CONCEPTS IN QUANTUM COGNITION: MODELING CONCEPTS AS UNITARY HILBERT VECTORS AND CONTEXTS AS THEIR LINEAR OPERATORS

JOSHUA HEATON

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Abstract. The processes of human cognition are of great interest to both psychologists and programmers working on systems of artificial intelligence. Despite extensive research into the topic, many of the specifics of how people think and how the brain functions, on a fundamental level, remain unknown. One of these fundamental functions is how the brain handles the cognition of concepts. The handling of concepts is essential to both higher order and automatic cognition done by the brain. Concepts and their cognition are involved in learning concepts in math, identifying members of categories or groups (both category and group theory are major fields of study themselves), to stereotypes, and just interacting with the world from day to day. Prior research gave simple models for concepts related to the theory of sets, a concept holding members that could be identified with it. Fuzzy set theory was used to identify membership and work with prevalences of items. More recently, models using vectors and linear transformations in Hilbert spaces have allowed for the inclusion of context and concurrent concepts to be modeled as well. This research paper explores the past of these models, new research and possible ways forward.

1. Introduction

The theory of concepts has been of interest to philosophers and psychologists for millennia now. Socrates' theory of ideas was that the world we live in is truly made up of ideas or concepts, things that we perceive around us and how exactly we perceive them. So while we live in a real world, the only things we can really interact with are the mental pictures we have of the world around us. While our cognition of the world around us is so important, psychology was largely a field of philosophy for most of history, and only recent developments have provided empirical data with the addition of science. Now, the fields of computer programming and specifically artificial intelligence have given rise to new interests in human psychology and detailed mathematical models of the mind. One area of interest is that of how the mind handles concepts, specifically how we process a concept and instances of a concept that are perceived in life. One main theory of explanation is Prototype Theory, that concepts are centered around a prototype, a perfect exemplar of the traits central to being a member of a concept. Then other instances are compared to this prototype to give a frame of reference for likelihoods, typicalities, etc. However, the work of prototype theory is incomplete. Programming has made use of vector spaces and linear transformations for computations, and quantum mechanics has begun to provide models for programs themselves and other aspects of cognition.

2. Prototype Theory

The initial theory that dominated early modeling of concepts was Prototype Theory. Appearing in the work of Posner and Keele, 1968 and Rosch, Simpson and Miller, 1976, the theory is based on the idea that a concept is centered around a prototype, some perfect exemplar of the characteristics that define a concept. For instance the prototype for the concept of dog could be a golden retriever or some not-entirely-real average of different dog parts, e.g. the most common color, tail length, snout shape etc.

2.1. **Definitions.** We define a few variables necessary for understanding the formalization of the model (from Osherson and Smith, 1981 [3]) . According to the theory, a concept can be modeled as a quadruple, based on a set of instances of a concept defining the concept itself. We give,
A quadruple $\langle A, d, p, c \rangle$, where
A is a set of readily envisionable objects (real or imagined) called a conceptual domain
d a function from $A \times A$ into the positive real numbers $\mathbb{R}^+$, the distance metric
p is the prototype of A
c is a function from $c : A \to [0, 1]$ called the characteristic function; with conditions

- **1:** $\langle A, d \rangle$ is a metric space, so $\forall x \in A$, $\forall y \in A$
  - **a:** $d(x, y) = 0 \iff x = y$
  - **b:** $d(x, y) = d(y, x)$
  - **c:** $d(x, y) + d(y, z) \geq d(x, z)$
- **2:** $\forall x \in A$, $\forall y \in A$
  $d(x, p) \leq d(y, p) \implies c(y) \leq c(x)$

The first condition simply sets up the metric space necessary for relevant calculations. Then an instance of a concept is defined as a vector x made up of weights to relevant characteristics. The second condition meanwhile establishes the function of the characteristic function. When an instance of a concept is "close" to the prototype of the concept, measured by d, then it is more characteristic of the concept with c returning a higher value. So the distance function measures similarity while the characteristic function measures exemplariness of the concept.

