Chinese Board Game ‘Go’

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Abstract

“Go” is a strategic board game with ancient origins dating back to about 3000 years ago in China. It is continuously played till today and is widely loved globally. Despite the seemingly simple rule of encircling more territory, people find the game to have much more content than expected.

This paper aims to provide an introduction to Go and explore the most significant mathematical studies related to it. Divided into five sections, it provides an examination of the game and further investigations. In the introductory section, I will introduce the reason Go was chosen as the topic and provide an outline of the paper. In the second part, I will present the history of this game. In the third part, I will demonstrate basic patterns and mathematical models inherent in gameplay. In the fourth part, I will introduce further studies about analyzing the game as a whole. In the final part, I will list out my reflections on the peer review, as well as some potential further studies.

1 Introduction

I decided to choose Go as a topic firstly due to its significance in my Chinese cultural background, where children often encounter the game in kindergarten as their earliest introduction to mathematics. Additionally, the rich historical content and the complexity behind the rule make Go a topic that is still being continuously studied in different fields, including history, philosophy, and mathematics. As such extensively studied competitive chess games, Go further piqued my interest, and stand out as a perfect topic both for personal preference and mathematical connection.

The next section will explain the history of this game and its international propagation, and expose the nomenclature behind the name Go. The two following two sections introduced related studies of Go. In the first part, I will explain the mathematical theories behind the basic tactic in Go circling the area, and demonstrate the special patterns in Go. In the second part, I will focus on the possible game problem of Go, and explain people’s attempts to optimize the strategy by AI approach. In the final section of this paper, I will reflect to the peer response and propose future potential studies related to Go.

In this paper, I aim to introduce Go as a board game, address and explain the basic key studies related to it, and inspire further inquiry into this charming game.

2 History and Rules

2.1 Origination

Go was first officially documented in the ancient historical chronicle ZuoZhuan about 3000 years ago. The recorded phrase " If a chess-player lifts his man without a definite object, he will not conquer his opponent”[1] portrays the act of relinquishing a gained territory in Go as a metaphor for the political act of supporting an emperor and later deposing him. This illustrates people had already shared widespread familiarity with Go during that era, and the strategies in Go show a resemblance to the tactics employed in the arts of contemporary politics and war.

The Chinese name for this game is Weiqi, where "qi" denotes chess and "wei" means surrounding. In fig.1, we can see the ancient Chinese character for "wei" resembles a depiction of a city being encircled. It is a hieroglyph—a pictorial symbol representing a word. This kind of word symbol is often employed in the study of mathematical history within Asian contexts.

Figure 1: The Chinese Ancient Character of 'wei'.

2.2 The Asian Spread

In the early 7th century A.D., Go gradually spread to other Asian countries, believed to have been introduced to Japan by the Japanese scholar Kibi no Makibi.[2] However, there also may be earlier records that serve as evidence of its earlier introduction to Japan. Initially, it was referred to by its Chinese pronunciation and later referred as 'igo'. Go gradually spread throughout Japan and the entire East Asian region, becoming quite popular.

2.3 The Western and International Spread

For a long time, Go was limited to Asia only. It wasn't until the 17th century that Go began to spread to the rest of the world. The first detailed Western record about Go was written by English linguist Thomas Hyde in his work *About Oriental Games*, where he referred to the game as Chinensium (Chinese encircling game).[3] Following this, German chemist Oskar Korschelt learned the game while working in Japan in 1878.[4] He later wrote the book *The Theory and Practice of Go*, in which he transliterated the Japanese sound 'go' into "Go" to represent the game. This book successfully popularized the game internationally, which is why the game is commonly known as 'Go' in Western contexts.

2.4 Rules

In Go, there are alternative board sizes available. For instance, beginners often start with smaller boards like 9x9 or 11x11. Additionally, alternative rules do exist. However, for the sake of consistency and as the most commonly used rule in related academic studies, this paper adheres to the internationally accepted standard.

The general rules of Go can be summarized as:

1. Two players, one holding Black and the other holding White, take turns to place pieces on the intersection of a 19x19 Go board.

2. If a piece or group of pieces on the board is completely surrounded on all adjacent sides, then those pieces are "captured" and removed from the board.

3. When every point is either surrounded or occupied by Black or White, the game ends.

4. The player with more territory wins (consisting of all the points the player has either occupied or surrounded).

As fig.2 shows, 'adjacent sides' refer to the intersection directly above, below, left, and right of the given point.

fig.3 shows an example of capturing the white piece.

