The 13th Workshop on Numerical Ranges and Numerical Radii

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ABSTRACTS as of June 11

Name: **Natalia Bebiano**, bebiano@mat.uc.pt

Affiliation: Department of Mathematics, University of Coimbra, Portugal

Title: Fields of values of linear pencils and spectral inclusion regions

Abstract: We propose efficient methods for the numerical approximation of the field of values of the linear pencil $A - \lambda B$, when one of the matrix coefficients A or B is Hermitian and $\lambda \in \mathbb{C}$. Our approach builds on the fact that the field of values can be reduced under compressions to the bidimensional case, for which these sets can be exactly determined. The presented algorithms hold for matrices both of small and large size. Furthermore, we investigate spectral inclusion regions for the pencil based on certain fields of values. The results are illustrated by numerical examples. We point out that the given procedures complement the known ones in the literature.

Name: **Wai-Shun Cheung**, cheungwaishun@gmail.com

Affiliation: Department of Mathematics, The University of Hong Kong, Hong Kong

Title: On pair of matrices with convex numerical range

Abstract: We will have a discussion about the search of pairs of matrices A and B with convex $W_A(B)$.

Name: **Mao-Ting Chien**, mtchien@scu.edu.tw

Affiliation: Department of Mathematics, Soochow University, Taiwan

Title: Determinantal representation and numerical range

Abstract: Let A be an $n \times n$ matrix. A ternary polynomial associated to A, defined by $F_A(t, x, y) = \det(tI_n + xH + yK)$, is hyperbolic with respect to $(1, 0, 0)$, where $H = (A + A^*)/2$, $K = (A - A^*)/(2i)$. Kippenhahn proved that the numerical range of A is the convex hull of the real affine part of the dual curve of the algebraic curve $F_A(t, x, y) = 0$. The Fiedler-Lax conjecture is recently affirmed, namely, for any real ternary hyperbolic form $F(t, x, y)$, there exist real symmetric matrices H and K such that $F(t, x, y) = F_{H+iK}(t, x, y)$. We construct real symmetric matrices for the determinantal representations of some ternary polynomials and algebraic curves.

Co-author(s): Hiroshi Nakazato

Name: **Man-Duen Choi**, choi@math.toronto.edu

Affiliation: Department of Mathematics, University of Toronto,Canada

Title: Bewildered in Squares and circles

Abstract: The easy geometry of squares and circles has appeared naturally in the difficult theory of non-commutative harmonic analysis. Surprisingly, there remain simple questions (with unknown depth) of matrix theory.

Name: **Hwa-Long Gau**, hlgau@math.ncu.edu.tw

Affiliation: Department of Mathematics, National Central University, Taiwan

Title: Numerical Ranges of Row Stochastic Matrices

Abstract: In this talk, we consider properties of the numerical range of an n-by-n row stochastic matrix A. It is shown that the numerical radius of A satisfies $1 \le w(A) \le (1 + \sqrt{n})/2$, and, moreover, $w(A) = 1$ (resp., $w(A) = (1 + \sqrt{n})/2$) if and only if A is doubly stochastic (resp.,

$$A = \underset{j\text{th}}{\begin{bmatrix} & 1 & \\ 0 & \vdots & 0 \\ & 1 & \end{bmatrix}}$$

for some j, $1 \le j \le n$). A complete characterization of the A's for which the zero matrix of size $n-1$ can be dilated to A is also given. Finally, for each $n \ge 2$, we determine the smallest rectangular region in the complex plane whose sides are parallel to the x- and y-axis and which contains the numerical ranges of all n-by-n row stochastic matrices.

Co-author(s): Kuo-Zhong Wang, Pei Yuan Wu

Name: **Peng-Ruei Huang**, h16ds202@hirosaki-u.ac.jp

Affiliation: Graduate School of Science and Technology, Hirosaki University, Hirosaki, Japan

Title: Product range of a 3×3 normal matrix

Abstract: The k^{th} product range of a matrix A is defined as the set of the products of the first k^{th} diagonal entries of all unitary similarities of A. In this talk, the convexity of the k^{th} product range of a 3×3 normal matrix will be discussed. The convexity of the k^{th} product range of a normal matrix whose eigenvalues form an acute-angled or right-angled triangle will be characterized.

Co-author(s): Hiroshi Nakazato (HiroDai).

Name: **Chi-Wai Leung**, cwleung@math.cuhk.edu.hk

Affiliation: Department of Mathematics, The Chinese University of Hong Kong, Hong Kong

Title: Norm-Attaining Property for a Dual Pair of Banach Spaces

Abstract: In this talk, we introduce the norm attaining property for a dual pair of Banach spaces. We will use this property to determine a quotient Banach space being a dual space. We will apply this to give another proof of the James's characterization theorem about the reflexivity of a separable Banach space. Also the norm attaining property of Fourier spaces is studied.
This is the joint work with Cheuk-Yin Lee.

