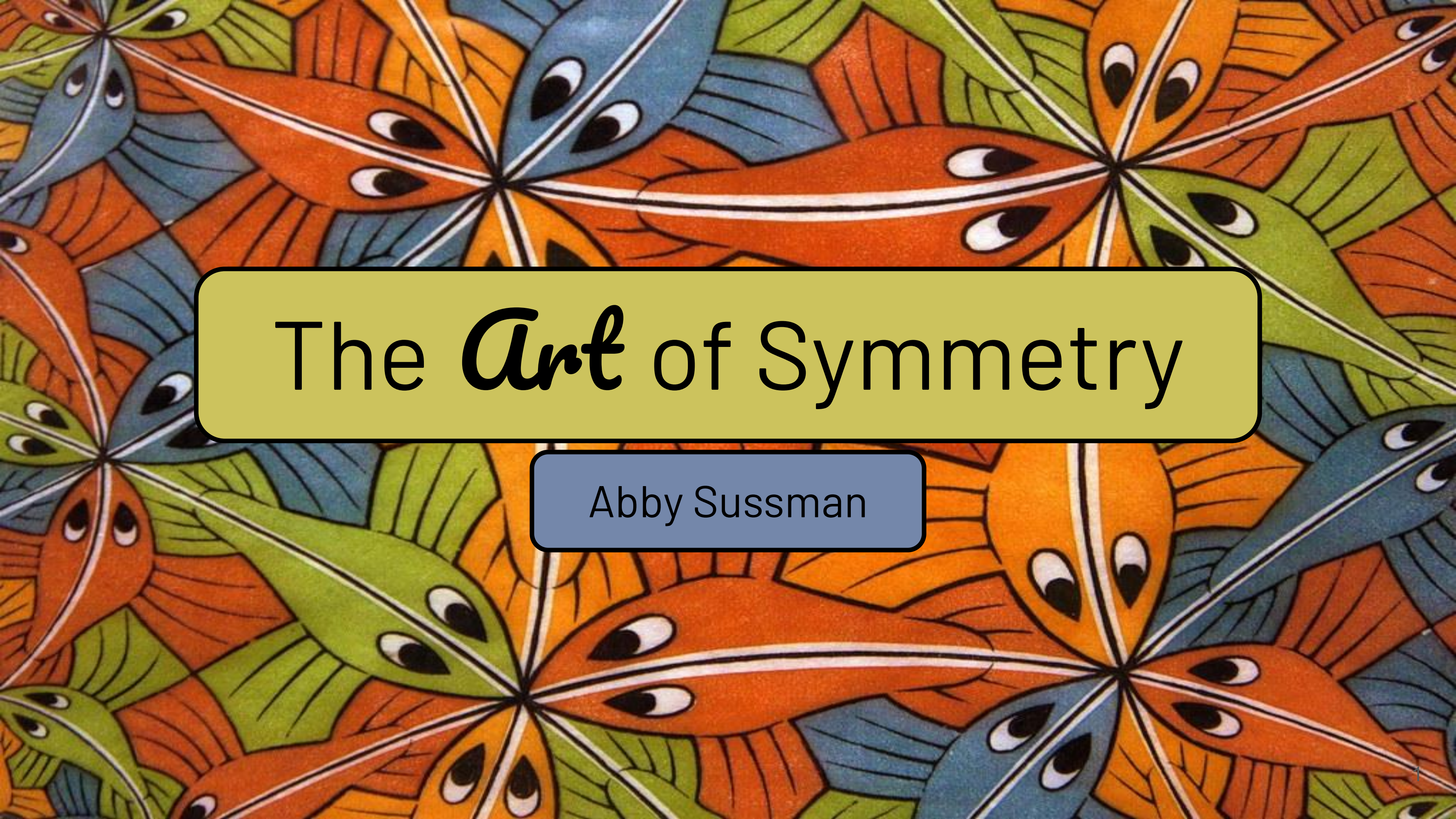
