

Casino games and related math

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Introduction

If you get to gamble in a casino, and believe that your fortune will make you money, there are chances that you make some solid profit at first. However, if you believe that this fortune is going to continuously make you money in casinos, then you have fallen into the traps the casinos have set for you. In this paper, we are going to examine the casino games, and see how they are designed to be profitable for the casinos and why most of us end up losing money in casinos. More importantly, we are looking deep into some specific games, and see if we can adopt certain strategies to get the odds more in our own favors.

House Edge

“It’s important that people don’t think that they can win, because you can’t win in gambling. You can win for a little while but you can’t win in the long run.”

The most fundamental term we use to quantify the advantage the casino has over the players is the House Edge. It is the term used to describe the mathematical advantage that the gambling game has over you as you play overtime.

The way to calculate House Edge is quite simple, it can be understood as the mathematical percentage loss of your bet in the long run. Say if the house edge is 5 percent, then in the long run if you play the game long enough, you will lose 5 percent of the money that you bet. This 5 percent is exactly what the casino aim to make over the players. Since the casino is the side who involves in a huge number of games, the

Large Number Theory applies perfectly in this scenario: the Casino will make 5 percent of the total bets on this game.

To understand the concept more clearly, we can draw a chart to see this more clearly.

Suppose we have a completely fair coin toss game, then we can have this chart:

Event	Odds	Payouts	Return
Win	50%	1	0.5
Lose	50%	-1	-0.5
		Player loss	0
		House edge	0

This is quite straightforward that the game is fair, so the house edge is 0.

However, what if the payouts for winning is 0.9?

Event	Odds	Payouts	Return
Win	50%	0.9	0.45
Lose	50%	-1	-0.5
		Player loss	-0.05
		House Edge	5%

Here, even though the percentage of winning and losing is strictly 50%, but since the payouts is changed, the game is in clear favor to the House.

Another way we can understand this is to use expected values. In the first game, if we bet one dollar and win, we will get two dollars. Then we time this total payout with

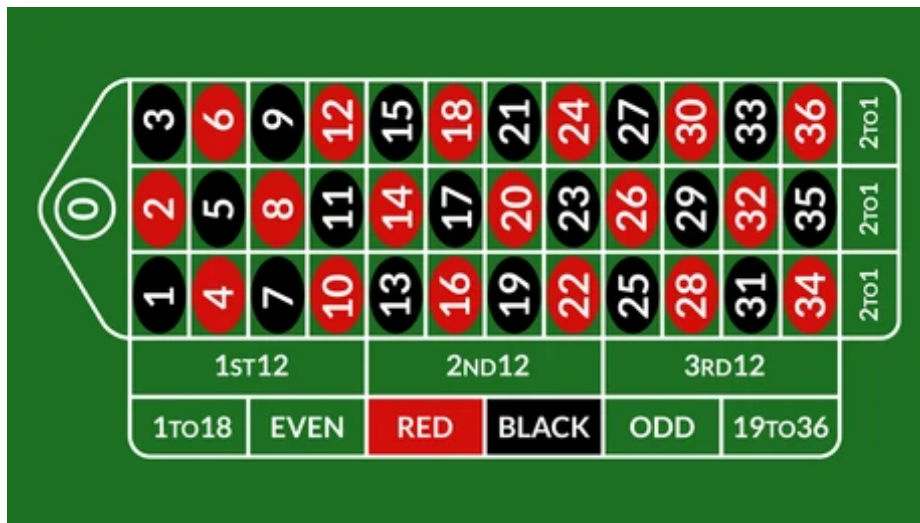
percentage of winning: 50%, which gets us 1 dollar as the expected value. This means that if we bet one dollar, we are expected to get 1 dollar back in the long run. However, in the second game, if we bet one dollar and win, the total payouts is 1.9 dollars, and the expected value is $1.9 \cdot \frac{1}{2} = 0.95 = 1 - 0.05$

This means that we will only get back 95 cents for every dollar we bet in the long run. Notice that this difference over 1 is exactly the house edge. So the House Edge can also be calculated by $(1 - ev(1)) \cdot 100\%$.

This is exactly how the casinos design the games. In most cases, the odds of a game is fixed, however, the payouts are designed in certain ways that will yield positive house edge which benefits the casino.

Examine the game of Roulette

Here is another popular game Roulette:



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The return, for betting on a single number is 35, and for the lower two rows are one to two and one to one, etc.

If we exclude the number 0, then the odds and the payout for roulette is a perfectly fair game. However, because of the addition of 0, and the payouts remaining the same, this game becomes unfair to players, and yields positive House Edge. (0 does not belong to either black or red, even or odd).

For example, if we bet 1 dollar on the “1st 12” slot, meaning that we believe the ball will fall on a number between 1 and 12. If we win, we will get 3 dollars back. However, the probability is actually $\frac{12}{37}$. Thus we can calculate the expected value to be

$$3 \cdot \frac{12}{37} = \frac{36}{37}$$

Then we can get the house edge to place this bet to be:

$$\left(1 - \frac{36}{37}\right) \cdot 100\% = 2.703\%$$

Type Of Casino Game	House Edge
American Roulette	5.26%
Baccarat	1.01% - 15.75%
Blackjack	0.5%
Craps	0 - 16.67%
Keno	20 - 40%
Slots	2 - 15%
Video Poker	0.46%
Caribbean Stud	5.22%
Hold'em	2.36%
Sic Bo	2.78 - 33.33%

The chart shows the House Edge of different casino games

A simpler way of seeing these numbers is that since the Large Number theory applies perfectly on the casino side, then for each game in the casinos, the house can make the corresponding house edge percentage of the total bets placed on that game. And of course, that profit comes straight from all the players, which explains why most players will end up losing in casinos.

Blackjack and basic strategies

Among those games, the game Blackjack is worth pointing out. It is stated in the chart that the House edge in this game is 0.5%, making it one of the fairer games in Casino. However, the true House edge for most of the players is 0.5%-2%. This variation in House Edge is based on uses of Basic Strategies in Blackjack.

However, to understand how this strategy cuts down the House edge, we must first understand the real objective of Blackjack. The real objective of blackjack is to beat the dealer's hand without going over 21, instead of aiming to get as close to 21 as possible. Therefore, in many cases, we can hold a lesser hand and expect the dealer to bust, as the dealer must hit on 16s and hold on 17s (some will hit on soft 17).

Thus, when we see the dealers have faced up card less than 7, we can take actions expecting the dealer to bust, as they must hit on these lesser hands, and more times of hits means the increasing chances to bust. And here goes our chart of basic strategies below:

BASIC STRATEGY FOR BLACKJACK

DEALER'S UP CARD

	2	3	4	5	6	7	8	9	10	A
17+	S	S	S	S	S	S	S	S	S	S
16	S	S	S	S	S	H	H	H	H	H
15	S	S	S	S	S	H	H	H	H	H
14	S	S	S	S	S	H	H	H	H	H
13	S	S	S	S	S	H	H	H	H	H
12	H	H	S	S	S	H	H	H	H	H
11	D	D	D	D	D	D	D	D	D	H
10	D	D	D	D	D	D	D	D	H	H
9	H	D	D	D	D	H	H	H	H	H
5-8	H	H	H	H	H	H	H	H	H	H
A8-10	S	S	S	S	S	S	S	S	S	S
A,7	S	D	D	D	D	S	S	H	H	H
A,6	H	D	D	D	D	H	H	H	H	H
A,5	H	H	D	D	D	H	H	H	H	H
A,4	H	H	D	D	D	H	H	H	H	H
A,3	H	H	H	D	D	H	H	H	H	H
A,2	H	H	H	D	D	H	H	H	H	H
A,A, 8,8	SP	SP	SP	SP	SP	SP	SP	SP	SP	SP
10,10	S	S	S	S	S	S	S	S	S	S
9,9	SP	SP	SP	SP	SP	S	SP	SP	S	S
7,7	SP	SP	SP	SP	SP	SP	H	H	H	H
6,6	SP	SP	SP	SP	SP	H	H	H	H	H
5,5	D	D	D	D	D	D	D	D	H	H
4,4	H	H	H	SP	SP	H	H	H	H	H
3,3	SP	SP	SP	SP	SP	SP	H	H	H	H
2,2	SP	SP	SP	SP	SP	SP	H	H	H	H
	2	3	4	5	6	7	8	9	10	A

HIT
STAND
DOUBLE DOWN
SPLIT

With the help of this chart, we can actually cut down the house edge to 0.5%.

However, that is still not fair enough for us to play. But is it possible to have zero or even negative house edge in blackjack?

Monty Hall Problem and card counting

In fact, the number 0.5% is based on betting equal amounts in every hands, and if we can know before each hands that our chance of winning varies by a certain amount, and then change the amount of bets accordingly, theoretically we can eliminate this remaining 0.5% house edge.

Before introducing this strategy, let us first look at a famous problem called the Monty Hall Problem:

“Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

Many people think that they should not switch, but the answer is to always switch the choice. We can compute with conditional probability: $P(A | B) = \frac{P(A \cap B)}{P(B)}$.

Here the problem is the probability of choosing a car given that we know an empty door.

$$P(A \cap B) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}, \text{ and } P(B) = \frac{1}{3}$$

$$\text{Thus } P(A | B) = \frac{1}{2}.$$

This indicates that if we make another choice after we are show an empty door, the chance of winning increase from one-third to one half.

This is the essential idea used in changing bets in Blackjack: If we know that more cards not in favor of the players has been dealt, then we can increase our bet as we have bigger chance of being dealt cards in favor of players.

In blackjack, smaller value cards are usually considered as not in favor to players, for example if I get a 10 and a 3, while dealer has an eight. Then according to the chart I should hit until I get to 17 or higher, but this will increase the chance to bust. On the contrary, if we get Q Q, we do not need to take any actions, and it will give us high chance of winning.

High-Low card counting strategy

But how do we keep count of favorable and unfavorable cards dealt? It is too hard to keep count how many cards of each value is left as the games is usually played in six decks.

A usual way is called the High-Low card counting strategy.

Essentially, we assign values to different cards :

Card value	Assigned value
2 3 4 5 6	1
7 8 9	0
10 J Q K A	-1

First we can set a base amount of bet. As we play along, we also keep this plus one,

minus one count in our mind. Before each hand, we calculate the $\frac{livecount}{\#decksleft}$ taking

the whole number part.

For example, if we have a live count of plus 17, and observe that there are approximately 4 decks left, then we will place four times of our base bet for the next hand.

Did this method actually work out?

A similar or even more advanced way of counting cards was used by the famous MIT Blackjack Team. Basically, they would deploy people to sit on every table and bet small, and when the running count is high, they would secretly call the “big better” to come and tell the “big better” the running count secretly. With this method, the group can get up to 2 to 4 percent edge in their own favor, and they have indeed made huge amount of money in this way.

However, an important training in this team is to not let the casinos to notice someone is collaborating and counting cards. Because even though card counting is legal, the casino still have rights to ban you from playing with no reasons. So for most individuals if they are counting card and then bet accordingly, the casino will quickly notice and throw you out of the game to keep the game profitable.

You can find find some further readings about this in this link:

<https://abcnews.go.com/Primetime/story?id=131939&page=1>

Conclusion:

Therefore, in conclusion, even though someone might be able to find ways to mathematically win over the casinos, the casinos still have every means to keep making the games favorable to the house. So, after all, it is still the best for us not to gamble in all ways possible.

Further readings and responses

Here is a further reading about a special rule in Blackjack called the insurance and why we should never have insurance in Blackjack:

<https://www.casino.org/blog/blackjack-insurance/>

Also in response to some comments that many people may at some time psychologically think that they have better chance of winning: In reality, the house edge for these games always exists. But I understand that our minds can be easily disturbed when we are in casinos, and maybe that is why the casinos can make money even though we know the strategies to win in blackjack. Sometimes if we believe in our fortune and take wrong decisions, the edge will go up and the game is no longer in our own favor.

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