Co-author(s): Cheuk-Yin Lee

Name: **Chi-Kwong Li**, ckli@math.wm.edu

Affiliation: Department of Mathematics, College of William and Mary, USA

Title: Numerical ranges and Parallel distinguishability of quantum operations

Abstract: Numerical range techniques are used to study the perfect distinguishability of two quantum operations by a parallel scheme. Recent results and open problems will be mentioned.

Co-author(s): Runyao Duan, Cheng Guo, and Yinan Li

Name: **Minghua Lin**, mlin87@ymail.com

Affiliation: Shanghai University

Title: On the geometric mean of accretive matrices

Abstract: A square (complex) matrix is accretive if its numerical range is contained on the right half of the complex plane. The geometric mean of accretive matrices was recently brought in by Drury [Linear and Multilinear Algebra 63 (2015) 296-301], in this talk, I will report a new property on the geometric mean. Some related results will be mentioned if time permits.

Name: **Hiroshi Nakazato**, nakahr@hirosaki-u.ac.jp

Affiliation: Department of Mathematical Sciences, Hirosaki University, Japan

Title: Riemann matrices for the Hyperbolic Curves

Abstract: The Riemann matrix of a Riemann surface provides a new information to classify the numerical ranges of matrices.

Co-author(s): This presentation is based on some joint works with Professor Mao-Ting Chien.

Name: **Rajesh Pereira**, pereirar@uoguelph.ca

Affiliation: Department of Mathematics and Statistics, University of Guelph, CANADA

Title: Numerical Ranges and Subspace approaches to Anticoherence and Spherical Designs.

Abstract: We study subspaces of the vector space of spin-s states which have the property that all of their elements are scalar multiples of anticoherent subspaces. We give a characterization of these anticoherent subspaces in terms of certain joint higher rank numerical ranges. We also look at examples of subspaces of the vector spaces of spin-s states all of whose elements have Majorana

representations that are spherical designs. Concrete examples of such subspaces are given.

Co-author(s): Connor-Paul Paddock.

Name: **Sarah Plosker**, ploskers@brandonu.ca

Affiliation: Department of Mathematics and Computer Science, Brandon University, Canada

Title: The probability of quantum state transfer: a matrix analysis approach

Abstract: A quantum spin chain has been proposed as a "data bus" for quantum state transfer within a quantum computer. This talk is an introduction to quantum state transfer, given from a matrix analysis point of view. Viewing quantum state transfer through the use of an adjacency or Laplacian matrix, we calculate bounds on the probability of state transfer in the presence of two types of errors: perturbations in readout time and perturbations in edge weight. At the heart of our analysis is the numerical range of a matrix and matrix norms.

Co-author(s): Whitney Gordon (Brandon University), Steve Kirkland (University of Manitoba), Chi-Kwong Li (College of William and Mary), and Xiaohong Zhang (University of Manitoba)

Name: **Edward Poon**, poon3de@erau.edu

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Title: Linear maps preserving a relative numerical range

Abstract: Given a subgroup G of the unitary group $U(n)$ and a Hermitian matrix C, the relative C-numerical range of a matrix A relative to G is the set of complex numbers $W_C^G(A) = \{trAUCU^* : U \in G\}$. In this talk, we characterize the linear preservers of a relative numerical range when G is a particular type of abelian group.

Name: **Yiu-Tung Poon**, ytpoon@iastate.edu

Affiliation: Department of Mathematics, Iowa State University, USA

Title: Normal compression and higher rank numerical range

Abstract: Given an $n \times n$ matrix A. A k-dimensional normal compression of

A is a normal matrix of the form U^*AU where U is an $n \times k$ matrix satisfying $U^*U = I_k$. We will study generalization of normal compression to an m-tuple of Hermitian matrices and its connection with the higher rank numerical range.

Co-author(s): Chi-Kwong Li and Nung-Sing Sze.