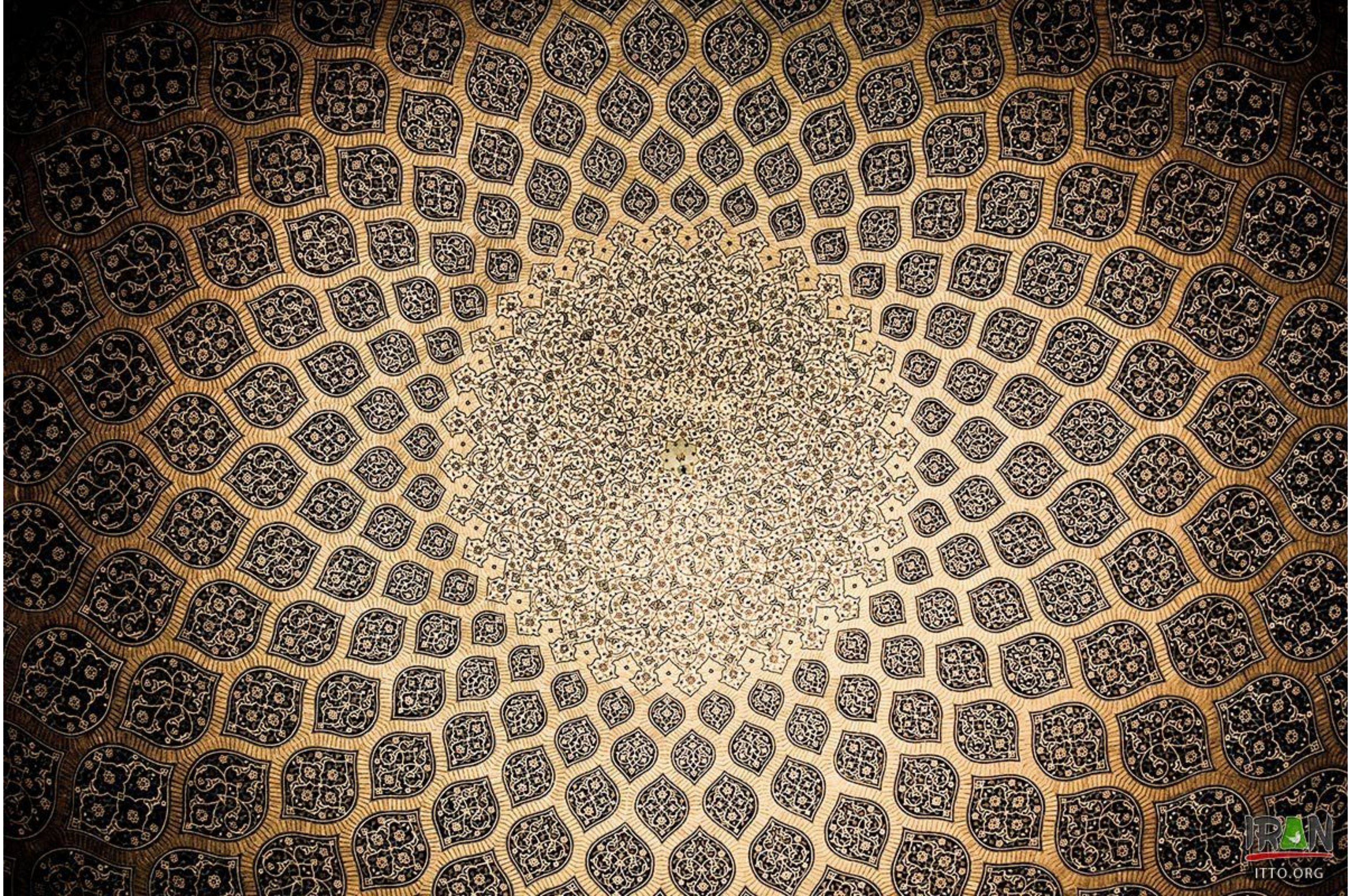
