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Poincaré's Philosophy of Mathematics

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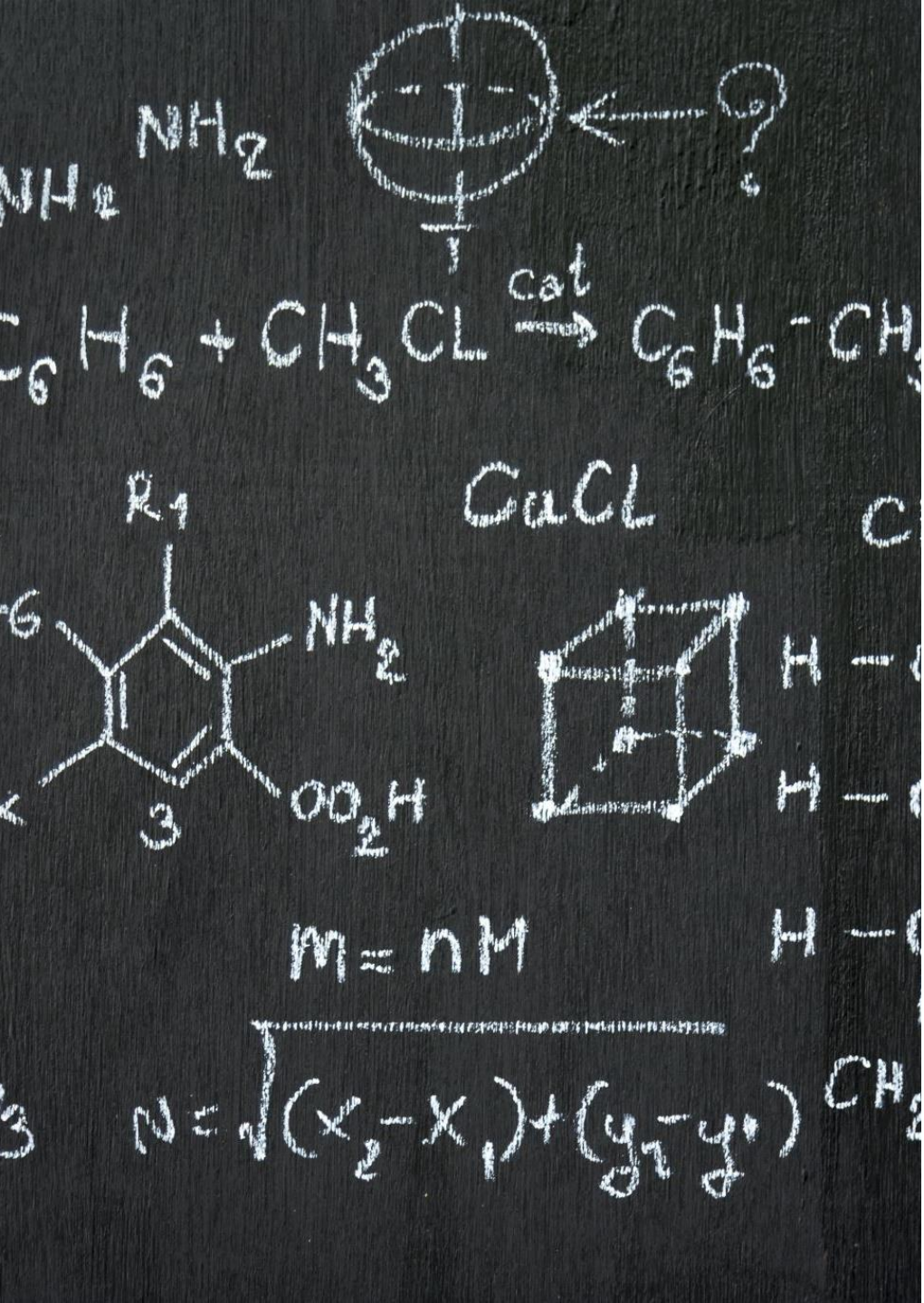
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Poincaré: Life and Work

Poincaré lived 1854-1912 in France. He was a polymath who worked primarily in math and physics. His biggest contributions include:

1. Originator of the modern theory of Lorentz transformations.
2. One of the founders of modern topology.
3. Formulated the Poincaré Conjecture (solved in 2002-3).
4. Contributor to special relativity and the theory of gravity.
5. Introduced group theory to physics.
6. Discovered the first chaotic deterministic system, laying foundations for chaos theory.



