Math 323 Operations Research Quiz 2 Name: Sample solution

Consider the optimization problem max $Z = c_1x_1 + c_2x_2 + c_3x_3$ subject to:

$$3x_1 + x_2 + x_3 \le 60, \quad x_1 - x_2 + 2x_3 \le 10, \quad x_1 + x_2 - x_3 \le 20, \quad x_1, x_2, x_3 \ge 0.$$

Using slack variable $s_1, s_2, s_3 \ge 0$, we get the initial tableau:

C_B	B	c_1x_1	$+c_2x_2$	$+c_3x_3$	$+0s_1$	$+0s_2$	$+0s_3$	constraints
0	s_1	3	1	1	1	0	0	60
0	s_2	1	-1	2	0	1	0	10
0	s_3	1	1	-1	0	0	1	20
	$\bar{C}$	c_1	c_2	c_3	0	0	0	$Z = 0$

$$B = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 1 & -1 & -2 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

1. Fill in the missing entries if the final tableau has the form:

C_B	B	c_1x_1	$+c_2x_2$	$+c_3x_3$	$+0s_1$	$+0s_2$	$+0s_3$	constraints
0	s_1	0	0	1	1	-1	-2	10
c_1	x_1	1	0	$1/2$	0	$1/2$	$1/2$	15
c_2	x_2	0	1	$-3/2$	0	$-1/2$	$1/2$	5
	$\bar{C}$	0	0	$c_3-\frac{(c_1-3c_2)}{2}$	0	$0-\frac{(c_1-c_2)}{2}$	$0-\frac{(c_1+c_2)}{2}$	

Note that the entries in the row $\bar{C}$ should be in terms of c_1, c_2, c_3.

$Z=0$ is impossible because $c_1 x_1 + c_2 \cdot x_2$
$= c_1 \cdot 15 + c_2 \cdot 5 = 0$
$\Rightarrow c_2 = -5c_1$
$\Rightarrow -(c_1+c_2) \ge 0$ or $-(c_1-c_2) \ge 0$

2. Determine the condition on c_1, c_2, c_3 such that the solution in the final tableau is unique.

$$c_3 - \frac{(c_1-3c_2)}{2} < 0, \quad -\left(\frac{c_1-c_2}{2}\right) < 0, \quad -\left(\frac{c_1+c_2}{2}\right) < 0$$

3. Give ONE example of c_1, c_2, c_3 such that the solution in the final tableau is NOT unique.

Let $c_1 = 1$, $c_2 = -1$, $c_3 = 3$. Then $\bar{C} = (0, 0, 0, 0, -1, 0)$

Pivot the $(3,6)$ entry, we get last column $\begin{pmatrix} 30 \\ 10 \\ 10 \end{pmatrix}$

So $(x_1, s_1, s_3) = (10, 30, 10)$ and $(x_1, x_2, s_1) = (15, 5, 10)$ gives the same $Z = 10$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & -1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & -2 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} \end{array}\right]; \quad B^{-1}\left(\begin{array}{ccc|c} 1 & 0 & 0 & 60 \\ 2 & 1 & 0 & 10 \\ -1 & 0 & 1 & 20 \end{array}\right) = \begin{pmatrix} 1 & -1 & -2 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}\left(\begin{array}{ccc|c} 1 & 0 & 0 & 60 \\ 2 & 1 & 0 & 10 \\ -1 & 0 & 1 & 20 \end{array}\right)$$