2.2. **Example.** This can be illustrated with the concept bird: $\langle B, d_{bird}, p_{bird}, c_{bird} \rangle$. We have then that B is the set of envisionable birds, e.g. a robin, sparrow, hawk or a penguin. d_{bird} returns as a function of two instances birds, a value of their similarity, say $d_{bird}(robin, sparrow) = 1.5$ while $d_{bird}(hawk, penguin) = 9.8$. Then p_{bird} is constructed or taken as the bird with values closest to all other instances $b \in B$ (usually the average of such values). Lastly, c_{bird} could be calculated by

$$c_{bird}(b) = 1 - \frac{d(b, p_{bird})}{\max_{b_i \in B}(d(b_i, p_{bird}))}$$

among other possible formulations (based on that of Osherson, Smith [3]). And according to the same paper, such modeling matches empirical findings using ratings of typicality and similarity between an instance and a concept or other instance.

3. Fuzzy Set Theory

So the characteristic function measures the level of membership of a certain instance or object to a concept. We can use this as a guideline for comparing and conjoining different conceptual domains. This is very important because as people we usually do not consider just one simple and isolated concept like "bird" or "dog" but rather a "red bird" or especially something like "mathematical psychology" or any relevant connection to concepts across various fields of study. The most relevant feature in the quadruple representing a concept is the characteristic function, or how we evaluate these conceptual combinations [3].

3.1. Defintions.

Since we are measuring inclusion in a set by the graded characteristic function that returns a value in $[0, 1]$, we have entered the realm of fuzzy set theory. Thus we can consider the characteristic function under a few rules.

Given a conceptual domain D and its subsets A and B, $\forall x \in D$

1: $c_{A\cap B}(x) = min(c_A(x), c_B(x))$
2: $c_{A\cup B}(x) = max(c_A(x), c_B(x))$
3: $c_{A^c}(x) = 1 - c_A(x)$

3.2. Example.

We take an example from Osherson and Smith [3]. Consider the domain A of animals with subsets D of dogs and F of females. Then for some animal Rover, or r, we suppose $c_D(r) = .85$ and $c_F(r) = .10$.

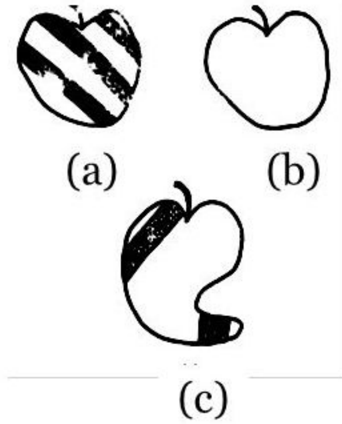
$$\begin{aligned}
c_{D\cap F}(r) &= min(c_D(r), c_F(r)) \\
&= min(1, 0) \\
&= 0 \\
c_{D\cup F}(r) &= max(c_D(r), c_F(r)) \\
&= max(1, 0) \\
&= 1 \\
c_{D^c}(r) &= 1 - c_D(r) \\
&= 1 - .85 \\
&= .15 \\
c_{F^c}(r) &= 1 - c_F(r) \\
&= 1 - .10 \\
&= .90
\end{aligned}$$

So far this system seems to work for the combination of concepts, but what about certain cases.

4. Gaps in the Theory

So what might be wrong with this model. Ultimately the model breaks down with greater development of conceptual combination. We demonstrate the problem with another example from Osherson and Smith [3]. Let F be the conceptual domain of fruit, consider the subsets

A of apples, S of striped fruit, and $S \cap A$ of striped apples. To really consider this we truly only need the intersection, being both striped and an apple.



