MATH 309, HW 2 solution

I.2 #7 Consider: $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, then $AB = 0$, which for sure gives a smaller column space.

I.3 #4 No, consider: $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ satisfies requirements, but it's not symmetric.

I.3 #7 Consider: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$. It is clear that they don't have the same null space.

I.3 #10 Let A $\in M_{m,n}$, then B $\in M_{m,3n}$. Let v_1, v_2, $v_3 \in C^n$. Then the vector $V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ will be in the null space of B if $v_1 + v_2 + v_3 = 0$.

I.4 #3 LU factorization: $A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{bmatrix}$, so $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$.

I.4 #5 (1) d has to be 0 since $1 \cdot d + 0 \cdot 0 = 0$. But $d \cdot l + 1 \cdot 0 = 2$, which means d can't be 0, so there's a contradiction and we cannot do LU factorization.

(2) From A, we can tell that d=1, e=1, l=1 and f=0. So $U = \begin{bmatrix} 1 & 1 & g \\ 0 & 0 & h \\ 0 & 0 & i \end{bmatrix}$. According to u_{31} of A, m=1. Then according to u_{32} of A, $1 \cdot 1 + n \cdot 0 + 1 \cdot 0 = 2$, which is impossible. Thus, LU decomposition is impossible.

I.4 #6 c=2 will lead to the second pivot position being 0, and c=1 will make the third pivot position 0.

I.4 #7 After doing LU factorization, we can tell the conditions from the pivot position: $a \neq 0$, and $a \neq b, b \neq c, c \neq d$.

I.4 #10 A_k factors into $L_k U_k$ because the upper left corner of L and U are derived from A_k.

I.4 #11 We can permute A by a column change and a row change: $P_1 A P_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$. Then, $A - \begin{bmatrix} 1 \\ \frac{3}{4} \end{bmatrix} \begin{bmatrix} 4 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$.

And we can get LU factorization: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{4} & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 0 & -\frac{1}{2} \end{bmatrix} = LU$