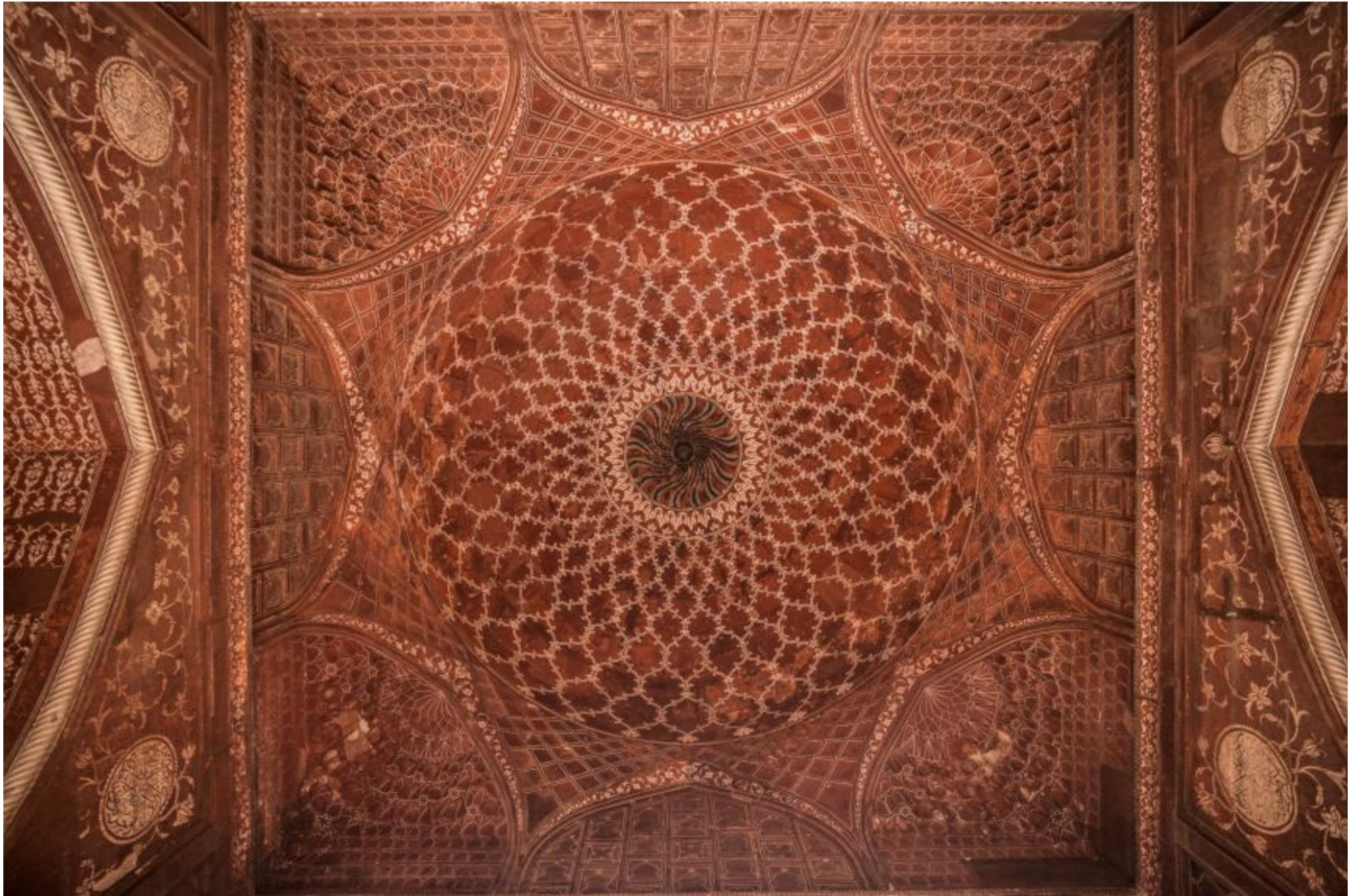