Overview of the Talk

Poincaré: Poincaré's Philosophy of Mathematics

1. What is science (mathematics)?
2. How are errors made in math?
3. What is logicism and what is wrong with it?
4. What kind of philosophical objections might there be to Russell's Paradox?

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Poincaré's Philosophy of Science

The Choice of Facts

Poincaré argues against the notion that science, mathematics included, can simply be described as a neutral discipline which takes up whatever facts present themselves and organizes them.

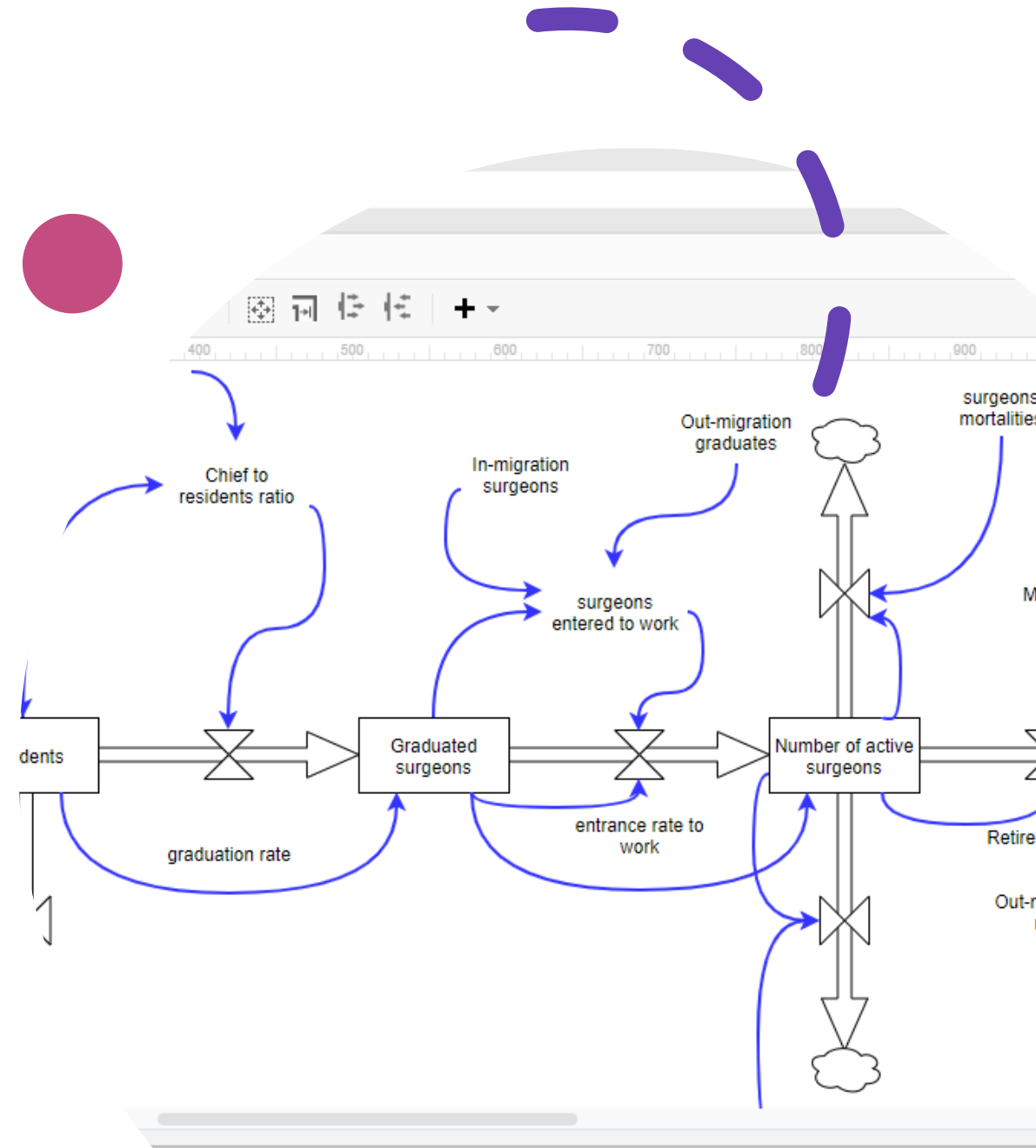
- There are *too many* facts.
- So, a *choice* of facts has to be made.