Taking the depicted apples in the figure, we see consider the apple a. We can find $c_{S \cap A}(a) = min(c_S(a), c_A(a))$. However, the strange stripes on the apple clearly means that $c_A(a)$ is quite low, certainly $c_A(a) < c_{S \cap A}(a)$. But this contradicts the calculation of $c_{S \cap A}(a) = min(c_S(a), c_A(a)) \leq c_A(a)$ [3]. So the object in question is more typical of the conjunction "striped apple" than of either constituent "striped" or "apple". This is exactly where this theory falls apart. Specifically the fuzzy set theoretical modeling of prototype theory seems to be inadequate for describing the complexity of combined concepts.

5. Hilbert Space Approach

Given the rise of quantum methods in artificial intelligence and in describing other aspects of human cognition, it has been put forward that such methods may be able to describe the cognition more accurately and completely than the combination of fuzzy set theory with prototype theory. For this model we consider states of a concept instead of instances, which collapse into other states under the influence of a context, with regards to the weights of varying salient properties as described by Aerts and Gabora [1] [2].

5.1. **Simple Definitions.** For this model we will be concerned with: a concept S, its set of states Σ and individual states $p, q, r \in \Sigma$, a set of contexts M or M^S with specific contexts $e, f, g \in M$, and a set of properties $\mathscr{L}$ with $a, b, c \in \mathscr{L}$ [1].

5.2. **Hilbert Space Definition.** A Hilbert space $\mathcal{H}$ is a complete inner-product space. That is a metric space with an inner-product which is complete with regards to the norm induced by the inner product. The inner product is a map $\langle\cdot|\cdot\rangle : \mathcal{H} \times \mathcal{H} \to \mathbb{C}$ such that

1: $\langle \lambda x + y | z \rangle = \lambda \langle x|z \rangle + \langle y|z \rangle$
2: $\langle y|x \rangle = \overline{\langle x|y \rangle}$
3: $\langle x|x \rangle \geq 0$ i.e. $\langle x|x \rangle \in \mathbb{R}_+$
4: $\langle x|x \rangle = 0 \iff x = 0$

We see that one of the simplest Hilbert spaces is indeed the vector space $\mathbb{C}^n$, equipped with the normal inner product $\langle x|y \rangle = \sum x_i \overline{y}_i$. So now we have the necessary formulation of the mathematical space in which we will operate.

5.3. **Quantum Understanding.** We consider a ground state of a concept $\hat{p}$, with respective states, contexts, and properties
$p_1, p_2, \ldots, p_n \in \Sigma$
$e_1, e_2, \ldots, e_m \in M$
$a_1, a_2, \ldots, a_k \in \mathscr{L}$
Then we give two functions, μ to calculate the probability of a state p becoming q under context e, and v giving the weight of a property to a particular state.
$\mu : \Sigma \times M \times \Sigma \to [0, 1]$ given by $\mu(q, e, p)$
$v : \Sigma \times \mathscr{L} \to [0, 1]$ given by $v(p, a)$ [1]
Now we consider a state $p \in \Sigma$. According to quantum mechanics this state can either be a pure state or a density or mixed state. If the state is pure, we represent p by a unit vector $|x_p\rangle \in \mathbb{C}^n$. If the state is mixed, p is represented by a self-adjoint linear operator ρ_p on $\mathbb{C}^n$, $\rho_p = \rho_p^*$ the conjugate transpose, with $Tr(\rho_p) = 1$.
A concept and each of its states also has relevant properties and weights that determine how important that property is to ratings of typicality. To model these properties, say a, we use orthogonal projection operators, that is a self-adjoint linear operator P_a such that $P_a^2 = P_a$. From quantum mechanics we have a way of calculating the weight of each property. That is for property $a \in \mathscr{L}$: if p a pure state:

$$v(p, a) = \langle x_p | P_a | x_p \rangle$$

And if p a density state, then:

$$v(p, a) = Tr(\rho_p P_a)$$

Finally, a context is generally a measurement in quantum mechanics, another self-adjoint linear operator on $\mathbb{C}^n$. For the purposes of this model, we break up the linear operator into the spectral decomposition of orthogonal projection operators, that is we consider the most basic forms of contexts. So then for conceptual states $p, q \in \Sigma$ and context $e \in M$ we define the ways in which context e changes concept S from state p to state q. Again we need to consider the two possible characterizations of p. If p a pure state, represented by the unit vector $|x_p\rangle \in \mathbb{C}^n$, then:

$$|x_q\rangle = \frac{P_e |x_p\rangle}{\sqrt{\langle x_p | P_e | x_p \rangle}}$$

such that the probability that the given change occurs is given by:

$$\mu(q, e, p) = \langle x_p | P_e | x_p \rangle$$

.

If p is a density state, represented by the orthogonal projection operator ρ_p, then:

$$\rho_q = \frac{P_e \rho_p P_e}{Tr(\rho_p P_e)}$$

such that the probability the change takes place is given by:

$$\mu(q, e, p) = Tr(\rho_p P_e)$$

5.4. **Describing the Vectors.** Finally we give a method for calculation. We know that any vector space can be described by a set of vectors $B = \{|u\rangle : |u\rangle \in \mathbb{C}^n\}$ that is an orthonormal basis for the space. That is $\forall |u\rangle in B$,

1: $\langle u|u\rangle = 1$
2: $\langle u|w\rangle = 0 \iff |u\rangle \neq |w\rangle$
3: $\exists! \alpha_1, \ldots, \alpha_n \in \mathbb{C}$ such that $|u\rangle = \sum_{|u\rangle \in B} \alpha_u |u\rangle$

Furthermore we use the orthogonality of the basis to establish that $\langle u|x\rangle = \langle u| \sum_{|w\rangle \in B} a_w |w\rangle = \sum_{|w\rangle \in B} \alpha_w \langle u|w\rangle = \alpha_u$. So,

$$|x\rangle = \sum_{|u\rangle \in B} |u\rangle\langle u|x\rangle$$

Following that

$$\sum_{|u\rangle \in B} |u\rangle\langle u| = 1$$

.

Taking the orthogonal projector P_u that projects onto the unit vector $|u\rangle$ by $P_u|x\rangle = \alpha_u|u\rangle$, we see that $P_u = |u\rangle\langle u|$. Thus,

$$\langle x|P_u|x\rangle = \langle x|u\rangle\langle u|x\rangle = \alpha_u \overline{\alpha}_u = |\alpha_u|^2$$

This presents a use for the coefficients in the linear combination or superposition of the vector $|x\rangle$ in an orthonormal basis B. Indeed, considering the orthogonal projection of a density operator ρ_p onto the unit vector $|x_p\rangle$, we can see that $Tr(\rho_p P_u) = |\alpha_u|^2$ [2]. These results, the coefficients in such a superposition, are the square root of the probability that a given state changes under influence of a context u.

6. Hilbert Space Results

In order to establish empirical results from the model, we need some more information about how to construct these vectors and linear operators and especially on how to calculate the weights and frequencies of specific states and exemplars of a given context. We use an ordering device to determine the strength of a given context. Indeed, we say that a context e is stronger than a context f if the eigenstates of f are contained in the set of eigenstates of e [2]. In other words, the strength is measured by the number of contexts which activate over the same conceptual states. It is hypothesized in the model that the frequency estimates reflect the presence of contexts that are stronger than those explicitly stated and that the distribution of such contexts reflects the frequencies measured in the experiment [2]. These frequencies as collected by the study are given in the table below.

exemplar	e_1		e_2		e_3		e_4		e_5		e_6		1	
	rate	freq	rate	freq	rate	freq	rate	freq	rate	freq	rate	freq	rate	freq
rabbit	0.07	0.04	2.52	0.07	4.58	0.15	1.77	0.05	0.15	0.01	0.10	0.00	4.23	0.07
cat	3.96	0.25	4.80	0.13	6.27	0.22	0.94	0.03	0.46	0.03	0.15	0.01	6.51	0.12
mouse	0.74	0.03	2.27	0.06	2.67	0.08	3.31	0.11	0.12	0.01	0.10	0.00	2.59	0.05
bird	0.42	0.02	3.06	0.08	0.63	0.02	1.41	0.04	2.21	0.17	0.15	0.01	4.21	0.08
parrot	0.53	0.02	5.80	0.16	0.44	0.01	1.57	0.04	6.72	0.63	0.16	0.01	4.20	0.07
goldfish	0.12	0.01	0.69	0.02	0.09	0.00	0.83	0.02	0.10	0.00	6.84	0.48	5.41	0.10
hamster	0.85	0.04	2.72	0.07	2.06	0.06	1.25	0.04	0.14	0.01	0.09	0.00	4.25	0.07
canary	0.26	0.01	2.73	0.07	0.23	0.01	0.86	0.02	1.08	0.07	0.14	0.01	4.79	0.08
guppy	0.14	0.01	0.68	0.02	0.09	0.00	0.83	0.02	0.10	0.00	6.64	0.46	5.16	0.09
snake	0.57	0.02	0.98	0.02	0.36	0.01	5.64	0.22	0.09	0.00	0.15	0.01	1.60	0.03
spider	0.26	0.01	0.40	0.01	1.05	0.03	5.96	0.23	0.09	0.00	0.09	0.00	1.22	0.02
dog	6.81	0.50	6.78	0.19	6.85	0.24	0.91	0.03	1.02	0.06	0.11	0.00	6.65	0.12
hedgehog	0.53	0.02	0.85	0.02	2.59	0.08	3.48	0.12	0.11	0.00	0.09	0.00	1.56	0.03
guinea pig	0.58	0.03	2.63	0.07	2.79	0.09	1.31	0.04	0.15	0.01	0.09	0.00	3.90	0.07

Figure 1 [1]

Such contexts that are stronger than those explicitly stated in the study are counted and used for calculation in the next table.

exemplar	e_1	e_2	e_3	e_4	e_5	e_6	1
	$n(E_1) = 303$	$n(E_2) = 495$	$n(E_3) = 500$	$n(E_4) = 101$	$n(E_5) = 200$	$n(E_6) = 100$	n = 1400
rabbit	$n_{13,1} = 12$	$n_{13,2} = 35$	$n_{13,3} = 75$	$n_{13,4} = 5$	$n_{13,5} = 2$	$n_{13,6} = 0$	$n(E_{13}) = 98$
cat	$n_{14,1} = 75$	$n_{14,2} = 65$	$n_{14,3} = 110$	$n_{14,4} = 3$	$n_{14,5} = 6$	$n_{14,6} = 1$	$n(E_{14}) = 168$
mouse	$n_{15,1} = 9$	$n_{15,2} = 30$	$n_{15,3} = 40$	$n_{15,4} = 11$	$n_{15,5} = 2$	$n_{15,6} = 0$	$n(E_{15}) = 70$
bird	$n_{16,1} = 6$	$n_{16,2} = 40$	$n_{16,3} = 10$	$n_{16,4} = 4$	$n_{16,5} = 34$	$n_{16,6} = 1$	$n(E_{16}) = 112$
parrot	$n_{17,1} = 6$	$n_{17,2} = 80$	$n_{17,3} = 5$	$n_{17,4} = 4$	$n_{17,5} = 126$	$n_{17,6} = 1$	$n(E_{17}) = 98$
goldfish	$n_{18,1} = 3$	$n_{18,2} = 10$	$n_{18,3} = 0$	$n_{18,4} = 2$	$n_{18,5} = 0$	$n_{18,6} = 48$	$n(E_{18}) = 140$
hamster	$n_{19,1} = 12$	$n_{19,2} = 35$	$n_{19,3} = 30$	$n_{19,4} = 4$	$n_{19,5} = 2$	$n_{19,6} = 0$	$n(E_{19}) = 98$
canary	$n_{20,1} = 3$	$n_{20,2} = 35$	$n_{20,3} = 5$	$n_{20,4} = 2$	$n_{20,5} = 14$	$n_{20,6} = 1$	$n(E_{20}) = 112$
guppy	$n_{21,1} = 3$	$n_{21,2} = 10$	$n_{21,3} = 0$	$n_{21,4} = 2$	$n_{21,5} = 0$	$n_{21,6} = 46$	$n(E_{21}) = 126$
snake	$n_{22,1} = 6$	$n_{22,2} = 10$	$n_{22,3} = 5$	$n_{22,4} = 22$	$n_{22,5} = 0$	$n_{22,6} = 1$	$n(E_{22}) = 42$
spider	$n_{23,1} = 3$	$n_{23,2} = 5$	$n_{23,3} = 15$	$n_{23,4} = 23$	$n_{23,5} = 0$	$n_{23,6} = 0$	$n(E_{23}) = 28$
dog	$n_{24,1} = 150$	$n_{24,2} = 95$	$n_{24,3} = 120$	$n_{24,4} = 3$	$n_{24,5} = 12$	$n_{24,6} = 1$	$n(E_{24}) = 168$
hedgehog	$n_{25,1} = 6$	$n_{25,2} = 10$	$n_{25,3} = 40$	$n_{25,4} = 12$	$n_{25,5} = 0$	$n_{25,6} = 0$	$n(E_{25}) = 42$
guinea pig	$n_{26,1} = 9$	$n_{26,2} = 35$	$n_{26,3} = 45$	$n_{26,4} = 4$	$n_{26,5} = 2$	$n_{26,6} = 0$	$n(E_{26}) = 98$