Name: **Panayiotis J. Psarrakos**, ppsarr@math.ntua.gr

Affiliation: National Technical University of Athens, Greece

Title: Birkhoff-James approximate orthogonality sets

Abstract: The *numerical range* of a square matrix $A \in \mathbb{C}^{n\times n}$ is the compact and convex set $F(A) = \{x^*Ax \in \mathbb{C} : x \in \mathbb{C}^n,\, x^*x = 1\}$, and it has been studied extensively for many decades and is useful in studying matrices and operators. Stampfli and Williams (1968) observed that $F(A)$ can be written

$$\begin{aligned} F(A) &= \{\mu \in \mathbb{C} : \|A - \lambda I_n\|_2 \geq |\mu - \lambda|, \forall\, \lambda \in \mathbb{C}\} \\ &= \bigcap_{\lambda\in\mathbb{C}} \{\mu \in \mathbb{C} : |\mu - \lambda| \leq \|A - \lambda I_n\|_2\}\,. \end{aligned}$$

Hence, $F(A)$ is an infinite intersection of closed (circular) disks $\mathcal{D}\left(\lambda, \|A - \lambda I_n\|_2\right) = \{\mu \in \mathbb{C} : |\mu - \lambda| \leq \|A - \lambda I_n\|_2\}$ $(\lambda \in \mathbb{C})$. Inspirited by this intersection property, Chorianopoulos, Karanasios and Psarrakos (2009) introduced a definition of numerical range for rectangular complex matrices. In particular, for any $A, B \in \mathbb{C}^{n\times m}$ with $B \neq 0$, and any matrix norm $\|\cdot\|$, the *numerical range of A with respect to B* is defined as

$$\begin{aligned} F_{\|\cdot\|}(A;B) &= \{\mu \in \mathbb{C} : \|A - \lambda B\| \geq |\mu - \lambda|, \forall\, \lambda \in \mathbb{C}\} \\ &= \bigcap_{\lambda\in\mathbb{C}} \mathcal{D}\left(\lambda, \|A - \lambda B\|\right). \end{aligned}$$

This set is obviously compact and convex, and it satisfies basic properties of the standard numerical range. Moreover, it is nonempty if and only if $\|B\| \geq 1$.

In this work, we introduce a new range of values for rectangular matrices and matrix polynomials, which is based on the notion of Birkhoff-James approximate orthogonality and generalizes the numerical range $F_{\|\cdot\|}(A;B)$. We show that the new set is quite rich in structure by establishing some of its main properties. We also study the boundary points and the case of matrix polynomials, and we give illustrative examples to verify our results.

Co-authors: Christos Chorianopoulos, Vasiliki Panagakou

Name: **Ilya M. Spitkovsky**, ims2@nyu.edu and ilya@math.wm.edu

Affiliation: New York University Abu Dhabi (NYUAD), UAE, and
The College of William and Mary, USA

Title: On the normalized numerical range

Abstract: The normalized numerical range of an operator A is defined as the set $F_N(A)$ of all the values $\langle Ax, x\rangle/||Ax||$ attained by unit vectors $x \notin \ker A$. We prove that $F_N(A)$ is simply connected, establish conditions for it to be star-shaped with the center at zero, to be open, closed, and to have empty interior. For some classes of operators (weighted shifts, isometries, essentially Hermitian) the complete description of $F_N(A)$ is obtained.

Co-author: Andrei-Florian Stoica (NYUAD).

Name: **Raymond Nung-Sing Sze**, raymond.sze@polyu.edu.hk

Affiliation: Department of Applied Mathematics, The Hong Kong Polytechnic University, Hong Kong

Title: Higher rank numerical range

Abstract: Fixed a positive integer k, the rank-k (higher rank) numerical range of a matrix A is the set of complex λ such that $PAP = \lambda P$ for some rank k projection P. The joint higher rank numerical range of matrices $A_1, \ldots, A_m$ can be defined in a similar way. In this talk, we will review some results on (joint) higher rank numerical range and discuss its possible open problems.

Name: **Bit-Shun Tam**, bsm01@mail.tku.edu.tw

Affiliation: Department of Mathematics, Tamkang University

Title: Nullities of graphs revisited

Abstract: By exploiting the canonical star associated with a rooted tree and the canonical unicyclic graph associated with a unicyclic graph, we offer more conceptual, more transparent or shorter proofs for many known results on nullities of unicyclic graphs. Some new results are also found.

This is a joint work with Tsu-Hsien Huang.

Name: **Tin-Yau Tam**, tamtiny@auburn.edu

Affiliation: Department of Mathematics and Statistics, Auburn University, USA

Title: Every Toeplitz matrix is unitarily similar to a complex symmetric matrix

Abstract: We prove that Toeplitz matrices are unitarily similar to complex symmetric matrices. Moreover, an $n \times n$ unitary matrix that uniformly turns all $n \times n$ Toeplitz matrices via similarity to complex symmetric matrices is explicitly given. When $n \leq 3$, we prove that each complex symmetric matrix is unitarily similar to some Toeplitz matrix, but the statement is false when $n > 3$. We make use of the convexity property of some generalized numerical range to obtain some results.