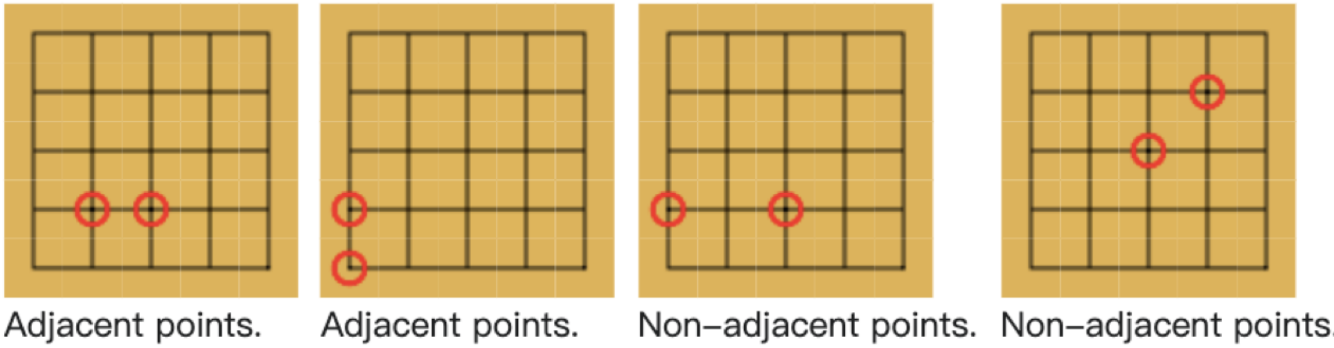

Figure 2: Adjacent and Non-adjacent Points

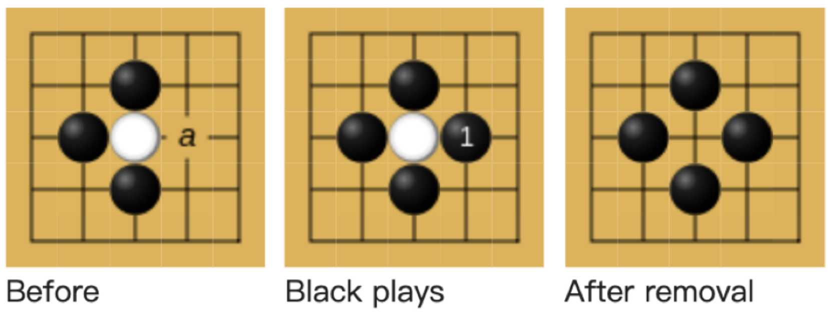

Figure 3: Simple Example of Capturing

3 Gameplay Studies and Important Patterns

3.1 Circling Area

The primary goal in Go was to circulate more area, and if we simulate the length that the pieces formed as a line, this question can be simplified to a simple conditional extreme value problem:

Problem: The perimeter C of a rectangle is a fixed value. Find the optimal side length of x, y to maximize the area S.

To simplify the problem, we use $C = 4$ for the following proof

Proof:

$$ProblemSet = \begin{cases} S = x \times y, \\ 2(x + y) = 4, \end{cases}$$

By representing y by x, we have:

$$y = 2 - x$$

The area is represented by:

$$S = 2x - x^2$$

Take the derivative with respect to x:

$$S' = 2 - 2x$$

Set equal to 0 and solve for x and y:

$$x = y = 1$$

Therefore, when $x = y = 1$, the maximum area S of the rectangle is 1.

Explanation: By maximizing the area of the rectangle given a fixed perimeter, we find that a square provides the maximum area for a set perimeter. The above proof shows that circling a square is the most efficient use of space in Go.

This simple proof demonstrates the math behind the most basic proverbs in Go for beginners: "Golden edge, silver sides, grass center(It is better to circulate on edges than in center).", and "Severn pieces aligning on edge, even alive then lose(Don't make inefficient usage of pieces by forming lines).", as shown in fig.4 and fig.5:

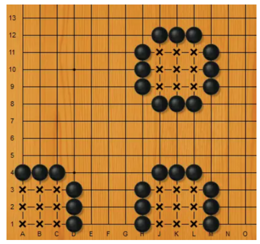

Figure 4: Better to Circulate in Center.