Figure 2 [2]

6.1. **Construction of Conceptual State Vectors in the Space.** So then how do we construct this Hilbert space. From Aerts and Gabora [2], first let it be of dimension 1400, that is $\mathbb{C}^n$, $n = 1400$. Then each basic context is represented by a member of the orthonormal basis B of $\mathbb{C}^n$ by the projector $\langle u|u\rangle$ such that $|u\rangle \in B$. We can write the ground state $\hat{p}$ of the concept "pet", which is represented as a unit vector $|x_{\hat{p}}\rangle$, as the linear combination of these basic contexts.

$$|x_{\hat{p}}\rangle = \sum_{u \in B} \alpha|u\rangle \text{ where } \alpha_u = \langle u|x_{\hat{p}}\rangle$$

From before we know that $|\alpha_u|^2$ is the probability that the concept pet changes to base state u. Given that each u is a base state in $\mathbb{C}^n$, $n = 1400$, we can see the probability of each base state under u is equivalent and $\forall u \in B$, $|\alpha_u|^2 = \frac{1}{1400}$. Thus, we can rewrite $|x_{\hat{p}}\rangle$ as

$$|x_{\hat{p}}\rangle = \sum_{u \in B} \frac{|u\rangle}{\sqrt{1400}}$$

So now, we want to compute the state of "pet" that the ground state changes into under the context e_1, represented by the projection operator $P_{e_1} = \sum_{u \in E_1} |u\rangle\langle u|$. By definition we have,

$$|x_{p_1}\rangle = \frac{P_{e_1}|x_{\hat{p}}\rangle}{\sqrt{\langle x_{\hat{p}}|P_{e_1}|x_{\hat{p}}\rangle}}$$

$$P_{e_1}|x_{\hat{p}}\rangle = \sum_{u \in E_1} |u\rangle\langle u|x_{\hat{p}}\rangle = \sum_{u \in E_1} \frac{1}{\sqrt{1400}}|u\rangle$$

and

$$\langle x_{\hat{p}}|P_{e_1}|x_{\hat{p}}\rangle = \sum_{u \in E_1} \langle x_{\hat{p}}|u\rangle\langle u|x_{\hat{p}}\rangle = \sum_{u \in e_1} |\langle x_{\hat{p}}|u\rangle|^2 = \sum_{u \in E_1} \frac{1}{1400} = \frac{303}{1400}$$