Co-author(s): Mao-Ting Chien, Jianzhen Liu, Hiroshi Nakazato

Name: **Ming-Cheng Tsai**, mctsai2@gmail.com

Affiliation: General Education Center, National Taipei University of Technology, Taiwan

Title: Numerical Ranges of 4-by-4 Nilpotent Matrices

Abstract: In their 2008 paper Gau and Wu conjectured that the numerical range of a 4-by-4 nilpotent matrix has at most two flat portions on its boundary. We prove this conjecture, establishing along the way some additional facts of independent interest.

Co-author(s): Erin Militzer, Linda J. Patton, Ilya M. Spitkovsky

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Name: **Nam-Kiu Tsing**, nktsing@gmail.com

Affiliation: Department of Mathematics, The University of Hong Kong, Hong Kong

Title: The L-numerical range

Abstract: Let H_n be the set of all $n \times n$ Hermitian matrices, and H_n^m be the set of all m-tuples of matrices in H_n. For any $\mathbf{A} = (A_1, A_2, \ldots, A_m) \in H_n^m$, we define the unitary orbit of $\mathbf{A}$ as

$$U(\mathbf{A}) := \{(U^*A_1U, U^*A_2U, \ldots, U^*A_mU) : U \text{ is an } n \times n \text{ unitary matrix}\}.$$

Noting that H_n^m is a real vector space, we consider real linear maps $L : H_n^m \to \mathbb{R}^\ell$ which map from H_m^n to $\mathbb{R}^\ell$, and define the L-numerical range of $\mathbf{A}$ as

$$W_L(\mathbf{A}) := L\left(U(\mathbf{A})\right),$$

which can be regarded as a generalization of the c-numerical range, the C-numerical range, the joint numerical range, etc. We shall present some results on the L-numerical range pertaining to its geometric properties such as convexity and star-shapedness.

Name: **Kuo-Zhong Wang**, kzwang@math.nctu.edu.tw

Affiliation: Department of Applied Mathematics, National Chiao Tung University, Taiwan

Title: Numerical ranges of the product of operators

Abstract: We study containment regions of the numerical range of the product of operators A and B such that $W(A)$ and $W(B)$ are line segments. It is shown that the containment region is equal to the convex hull of elliptical disks determined by the spectrum of AB, and conditions on A and B for the set equality holding are obtained. The results cover the case when A and B are self-adjoint operators extending the previous results on the numerical range of the product of two orthogonal projection.

Co-author(s): Hongke Du, Chi-Kwong Li, Yueqing Wang, Ning Zuo

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Name: **Ya-Shu Wang**, yashu@nchu.edu.tw

Affiliation: Department of Applied Mathematics, National Chung Hsing University, Taiwan

Title: Holomorphic maps between Fourier algebras

Abstract: In this talk, we will introduce the Fourier algebra $A(G)$ and the representation of linear orthogonally multiplicative maps on $A(G)$. We also present the representation of the holomorphic maps on the Fourier algebras $A(G)$.

Co-author(s): Ngai-Ching Wong

Name: **Pei Yuan Wu**, pywu@math.nctu.edu.tw

Affiliation: Department of Applied Mathematics, National Chiao Tung University, Taiwan

Title: Extremality of Numerical Radii of Matrix Products

Abstract: For any two n-by-n matrices A and B, it is known that (1) $w(AB)$, the numerical radius of the product of A and B, is at most $4w(A)w(B)$, (2) $w(AB)$ is at most $2w(A)wB)$ if A and B commute, and (3) $w(AB)$ is at most $w(A)w(B)$ if A and B are 2-by-2 matrices or are n-by-n matrices with one of them normal. In this talk, we give complete characterizations of A and B for which the inequalities become equalities.

Name: **Fuzhen Zhang**, zhang@nova.edu

Affiliation: Department of Mathematics, Nova Southeastern University, Fort Lauderdale, Florida, USA.

Title: Schur power matrix and some related research problems

Abstract: Let A be an n-by-n square matrix. The Schur power matrix of A, denoted by S(A), is an n!-by-n! matrix whose entries are indexed by the elements of the symmetric group of degree n. Soules introduced such notion in order to prove the permanent dominance (Liebs) conjecture which asserts that, for positive semidefinite A, the permanent of A dominates all normalized generalized matrix functions (in terms of subgroup and character). Soules conjectured that the largest eigenvalue of the Schur power matrix is the permanent of A (this is the so-called permanent-on-top, POT, conjecture). Very recently, a counterexample was provided to disprove the POT conjecture. However, as a principal submatrix of the n-fold tensor product of A, S(A) has many important and nice properties; for instance, both determinant and permanent are eigenvalues of S(A).

We will revisit the power matrix S(A) and propose several research problems.

Name: **Karol Zyczkowski**, karol.zyczkowski@uj.edu.pl

Affiliation: Jagiellonian University, Poland

Title: On joint numerical range of three hermitian operators of order three

Abstract:
