Quantum Tomography: Theory and Practice

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Ongoing project with

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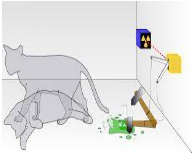

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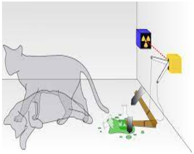

- However, in the quantum environment, the photon is in the superposition state $|\psi\rangle = a|0\rangle + b|1\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$ with $a, b \in \mathbb{C}$, $|a|^2 + |b|^2 = 1$.

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- Let us do some simple experiments.

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- After P_1, Q_1, P_2 measurements, we get a quantum state $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

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$$|a + ib - c|^2,\ |a + ib - c|^2 = 1||a + ib - c|^2.$$
- We can then solve for a, b, c.

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