The value-laden character of science is an idea found in other philosophers. Jurgen Habermas comes to mind.

“Economy of Thought”

What is science? How does science function?

- Poincaré's answer: science is an “economy of thought”.
 - Think of ‘economy’ abstractly as a dynamic system of transfers or exchanges, creation and satisfaction of demands, and the production of products.
 - Science is such an economy, but not of goods. It is an economy of *ideas*, of *thought*.
- The suggestion here is that the scientific (mathematical) value of a fact is determined by its “productivity”, as Poincaré says, to ‘stimulate’ the economy of thought, so to speak.



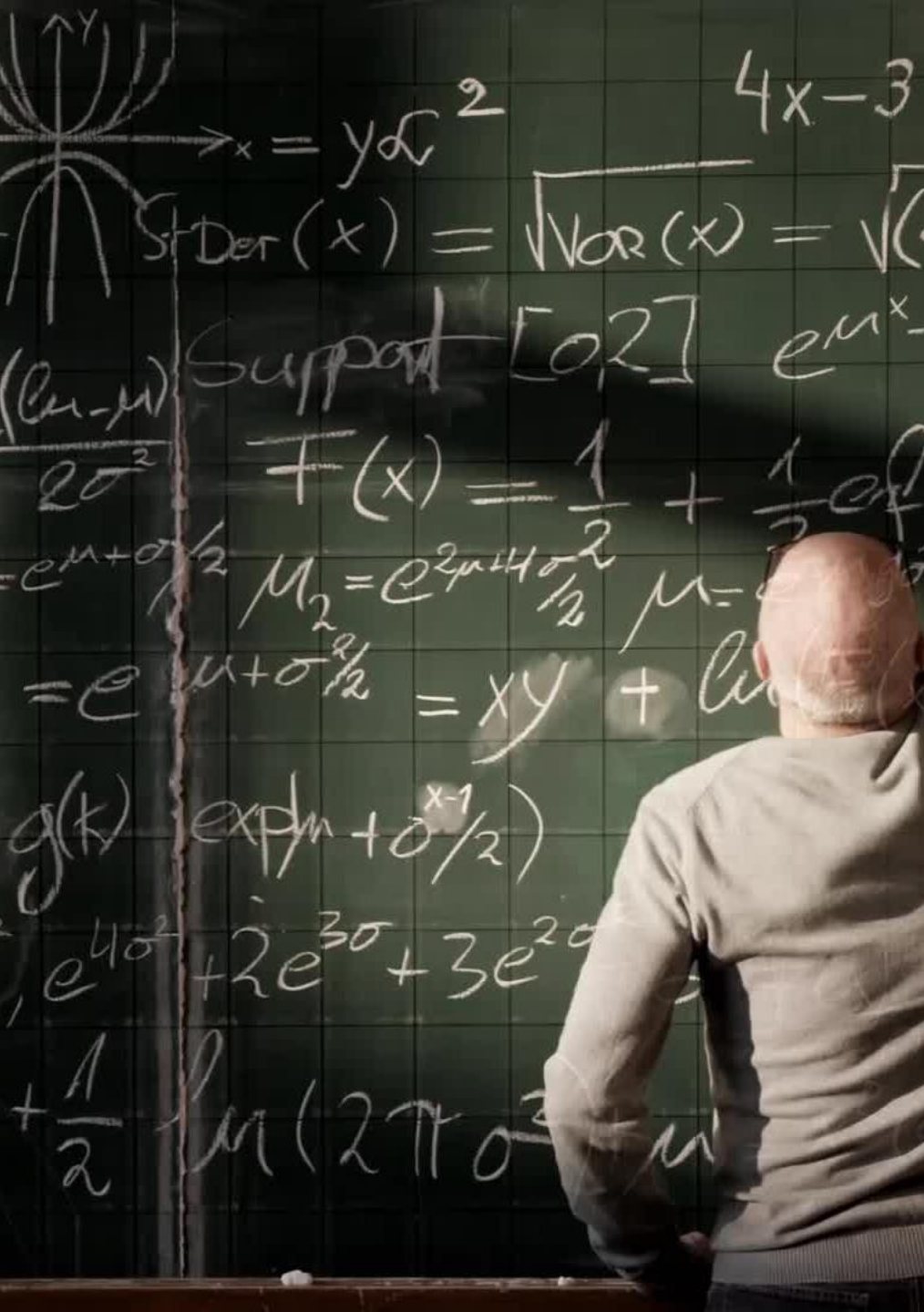


Questions & Discussion?

1. What do you think about the idea that science and math are already 'value-laden' in virtue of the "choice of facts"?
2. What do you think of Poincaré's account of mathematics and science as each consisting in an "economy of thought"?



Error & Fallibility



The Source of Error in Mathematics

- Epistemology: The study of knowledge.
 - What can we know and how?
- Mathematics has traditionally been thought to be knowable *a priori* (justified and true independently of experience) and *apodictic* (all its claims are either demonstrable, necessary, or self-evident).
 - The question then arises: How does it come to pass that mathematicians, even the most skilled ones, *make errors*?
- Poincaré argues that the source of error in mathematics is memory.
 - Example.
 - Compare with Descartes's account of error (in reasoning), for example.



Questions & Discussion?

What do you think the main source of error in mathematical reasoning is? Is reason, as far as math is concerned, infallible?



Logicism & Objections

The Logician Movement and Project



There are multiple versions of logicism, but the general view can be described as follows.

Logicism: Logicism about mathematics is the idea that mathematics can be 'reduced to logic', i.e., that all mathematical truths are propositions which are either true by definition or by deductive inference from definitions.



Logicism was a popular idea in the turn of the 20th Century. Bertrand Russell is one of the most famous examples of a logicist in this period.

The goal of the movement was, in general, to show that mathematics is, indeed, something like a priori and apodictic by showing it contains nothing beyond logic.

Poincaré's First Objection to Logicism: Mathematical induction is not logically derived