The *Art* of Symmetry

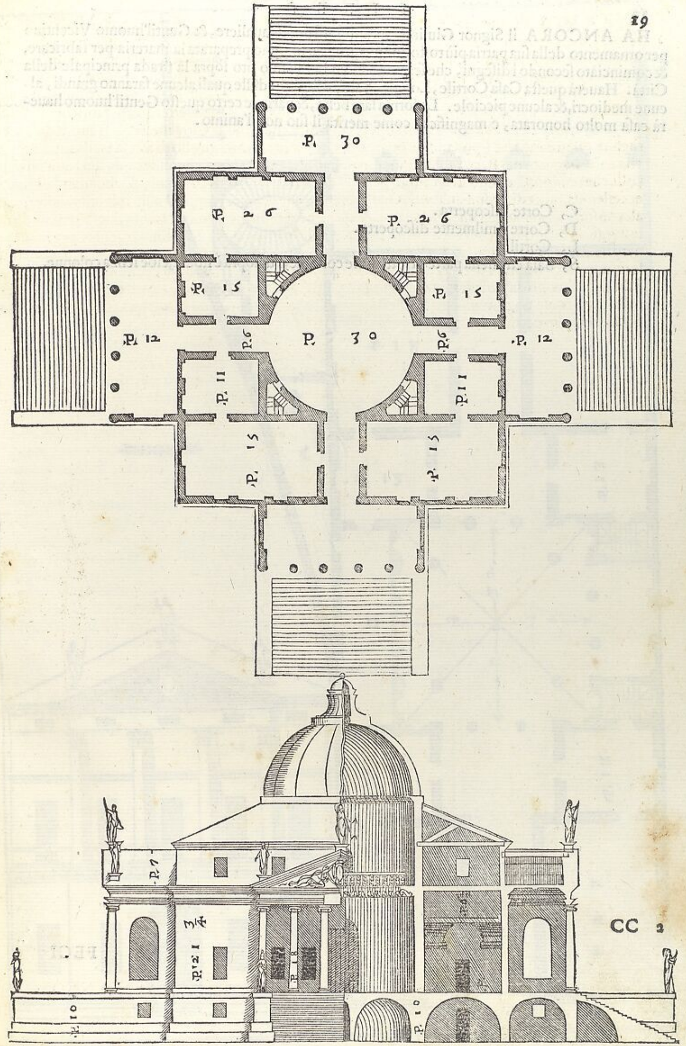
Abby Sussman

Pretty Pictures


ITTO.ORG




474
אני שמואל בן יעקב
כתבתי ונקדתי ומס׳
זה המצחף לכבוד
רבנא מבורך הכהן
בן יוסף הידוע בן
אדאי יברכהו וחי



CC 2

What Is Symmetry?

Symmetry

In the simplest (math) terms, a symmetry is a distance preserving bijection.

- If you do something to an object, it looks the same afterwards.

Three main types of symmetry

- Reflectional: line dividing an object into two mirror halves
- Rotational: an object can be rotated without changing the overall shape
- Translational: an object can be moved without changing its overall shape

