Math 214 – Foundations of Mathematics  Homework 1  Your name

Solve the following problems. Six points for each problem.

1. Write each of the following sets as specified.
   (a) List the elements in the set \( A = \{ n \in \mathbb{N} : n^3 < 1000 \} \).
   (b) Describe the set \( B = \{-2, -1, 0, 1, 2, 3, 4\} \) using the notation 
       \( B = \{ n : p(n) \} \), where \( p(n) \) specifies the property of element \( n \).

2. Recall that for a set \( A \), \( \mathcal{P}(A) \) denotes the power set of \( A \).
   (a) Find \( \mathcal{P}(\mathcal{P}(\{a\})) \) and its cardinality.
   (b) Give an example of a set \( S \) such that \( S \in \mathcal{P}(\mathbb{N}) \) and \( |S| = 6 \).
   (c) Give an example of a set \( S \) such that \( S \subseteq \mathcal{P}(\mathbb{N}) \) and \( |S| = 6 \).

3. The following problems involve set operations.
   (a) Give an example of three non-empty sets \( A, B \), and \( C \) such that \( B \neq C \) but \( B - A = C - A \).
   (b) Let \( A = \{\emptyset, \{\emptyset\}\} \). Find \( \mathcal{P}(A) - A \).

4. For a real number \( r \), define \( S_r \) to be the interval \([r - 1, r + 2)\). Let \( A = \{1, 3, 4\} \).
   (a) List the intervals \( S_r \) for \( r \in A \).
   (b) Determine \( \bigcup_{a \in A} S_a \) and \( \bigcap_{a \in A} S_a \).
   (c) (Extra 3 points) Let \( B = [0, 1] \). Determine \( \bigcap_{r \in B} S_r \) and \( \bigcup_{r \in B} S_r \). (Give an explanation.)

5. For two sets \( A \) and \( B \), let \( A \times B = \{(a, b) : a \in A, b \in B\} \) be their Cartesian product.
   (a) Let \( A = \{a, b\} \), \( B = \{c, d\} \), and \( A \times B \). Determine \( A \times \mathcal{P}(A) \).
   (b) Let \( A = \{0, 1\} \) and \( B = [0, 2] \cap [1, 3] \). Describe geometrically the set \( A \times B \) in \( \mathbb{R}^2 = \mathbb{R} \times \mathbb{R} \).
      (The set is line segment, two line segments, a circular arc, or ???)
   (c) Let \( A = \{0, 1\}, B = (0, 1) \cap A \) and \( C = \mathbb{R} \). What is \( A \times B \times C \)?

6. Determine all different partitions of the set \( \{1, 2, 3\} \).

Extra credit problem. (3 points) If a set \( A \) has \( n \) elements, show that \( \mathcal{P}(A) \) has \( 2^n \) elements.