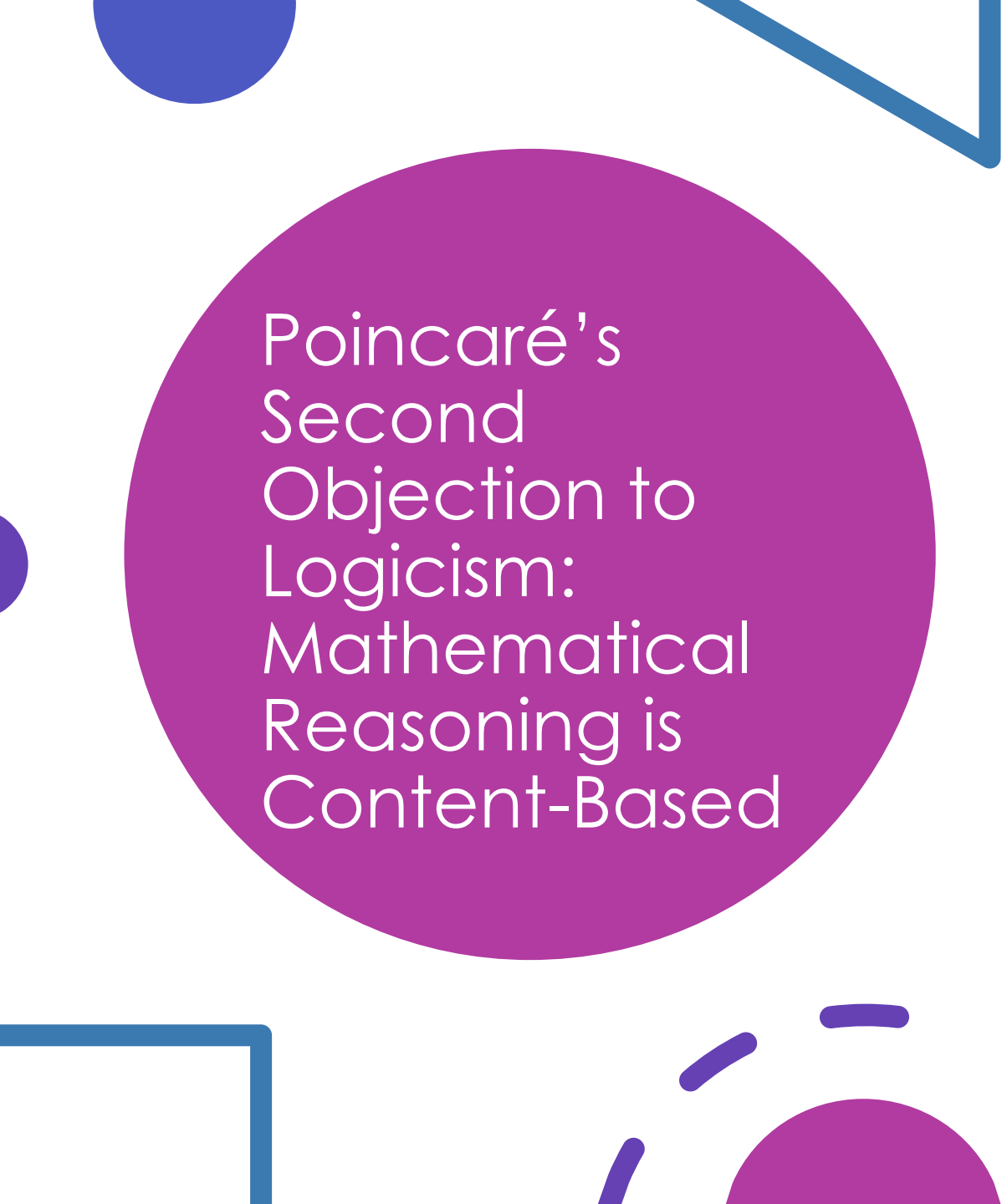
Poincaré's objections to logicism are really just arguments that intuition, by which we understand *immediate knowledge*, plays an important role in mathematical reasoning.

- The first argument is quite simple and interesting. Poincaré claims that mathematical induction cannot be justified by formal-logical means; it requires an intuitive leap.
 - Recall: Mathematical induction.
(1) Prove $P(1)$. **(2)** Prove: $\forall k \in \mathbb{N}, P(k) \rightarrow P(k + 1)$. **(3)** Therefore, $\forall n \in \mathbb{N}, P(n)$.
 - What is the problem with this? The jump from the sequence of inferences to the claim that the statement is true is an immediately grasped leap which is never justified in logical terms.



Questions & Discussion?

What do you think about Poincaré's claim that mathematical induction requires intuition?

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Poincaré's Second Objection to Logicism: Mathematical Reasoning is Content-Based

Poincaré also argues against the idea that mathematical knowledge is (most) accurately understood in a formal-logical way.

1. In formal logic, propositions are assigned true or false, and the content they contain is irrelevant to their logical properties.
2. Poincaré, by contrast, claims that mathematical reasoning depends deeply on the *content* of mathematical propositions, not just their formal structure as propositions.

(Relates back to intuition's use in the previous argument and to his general view of mathematics as science.)



Questions & Discussion?

What do you think about Poincaré's claim that mathematics requires content-based, not merely formal, reasoning?



Russell's Paradox

Poincaré's Contribution on Impredicative Definitions

- Poincaré left an objection to Russell's Paradox which he hoped would allow us to avoid paradoxes like Russell's Paradox.
 - Russell's Paradox: $S = \{s: s \notin s\}$. (A presupposition of this is that $\exists s, \forall z, (z \in s) \leftrightarrow P(z)$ for some predicate P .)
- Poincaré objects that this presupposition involves a conceptual incoherence: that there is a vicious circularity in a predicative definition like this one.
 1. Predicative: A definition is predicative if the terms used to define something somehow presuppose the thing being defined.
 2. Impredicative: A definition is impredicative if it is not predicative.
- Poincaré noticed that the presupposition Russell uses to define the paradoxical set S is one that allows predicative definitions.



Concluding Discussion

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Images:

<https://online.visual-paradigm.com/diagrams/features/stock-and-flow-diagram-tool/>

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