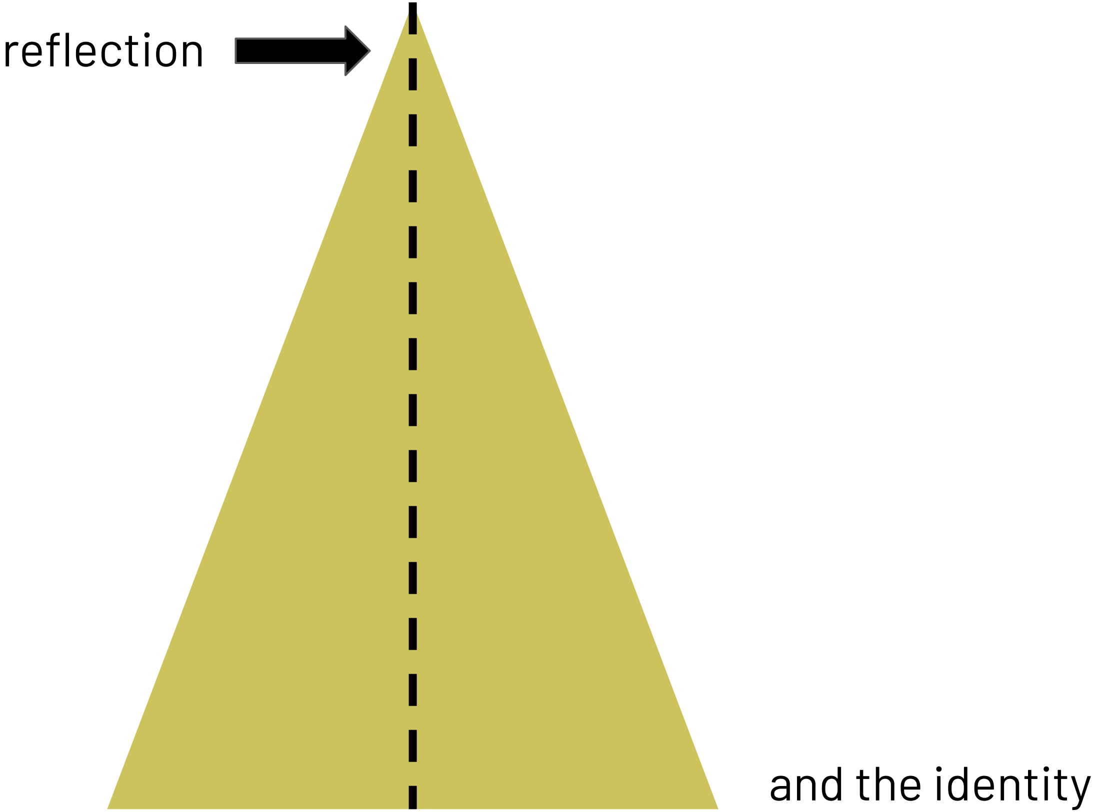
Symmetry

Other types of symmetry

- Glide reflection: a reflection and a translation
- Helical: an object can be simultaneously rotated and translate in 3D space
  - Screw and springs
- Scale: contraction or expansion of an object doesn’t change its overall shape
  - Fractals!
- Identity: do nothing ☺️
  - Every object has this symmetry