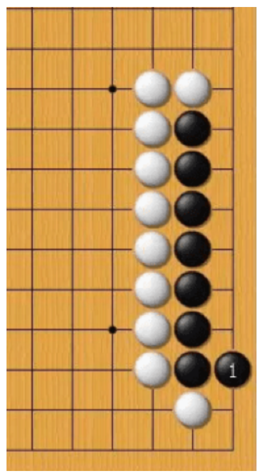

Figure 5: Inefficient Lines.

3.2 The Two Eyes Structure

The two eyes structure is a very important basic pattern in Go.
A single empty space inside a group of pieces is called an eye, and is possible to form a group of pieces that cannot be removed by creating two or more "eyes". Such unmovable patterns are called "alive".

In fig.6, the Black pieces surrounded by the White cannot be captured.
White cannot place a stone in both locations in the same turn, therefore it is impossible for White to completely surround the Black. Creating stable patterns such as the two eyes structure is crucial to winning the game.

3.3 The Seki Structure

There also exist special combinational patterns that cannot be simply considered as being alive or dead for either Black or White called 'Seki', which can be interpreted as a mutual life of black and white pieces, or a stalemate in the game.

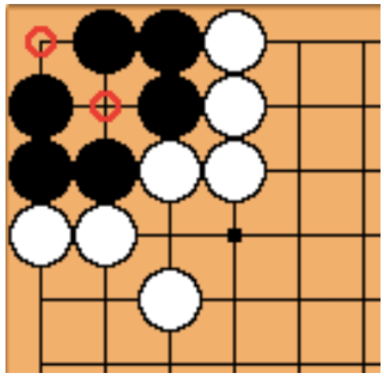

Figure 6: Example of the Two Eyes Structure

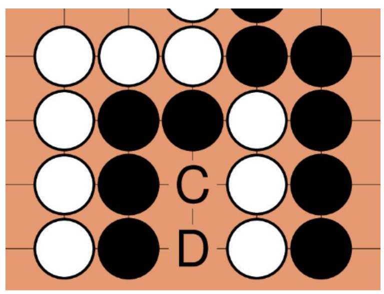

Figure 7: Example of Seki[5]

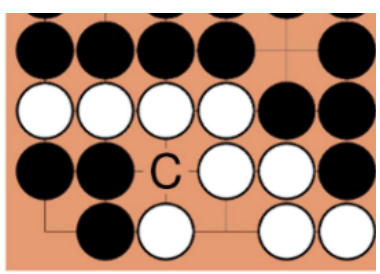

Figure 8: Example of Seki: Stable Structure with One Eye[5]

Here are some examples of Seki:

In fig.7, both the Black and White pieces in the middle are surrounded, and neither of them has the two-eyes structure. However, if Black attempts to capture White by playing on either C or D, White can play on the other spot and successfully capture Black instead. While the situation is exactly the same for White. In this case, trying to capture the opponent will lead to one's own death.

In fig.8, both the Black and White pieces in the middle have one eye. If any of them try to capture the other by playing on C, it will lead the result to self-capturing as well. With a seki structure, it is also possible to maintain a group of pieces only with one eye.

3.4 Capturing Race

Understanding patterns is valuable in actual gameplay, but math and calculation also play a role in addressing certain challenges, such as capturing race. A capturing race involves competing to capture each other's groups of pieces with an existing pattern.[6].

In a capturing race, players assess the liberties (empty adjacent points) of their groups to determine if capturing is possible and set the optimal strategy to capture them. These liberties can be categorized as inner liberties within enclosed areas, outer liberties outside the groups, and eye liberties that are both inside liberties and considered eyes.

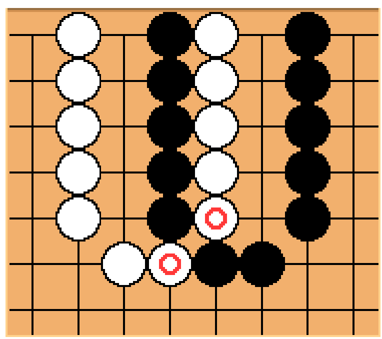

Figure 9: Simplest Capturing Race[6]

The simplest capturing race shown in fig.9 involves five inner liberties against five inner liberties, where the first player to move wins. However, more complex patterns require extensive calculations.

Since capturing races varies based on factors like the number of eyes, the calculation approach differs. There are scenarios like no eye versus no eye, eye versus no eye, and eye versus eye capturing races, each with its own considerations. Due to space constraints, more detailed information about capturing race can be found in [6].

