 5: Result of color restoration

Channels	Fig5(b)(KL-divergence)	Fig5(d)(KL-divergence)
Red channel	0.18	0.38
Green channel	0.15	0.62
Blue channel	0.50	0.81

Table 7: KL-divergence values for the manually cleaned painting and the restored painting with reference to old painting for Fig 5



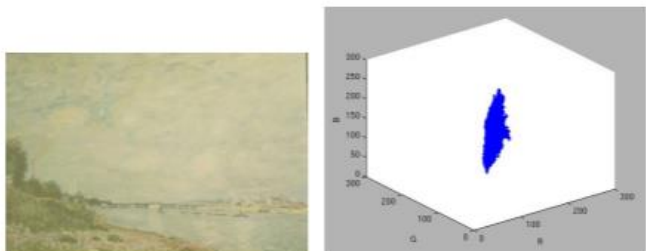
(a) Original painting (b) Chemically cleaned painting



(c) Example painting (d) Restored result  
Figure 7: Result of color restoration

Channels	Fig7(b)(KL-divergence)	Fig7(d)(KL-divergence)
Red channel	0.17	0.08
Green channel	0.24	0.21
Blue channel	0.28	0.13

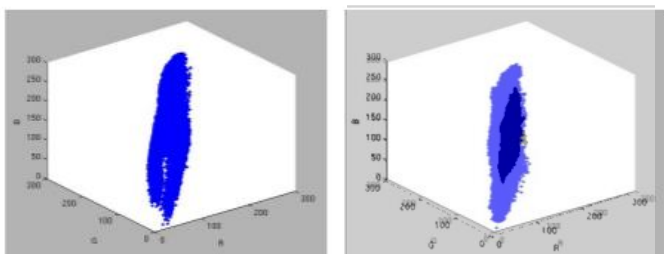
Table 8: KL-divergence values for the manually cleaned and the restored painting with respect to the old painting for Fig 7



(a) Original painting (b) Point cluster of (a)



(c) Example painting1 (d) Result using (c)

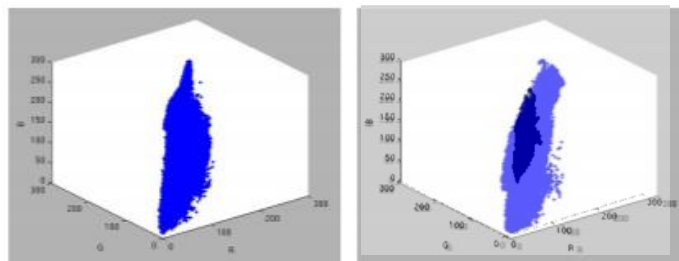


(e) Point cluster of (c) (f) Point cluster of (d)

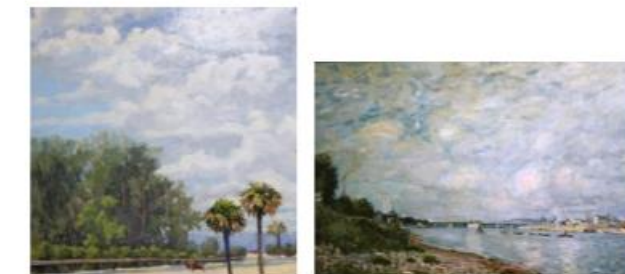
## SAME PAINTING RECOLORED WITH DIFFERENT EXAMPLE PAINTINGS



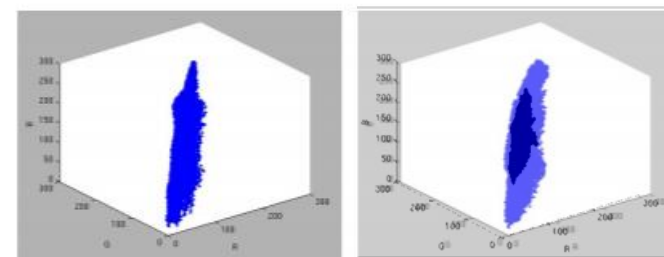
(g) Example painting2 (h) Result using (g)



(i) Point cluster of (g) (j) Point cluster of (h)



(k) Example painting3 (l) Result using (k)



(m) Point cluster of (k) (n) Point cluster of (l)

Channels	Fig3(c)(KL-divergence)	Fig3(g)(KL-divergence)	Fig3(k)(KL-divergence)
Red channel	0.57	0.41	0.31
Green channel	1.43	0.52	0.74
Blue channel	1.51	1.87	0.36

Table 5: KL-divergence values for the old painting and the chosen sample paintings for Fig 3

## SAME PAINTING AFTER BEING CHEMICALLY CLEANED


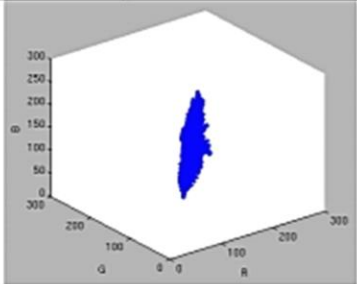

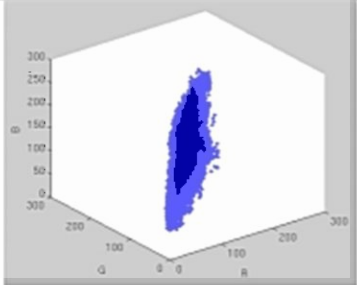
Image	Point cluster	Red Mean	Red Std. Dev.	Red Entropy	Green Mean	Green Std. Dev.	Green Entropy	Blue Mean	Blue Std. Dev.	Blue Entropy
		184.6	20.8	6.032	182.2	19.5	5.825	144	22.7	5.983
		155.9	54.1	7.386	158.2	54.3	7.287	158.0	66.3	7.278

Table 1: Statistics of a pair of degraded and clean paintings

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# OTHER APPLICATIONS OF K-L DIVERGENCE

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AND APPLICATIONS OF PREVIOUSLY MENTIONED CONCEPTS TO ART RESTORATION

## ANOTHER USAGE OF NEURAL NETWORKS: IDENTIFYING BRUSH STROKES & CRACKS

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(a) Old painting



(b) Color restored painting

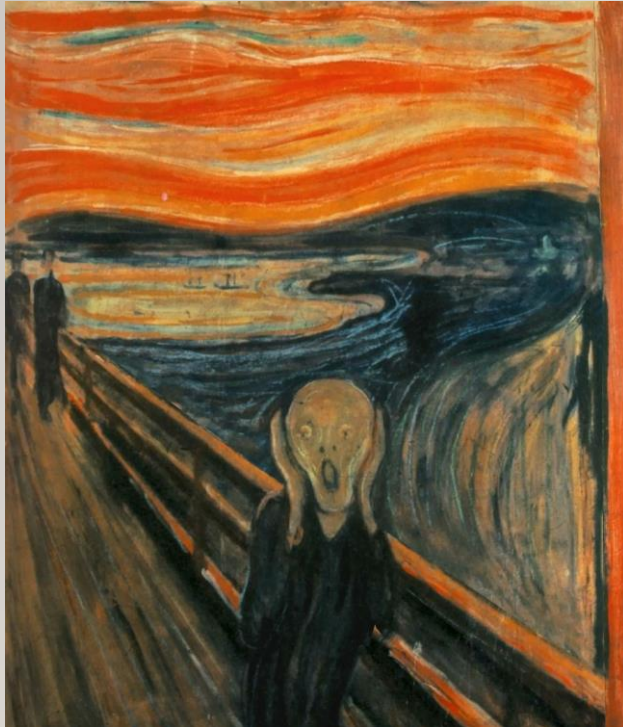


(c) Mask



(d) After crack filling

Figure 8: Integration of color restoration and crack filling



The Scream, by Edvard Munch

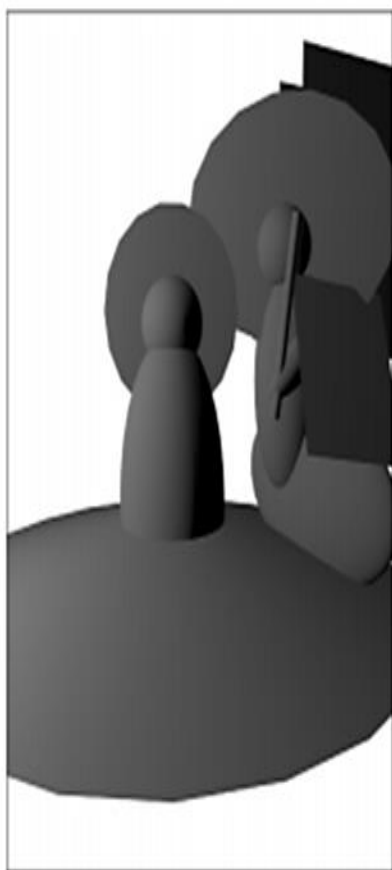


Annunciation, by Simon Bening

## 3D CONSERVATION UNVEILING THE INVISIBLE (CALATRONI,..)

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- Step 1: Generate rough 3d model of scene with plausible parameters.
- Step 2: Generate **masks** for all objects in painting (by hand or segmentation)
- Step 3: Project masks onto 3d model and project original image as texture onto objects.
- Step 4: Add virtual cameras
- Step 5: Inpaint what was hidden from view in original scene.



**a** Rough 3D models approximating the Angel Gabriel and Virgin Mary.



**b** High accuracy segmentation of the Angel Gabriel and Virgin Mary.



**c** 3D models of Gabriel and Mary after “clipping” using the segmentation above.



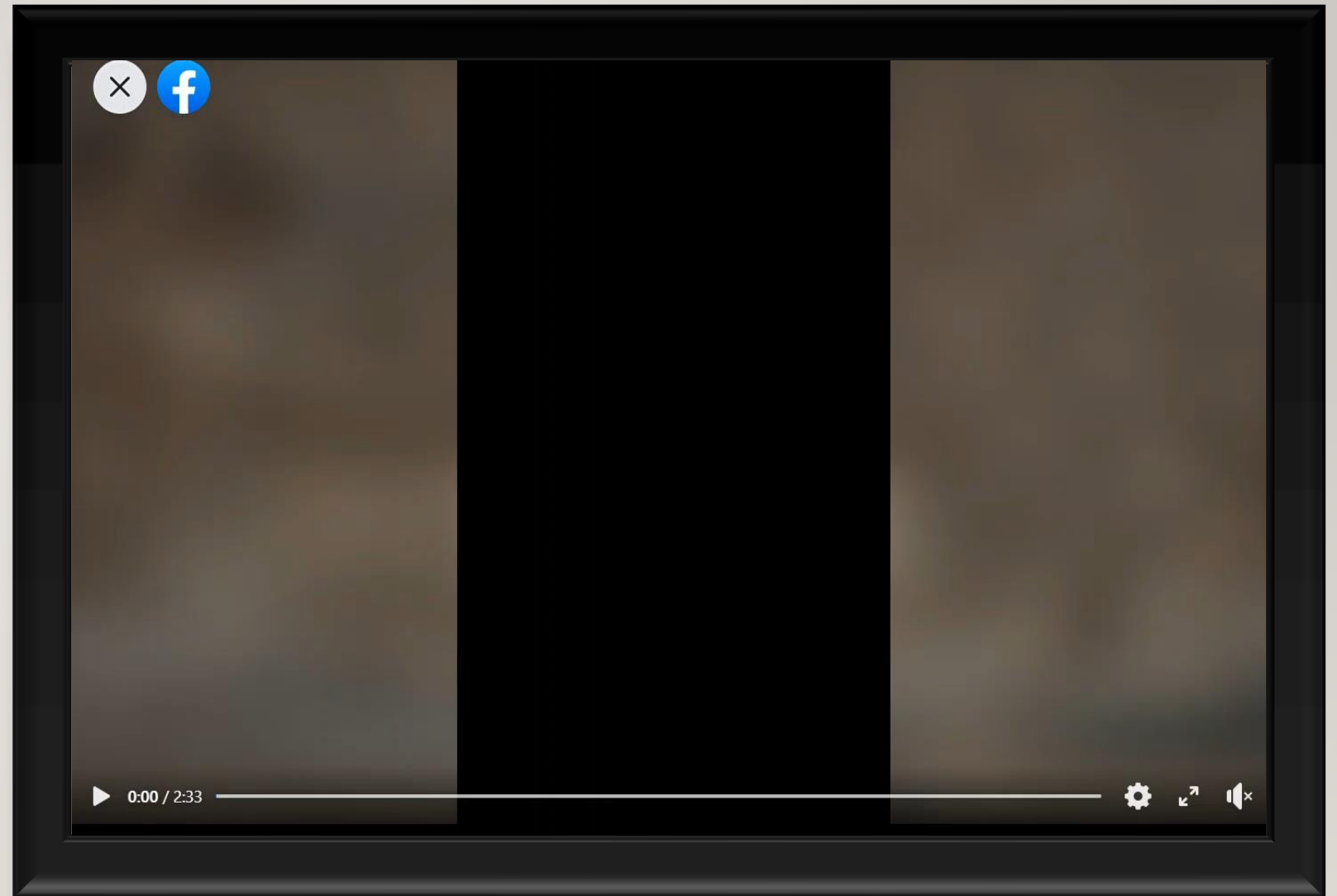
**d** 3D models after clipping and after “clipping” using the segmentation above. projecting the original manuscript onto them.

**Fig. 11** 3D conversion Pipeline. Here we illustrate steps one to three of the 3D conversion pipeline presented in the “[Overview of a 3D conversion pipeline](#)” section. First, in **a**, rough 3D geometry is generated for all objects in the scene (here, only the Virgin Mary and Angel Gabriel are shown). Next, in **b**, accurate masks are generated for all objects (again, only Mary and Gabriel are shown). In **c**, the camera is turned into a projector and the masks from **b** onto the rough 3D geometry from **a**. This projection is then used to “clip” the 3D models by throwing away the portion of the geometry not falling within the projection. Finally, in **d**, the clipped geometry is “painted” by projecting the original image onto it



# THE BRITISH MUSEUM: HUTONG GALLERY'S

“READING IN THE AUTUMN MOUNTAINS”,  
By Xiang Shenmo (項聖謨)



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