MATH 309 HW9 Solution

I.12 #1 $||A - UV||_F^2 = a^2 - 2av_1u_1 + v_1^2u_1^2 + b^2 - 2bv_1u_2 + v_1^2u_2^2$.

If we want the minimum, we can take the derivative with respect to v_1, and set it to be zero: $-2au_1 + 2v_1u_1^2 - 2bu_2 + 2v_1u_2^2 = 0$.

So $v_1(u_1^2 + u_2^2) = au_1 + bu_2$.

I.12 #2 The point that $a_1 - v_1u$ is perpendicular to v_1u minimizes $||a_1 - v_1u||^2$. Any point else will result in a larger length of $a_1 - v_1u$. This basically means that $a_1 - v_1u$ is perpendicular to u. Thus, $u^T(a_1 - v_1u) = 0$, and we get $u^Ta_1 = u^Tuv_1$ as stated in the first problem.

I.12 #3 Using the same method as #1, we can see v_2 that satisfies $u^Ta_2 = u^Tuv_2$ will minimize $||a_2 - v_2u||^2$.

I.12 #4 We can again expand $||A - UV||_F^2$ and take derivative with respect to u_1 and u_2. This will give us $u_1 = \frac{av_1+cv_2}{v_1^2+v_2^2}$ and $u_2 = \frac{bv_2+dv_2}{v_1^2+v_2^2}$.

Another way to achieve this: think of the first row: the best u_1 will occur when $a_1^Tv = (v^Tv)u_1$, and the best u_2 will occur when $a_2^Tv = (v^Tv)u_2$. Thus, we can solve $(V^TV)U = AV$ to get the best U.