It’s Not Magic, It’s Math

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Introduction

The focus of this paper is on the mathematics behind two different card tricks, AweSum 10 and The Final Three. Both of these tricks employ the use of fairly simple mathematics but are nonetheless a fun exercise in connecting mathematics to something seemingly unrelated. In this paper, I will also give a very brief history of card tricks, as well as talk about Persi Diaconis and his connection to mathematics and magic.

Background

It is difficult to say for certain when exactly playing cards first appeared in the world, some historians even date them to 9th century China. However, the consensus is that playing cards became *popular* some time during the mid to late 14th century. We know this because of a German monk, Johannes, who wrote about the cards and the various games played with them. Cards quickly became popular all over the world and as their popularity grew, so did the tricks people were doing with them. Until around the 18th century, card tricks were mostly used by gamblers to cheat, however, Giovanni Giuseppi Pinetti, an Italian magician, changed this. Pinetti used card tricks to impress large crowds of people, including the king of France at the time and he brought card tricks to prominence among magicians. After Pinetti, many more magicians followed and developed card magic into what we know today.

Persi Diaconis, a professor of statistics and mathematics at Stanford University, found his interest in mathematics through magic. Diaconis dropped out of school when he was 14 to travel with Dai Vernon, a professional sleight of hand magician. He traveled with Vernon and performed his own tricks for about a decade before returning to school and earning his PhD in in statistics from Harvard in 1974.

The AweSum 10

This card trick is the simplest of the two that I will be discussing. In this game, a volunteer chooses two cards. The cards must be between an Ace (1) and a 9, additionally, the cards chosen cannot add up to 10 (suit cards have a value of 10 in this game). After the volunteer has chosen the cards, the remaining 50 cards are sorted into piles. Every time a suit card or a 10 is put down, another card is put on top of it. Every time two cards are seen that add up to 10, a card is placed on each of those two cards. After all 50 cards are sorted, the piles that have a 10 or suit card on top are removed. Any two piles that add up to 10 are removed. At this point, there should only remain two piles and by subtracting the value of the card on each of these piles from 10, the value of the two cards picked by the volunteer is revealed. If sorting is done correctly, this trick works every time.

After seeing the game in action, the mathematics behind it becomes obvious. This is essentially a summation game. Since the suit cards have a value of 10, every card can be sorted such that it is equal to 10 in the case of a suit card or a 10, or there exists two cards which sum to 10. This *must* be true for every pile except for two, since the cards that complete these two piles are hidden. Since we know that $a+b=10$, where a is the value of one of our last two piles, it is easy to solve for b, the value of our hidden card.

The Final Three

For this card trick, a volunteer chooses any three cards from the deck. After choosing, the remaining 49 cards are sorted into piles of 10 (pile 1), 15 (piles 2 and 3), and 9 (pile 4). Volunteer takes their first card and puts it on pile 1, cuts pile 2 and puts it on pile 1, places their second card on pile 2, cuts pile 3 and places it on pile 2, then places their third card on pile 3. The piles are then stacked on top of each other in a 4-3-2-1 order, where pile 4 is on top and pile 1 is on the bottom. After this, the cards are dealt out in four rounds. Rounds one, two, and four are dealt out in a face up face down order, while round three is dealt out in a face down face up order. Cards that are face up are discarded after each round. In round four, there will be three cards dealt out that are face down, these will be the three cards selected by the volunteer.

This trick is all about the ordering of the cards. Since the piles are laid out in stack of 10, 15, 15, and 9, and then the selected cards (now referred to as *a,b,c*) are placed on the first three piles, there will always be exactly 15 cards between cards *a* and *b* and *b* and *c*, for the first dealing. These cards will be in positions 10, 26, and 42 in the deck. After dealing the cards in a face up face down order, *a,b,* and *c* are now in positions 6, 14, and 22. Dealt out in the same manner again, they end up in positions 3, 7, and 11. Notice that the cards are now in odd positions in the deck. Due to their odd positions, we must now switch the order in which we deal the cards out to a face down face up order to correct the positions. After this dealing, they are once again in even positions 2, 4, and 6. One more face up face down dealing is done, and cards *a,b,* and *c* will be the final three face down cards.

From this, we can conclude that each dealing is simply a permutation of the original hand of cards. Permutations are seen everywhere in life since it is essentially a way of grouping/arranging a particular set. We see permutations in everything from the Rubik's Cube to quantum physics to this card trick. The Final Three trick also very likely has roots in combinatorics, although I haven't studied combinatorics enough to give an informed interpretation of the trick through that lens.

Discussion

First, I would like to thank everyone for their participation during my presentation and the feedback given to me. In class, there was discussion of extending the Final Three to a Final Four, Final Five, etc. Certainly, there is a way for this to be done. Since the basis of the trick is ordering, there must exist a certain number of piles, cards per pile, and dealing order that could be used for a four card, five, card, six card, etc. trick. I have not been able to find anything of this nature in my research, but with time and a more advanced mathematical background, it is surely possible.

One thing that I found interesting is how many of us presented on topics that had to do with gambling/tricks of some kind. I think that this goes to show how broad reaching mathematics is and how it is used for "entertainment". However, an important thing that we learned through these presentations, is that the house, or the person performing the trick, is always in control and no matter what, they will always "win".

References:

"Dai Vernon." *Wikipedia*, Wikimedia Foundation, 4 Mar. 2022, https://en.wikipedia.org/wiki/Dai_Vernon.

Inc., Vanishing. "History of Card Tricks." *Vanishing Inc.*, Vanishing Inc. Magic, 16 Oct. 2020, https://www.vanishingincmagic.com/learn-card-tricks/history-of-card-tricks/.

Klarreich, Erica. "For Persi Diaconis' next Magic Trick ..." *Quanta Magazine*, 25 Jan. 2019, https://www.quantamagazine.org/persi-diaconis-mixes-math-and-magic-20150414/.

"Persi Diaconis." *Wikipedia*, Wikimedia Foundation, 10 Feb. 2022, https://en.wikipedia.org/wiki/Persi_Diaconis.