In summary, the rules of Go are simple and easy to understand, based on the basic principle of circling space. However, as seen from the patterns displayed above, real-game scenarios often lead to much more intricate situations than simple patterns. When stones connect over a large area on the board, it can sometimes be challenging to determine who is capturing whom. Surrounding an opponent's stones to capture them can also backfire, and at times, abandoning an attack to achieve mutual life in a seki structure can yield greater benefits. We can also incorporate some mathematical skills, for example, the capturing race calculation, to help judge some situations and aid decision-making. Go involves both competing and reaching equilibrium and is a game with huge strategic depth and possibilities.

4 Game Study

4.1 Possible Games

Apart from how to play Go, people have also gradually started studying Go as a whole system. One important question about Go is: With a total of 361 points to play on, how many possibilities are there in Go in total? Studies made about Possible Games in Go are often in two approaches:

4.1.1 Possible End Patterns

Considering only the end pattern on a finished Go board, but not the process in each step, we can get a general approximation of the possible end patterns:
On a final board, one spot is either occupied by white, occupied by black, or unoccupied by both. With three possibilities on each spot, giving a: $3^3 61$ total results.
However, there are major flaws in this approximation:

1. There exist many illegal patterns according to the rule. For example: In fig.3, the pieces captured will not be presented on the board in any case but is included in the results.

2. End patterns that are considered as the same by rule, but counted for multiple times. For example, in fig.6, the two-eye pattern is already formed, there is no need to fill in the two red spots to maintain the area. Filling and not filling the red spots is considered as the same end pattern, but calculated for three times in the above approximation.

Since it is impossible to find all legal patterns with the current calculation capacity, Mathematicians started studying with smaller boards. In 2016, Mathematicians John Tromp and Gunnar Farnebäck observed that the percentage of legal patterns decreases as the size of the Go board increases, and tried to derive a recursive formula for legal positions of a rectangle board with length m and n with data collected. By improving the formula, and using computer simulations, they managed to approximate that the percentage of legal positions on a 19 x 19 Go board was close to 1.2 percent[7].

Probability	Illegal Positions	Legal Positions	Board Size
0.333333	2	1	1×1
0.70304	24	1	2×2
0.643957	7008	1	3×3
0.564925	18728556	1	4×4
0.527724	1646725708	1	4×5
0.235	2	approximation	9×9
0.087	2	approximation	13×13
0.012	2	approximation	19×19

Table 1: John and Gunnar's Estimation[7].

Interestingly, the legal pattern's percentage shows an exponential decay in fig.10. However, since only the first few points are collected experimental data, it is not reliable to make connections to the exponential decay model without further proof.
The number of end pattern of Go is estimated to be: $0.012 \times 3^{361} \sim 2.08910^{170}$

4.1.2 Possible Games

Considering the number of possible games, in which each step is given account, we can approximate first with the simplest idea:
In the first, move, there are 361 points to choose from and begin with. In the second move, we choose from the 360 remaining points... In the end, it gives a 3^{361} possible games in Go.

However, there are major flaws in this approximation:

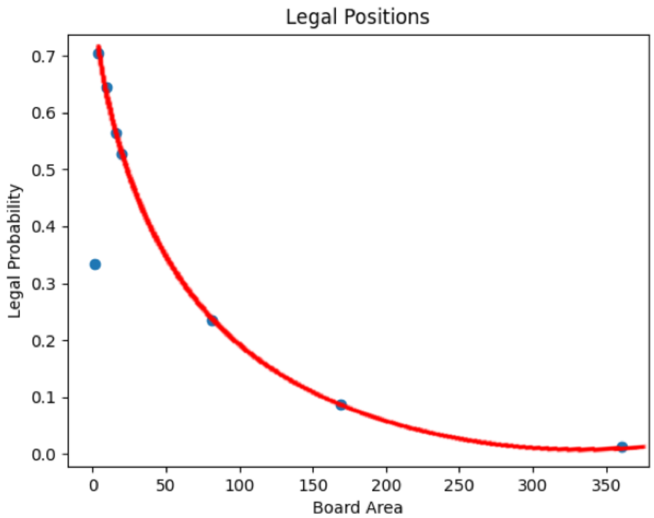

Figure 10: Legal Probability vs Board Area

1. Illegal games exist, and are much more complicated than in the end pattern case.
2. Go games don't need 361 moves to finish, or even rarely last for 361 moves.
3. Similar simulation as predicting the end pattern has not been successfully utilized