Giving

$$|x_{p_1}\rangle = \sum_{u \in E_1} \frac{1}{\sqrt{303}}|u\rangle$$

6.2. **Calculating Frequencies Under Effect of Relevant Contexts.** Finally, we take an example of finding the frequency or typicality rating for each exemplar under a given context. We calculate the likelihood that from state p_1 "the pet chews a bone" (resultant from context e_1) the state collapses into state p_14, "the pet is a cat". So

$$\mu(p_14, e_14, p_1) = \langle x_{p_1}|P_{e_14}|x_{p_1}\rangle$$

And we have that,

$$\begin{aligned}\langle x_{p_1}|P_{e_14}|x_{p_1}\rangle &= \sum_{u \in E_14} \langle x_{p_1}|u\rangle\langle u|x_{p_1}\rangle = \sum_{u \in E_14}\sum_{v \in E_1}\sum_{w \in E_1} \frac{1}{303}\langle v|u\rangle\langle u|w\rangle \\ &= \sum_{u \in E_1 \cap E_14} \frac{1}{303} = \frac{75}{303} = 0.25\end{aligned}$$

And .25 matches exactly the corresponding state in Figure 1 [2].

7. What This Means

Clearly the Prototype theoretic approach alongside Fuzzy Set Theory can not handle the complexities of real world applications. This approach was very simplistic and focused almost solely on some average instance of a concept and upon a function for assigning membership to a concept. This results in both a lack of flexibility over complexities in a concept and difficulty in computation. However, the quantum formulation of concepts as states, which collapse into others under the influence of contexts, gives much needed capability to any such model of cognitive function. Both seem very similar, but there is a clear breadth and depth to calculations that is supported by this quantum model. Indeed, the central element of the quantum formulation is the contexts that act upon the states of a concept S. This fundamentally changes the way we model, not only mathematically but psychologically, the cognition of concepts. Prototype theory presupposes that a concept is centered around a prototype, with other surrounding instances readily available. This seems unreasonable, things generally do not all pop into your head at once. Indeed, after summoning a concept or given context, usually only one or maybe two instances of a concept come to mind. This fits much better with the quantum formulation. Presented a concept, and a context, the concept collapses into a certain state. Thus, we only need to store the memories already stored in our brain, and the method by which a context effects any state of concept (not even the contexts themselves). This would be called the schema by which the concepts are handled in cognition, the structure of ideas and memories that lends itself to salient thoughts and judgments. Furthermore, the quantum model lends itself perfectly to conceptual combination through the use of tensor products (not discussed here). Overall, we have seen a more complete and resilient model of conceptual cognition, better handled to empirical results and calculation.

8. Questions and Future Research

One thing that I had not touched on was the true combination of concepts in the quantum model using the tensor product. I indeed have material on this approach and would look forward to expanding on one of the prime reasons for progressing to the quantum model in the future. Based on peer feedback, the method of cognitive modeling seems at first glance to be a more or less complex. I hope to have addressed this issue in the above paper, realizing that while there is a higher number of definitions and variables to track, the mathematics are fairly simple, and easy to use. These methods are especially easy to to program, translating directly into existing methods. This brings us to another key part of the feedback received upon presentation. The potential applications for this model, and models like this in general are indeed vast. They range from easy applications to recognition systems or other models of intelligence that would mimic human cognition, to outright psychology and better understanding of how the brain works. One area in specific that stands out to me is understanding the concept of self in human cognition. Many Eastern philosophies, and even of schools modern psychology either expound or admit that much of what we call the self is an illusion of thought and cognition, a concept constantly referred back to when interpreting the world “outside”. This idea of the self being a pure concept, clearly representative of some real world entity, adheres to the model. Changing in different contexts, when goal striving and needing confidence, after failures, or in work vs around friends. Trying to model the self with these ideas would be a vastly interesting topic worthy of future research.

Reference

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