Sufficient Condition for $AB = BA$
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Let $\mathcal{H}$, $\mathcal{B}(\mathcal{H})$ be separable complex Hilbert space and the set of all bounded linear operators on $\mathcal{H}$, respectively. The elements of $\mathcal{E}(\mathcal{H}) = \{A \in \mathcal{B}(\mathcal{H}) : 0 \leq A \leq I\}$ are called quantum effects. For $A, B \in \mathcal{E}(\mathcal{H})$, the sequential product of $A$ and $B$ is $A \circ B = A^{1/2}BA^{1/2}$ and the Jordan product of $A$ and $B$ is $A \ast B = \frac{AB + BA}{2}$. Many of results show that algebraic conditions on $A \circ B$ or $A \ast B$ imply that $AB = BA$ [?]-[?]. There are some questions as following.

1. Can we get that $A_i, 1 \leq i \leq n$ are commutative if sequential product $A_n \circ A_{n-1} \circ \cdots \circ A_2 \circ A_1 = A_n^{1/2}A_{n-1}^{1/2} \cdots A_2^{1/2}A_1^{1/2} \cdots A_{n-1}^{1/2}A_n^{1/2}$ satisfies certain distributive or associative laws.

2. Can we get that $A_i, 1 \leq i \leq n$ are commutative if sequential product $A_n \ast A_{n-1} \ast \cdots \ast A_2 \ast A_1$ satisfies certain distributive or associative laws.

References