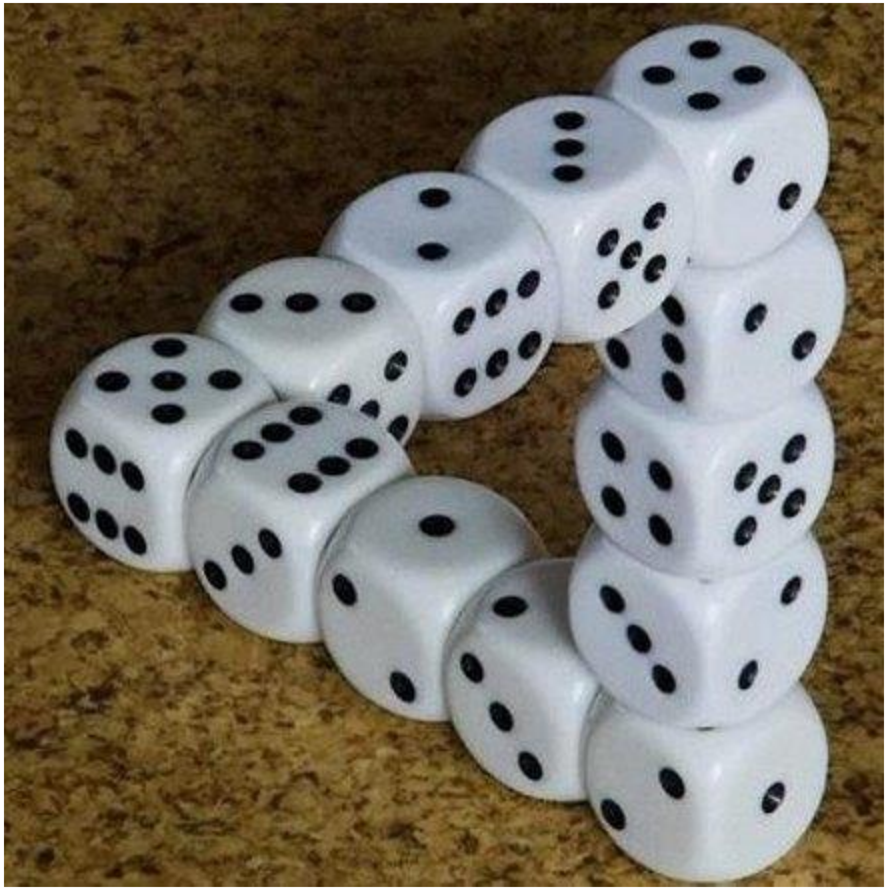
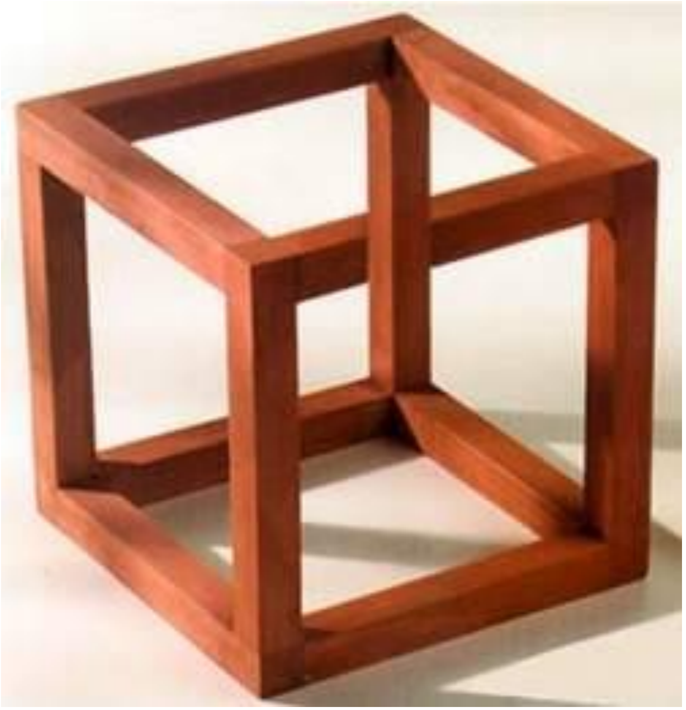
PARADOX

Chi-Kwong Li

A 'VISUAL' PARADOX: ILLUSION





Falsidical paradox

- A proof that seems right, but actually it is wrong!

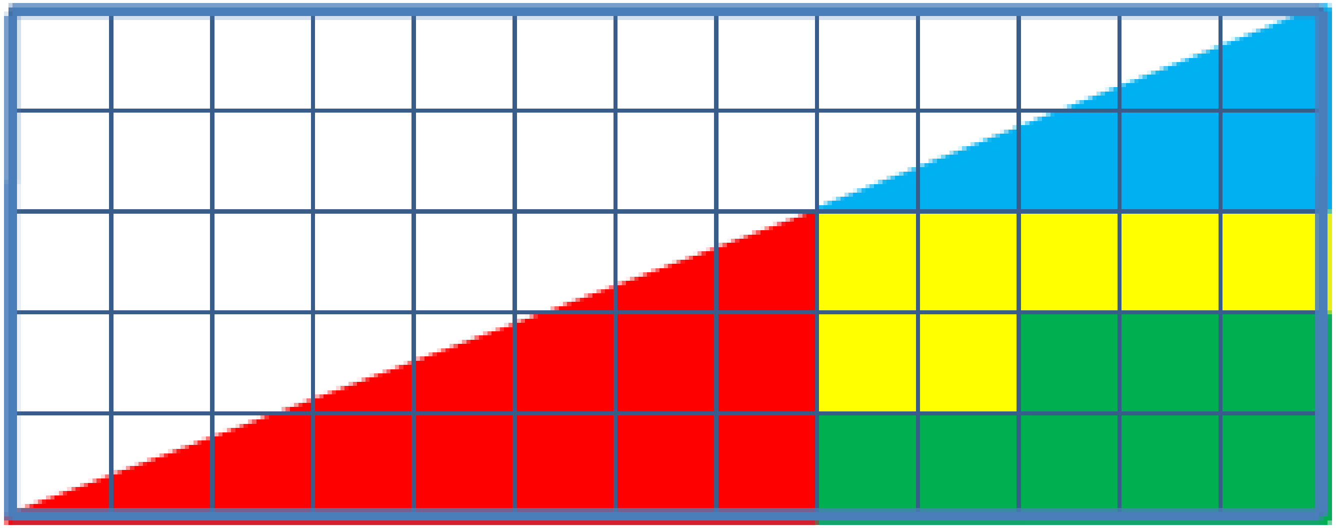
- Due to:
  Invalid mathematical proof
  logical demonstrations of absurdities

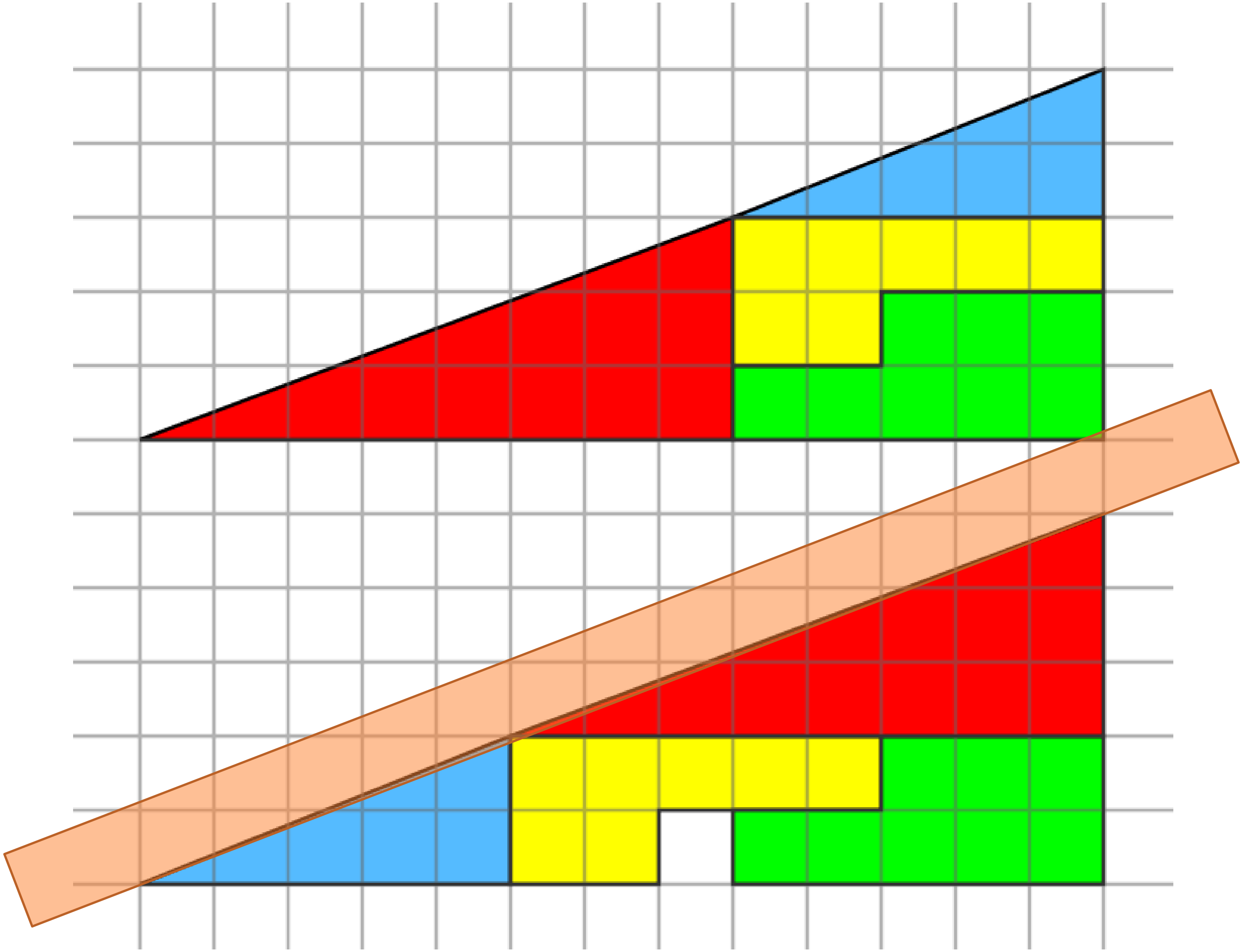
Example 1: 1=0 (?!)

- Let $x=0$
- $x(x-1)=0$
- $x-1=0$
- $x=1$
- $1=0$

What went wrong?

EXAMPLE 2: THE MISSING SQUARE (?!)





Mathematical Induction

- The principle of mathematical induction:
  For a statement involving positive integer n.
  a) check that the statement is true for $n = 1$.
  b) check that if the statement is true for $n = k$,
  it will ensure that $n = k+1$ is true.
  Then the statement is true for all positive integer n.
- Suppose there are n balls in a box such that.
  If you are ensured that you pick a ball from the box with a certain color, then the next ball must be of the same color. The first ball you pick is a red ball. Then ……

A wrong induction proof

- If there are n (> 0) people in the this room, then they are of the same gender.

Proof by Induction

- If there is one person only, then the statement is true.
- We show that if k people in this room have the same gender, then k+1 people in this room will have the same gender.

  Proof. For k+1 people, ask one person to leave the room. Then the k remaining people have the same gender.

  Now, ask the outside person to come back, and ask another person to leave the room. Then again the k remaining people have the same gender.

  So, …..

But we know, not all people in this room have the same Gender!

- What is wrong?

BARBER PARADOX (BERTRAND RUSSELL, 1901)

- Once upon a time… There is a town…
  - no communication with the rest of the world
  - only 1 barber
  - 2 kinds of town villagers:
    - Type A: people who shave themselves
    - Type B: people who do not shave themselves
  - The barber has a rule:

    He shaves Type B people only.

QUESTION:
WILL HE SHAVE HIMSELF?

- Yes. He will!
- No. He won't!

What's Wrong?

- **Which type of people does he belong to?**

ANTINOMY

- p -> p' and p' -> p
- p if and only if not p
- Logical Paradox

- More examples:
- **(1) Liar Paradox**
- *"This sentence is false." Can you state one more example for that paradox?*

- **(2) Grelling-Nelson Paradox**
- "Is the word 'heterological' heterological?"
- heterological(adj.) = not describing itself

- **(3) Russell's Paradox:**
- next slide....

RUSSELL'S PARADOX

- Discovered by Bertrand Russell at 1901
- Found contradiction on Naive Set Theory

If we define all mathematical entities as sets, and assume that there is a universal set U containing every sets.

Problem. Define a set R to be the elements in U such that x is not an element x.

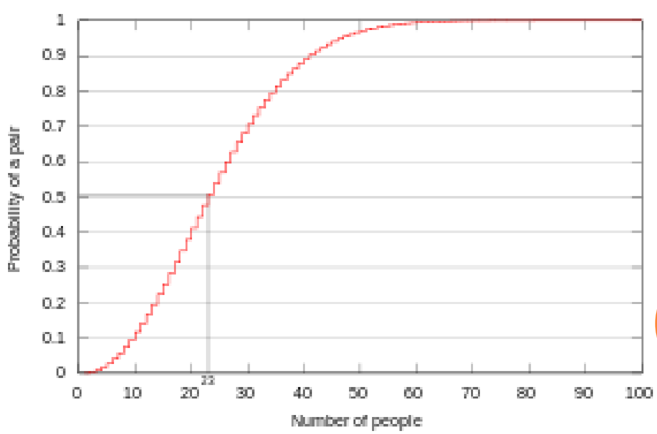
Question: Is R an element of R?

Birthday Paradox

- How many people in a room, that the probability of at least two of them have the same birthday, is more than 50%?

- Assumption:
  1. No one born on Feb 29
  2. No Twins
  3. Birthdays are distributed evenly.

  Formula: ???



3 Types of Paradox

- *Veridical Paradox*: contradict with our intuition but is perfectly logical

- *Falsidical paradox:* seems true but actually is false due to a fallacy in the demonstration.

- *Antinomy:* be self-contradictive

ADDITIONAL PARADOX

- Surprise test paradox

  The instructor says that he will give a surprise test in one of the lectures. Then ….

- Zeno's paradox (Zeno of Elea, 490–430 BC)

  In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead.

HOMEWORK

1. People from H village always tell the truth; people from L village always lie. If you have to decide to go left or go right to visit the H village, and seeing a person at the intersection who may be from H village or L village. What question should you ask the person to ensure that you will be told the right direction to the H village.

2. Consider the following proof of 2 = 1
   - Let $a = b$
   - $a^2 = ab$
   - $a^2 - b^2 = ab - ab^2$
   - $(a-b)(a+b) = b(a-b)$
   - $a + b = b$
   - $b + b = b$
   - $2b = b$
   - $2 = 1$

Which type of paradox is this?
Which part of the proof is wrong?


The End