Until now, the most credible prediction of the possible games was made by Victor Allis. He incorporated game-tree complexity to conclude the average branching factor (average move in a step) in Go is 250, while the average game length is 150 plays. Which gives a total possible game close to 10^{360}.[8]

4.1.3 The Scale of Possible Games

There are approximately 2.089×10^{170} potential end patterns and 4.910^359 possible games in Go. These numbers are so vast that it's hard to comprehend at first sight, but they can be more directly illustrated in the following example:

1. Consider that there are around 10^{50} atoms on Earth, and the Universe has existed for about 1.38×10^{10} years.
2. Imagine a computer that can generate a possible game in one nanosecond ($10^{-9} seconds$) and has been running since the beginning of the universe. Despite this, it has only managed to generate less than 0.000000000000000000000000000000000000000000000 00000000000000000000000000000000000000000000000000000000000000000000000000001 percent of all possible games in Go.
3. If each possible game could be stored in an atom, it would require atoms from 10^{100} Earths to store all possible games.

This immense scale highlights why it's practically impossible for computers to explore every potential game in Go.

4.2 AI Go

Despite being unable to generate all possible games, in recent years, the field of artificial intelligence (AI) rapidly developed in playing Go. With the development of deep learning algorithms and neural networks, AI has achieved unprecedented levels of performance in Go, surpassing even the best human players several

years ago.
The first version of AlphaGo's function is as follows:

Step 1: Using Deep Convolutional Neural Network, AlphaGo gathers Go game records played by human players, where each move made by a human creates a natural training sample. This process generates around 30 million training samples. By leveraging this massive dataset, it simulates human gameplay.[9]

Step 2: By employing Monte-Carlo Tree Search, AlphaGo simulates games using a random approach. Before making each move, it considers several future moves(branches). Even if it cannot generate all possible subsequent possibilities, it can generate a few steps ahead, and follow those steps. Different moves are assigned scores based on their potential outcomes. Through continuous self-play and simulations), AlphaGo improves the quality of its moves. After playing nearly 100,000 games, it selects the move with the highest score in a given state and gives the optimal one future move.[10]

Step 3: Utilizing Reinforcement Learning by Value Network, AlphaGo designs an evaluation function to quantify and assess Go game positions. This function allows AlphaGo to evaluate game positions without exploring the entire board, thus increasing computational efficiency. The evaluation function continuously evolves offline, enabling AlphaGo to improve its assessment of game positions over time.[11]

In summary, AlphaGo integrates these three technologies: simulating human strategies, using random simulations to determine optimal future moves, and evaluating game positions to enhance efficiency and self-improvement through continuous self-play. Although Go cannot be tackled on such a total scale, the development of AI Go stands out as one human approach to search for the optimal ways to play the game in a reachable finite scale.

5 Reflection

5.1 Presentation Reflection

After reviewing the presentation reflections on the Discussion Board, I observed that most classmates, even those encountering Go for the first time, reflected on gaining a good basic understanding of Go. Many posts specifically mentioned the simile to atoms in the universe was astonishing, and intuitively demonstrated the beyond-computable possible game in Go. It enhanced my thought that making connections to familiar concepts is effective when introducing unfamiliar subjects. Furthermore, peers also mentioned interest in the history and evolution of Go's name, which intrigues me to delve deeper into Go's historical records and incorporate them into this paper.

I am pleased to have introduced Go by this chance, and in future presentations, I will remember to utilize examples to illustrate mathematical concepts and numbers, while also exploring more math connections related to history and other aspects.

5.2 Related Potential Further Research

In exploring the peer review feedback received, I encountered several intriguing suggestions for potential research about Go.

One classmate mentioned exploring possible games in another board game called Backgammon, which could lead to further investigations into possible games in various competitive board games. Despite different boards and rules, if these games are confined to specific spaces without forming loops, efforts might be made to predict their possible games as well. Another review suggested a future study on reverse speculation, predicting the game-winner by observing only the initial minimum steps, which can be closely related to the value work mentioned in the AI Go part. What's more, a later presentation demonstrated different computing algorithms and their functionalities in generating optimal results. Professor Li proposed linking these algorithms to AI in Go, predicting possible games several steps ahead. Potential research could also explore how different algorithms influence computer strategies in playing Go.

Lastly, I would like to delve more into the Endgames problem in Go about the final stage where players occupy the last few points on the board. In recent years, mathematical research on this topic has rapidly grown and achieved significant results. While many endgame-related questions have been addressed, they represent a small but crucial step towards a deeper mathematical understanding of Go.

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