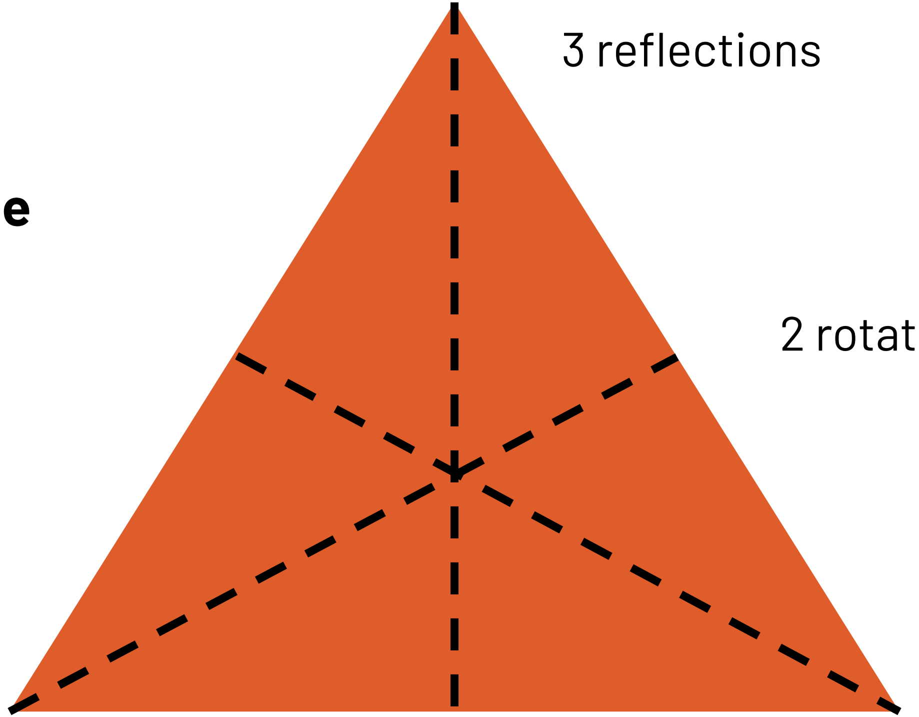
Isosceles triangle

- 2 symmetries



Equilateral triangle

- 6 symmetries



3 reflections

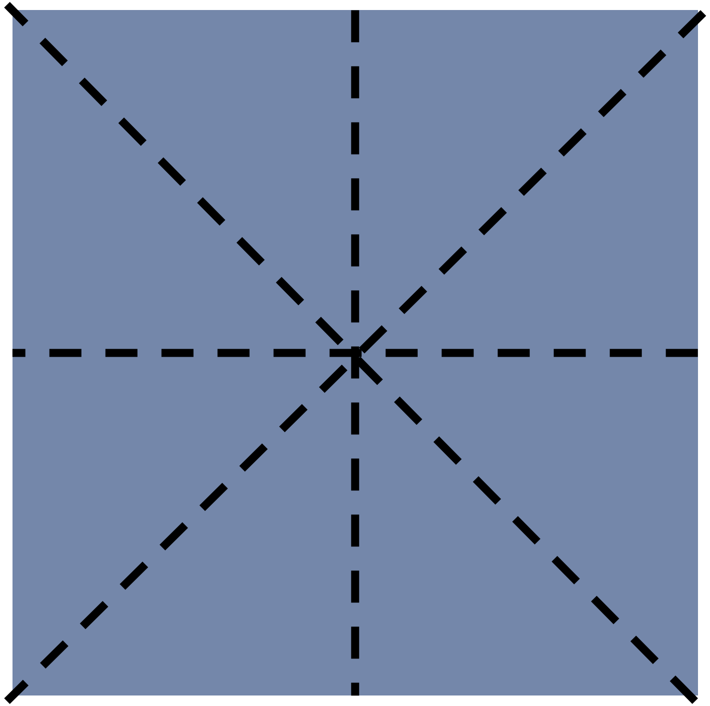
2 rotations

and the identity

Square

- 8 symmetries

4 reflections

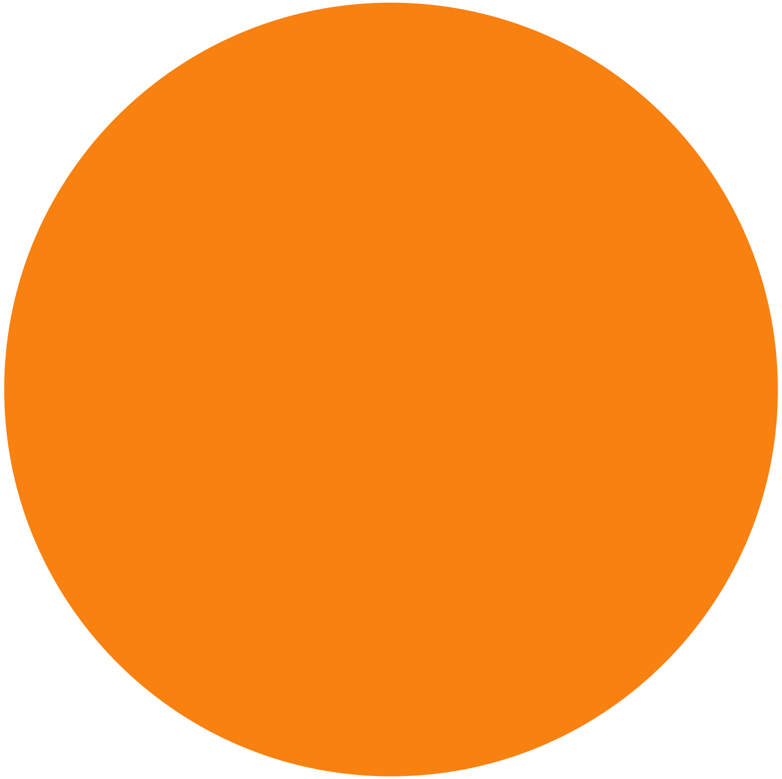


3 rotations

and the identity

Circle

- Infinite symmetries

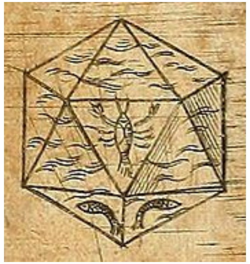


3D Symmetry

The most symmetrical 3D objects are the platonic solids (plus the sphere)

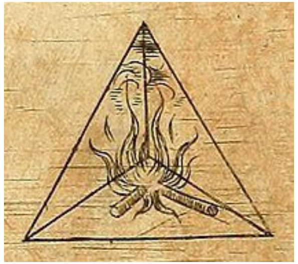
- Regular polygon faces
  - Same length edges
  - Same angles
  - Same amount of faces around every vertex

cube

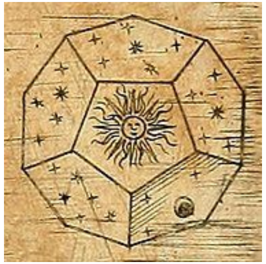


icosahedron

octahedron



tetrahedron



dodecahedron

What Is a Group?

Groups

A group is a non-empty set G with a binary operation "+" such that the following axioms are satisfied:

- Closure: for all a,b in G, $a + b$ is also in G
- Associativity: for all a,b,c in G we have $(a + b) + c = a + (b + c)$
- Identity: there exists an e in G such that for all a in G we have $a + e = a$
- Inverse: for all a in G we have a unique $-a$, or a^{-1}, such that $a + (-a) = e$
    - Combining an element with its inverse gets you the identity

Some symmetries (reflections, 180 degree rotations) are their own inverses

- These are called "involutions"
- If G is a finite group, with an even number of elements, then G has an involution

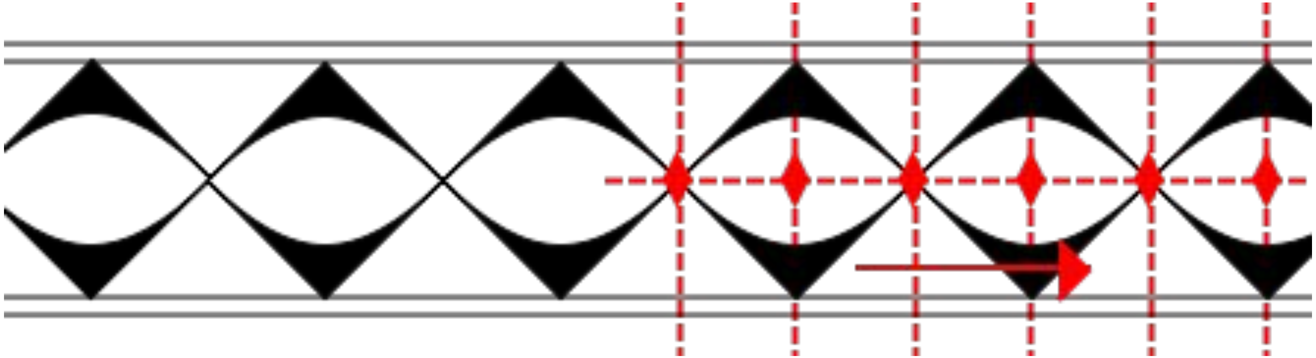
Groups

Examples of groups:

- The integers under addition (Z, +)
- Dihedral groups
  - A regular *n*-gon has 2*n* symmetries
  - *n* reflections, and *n* rotations (the identity is rotating the shape all the way around)
- Frieze groups
- Wallpaper groups

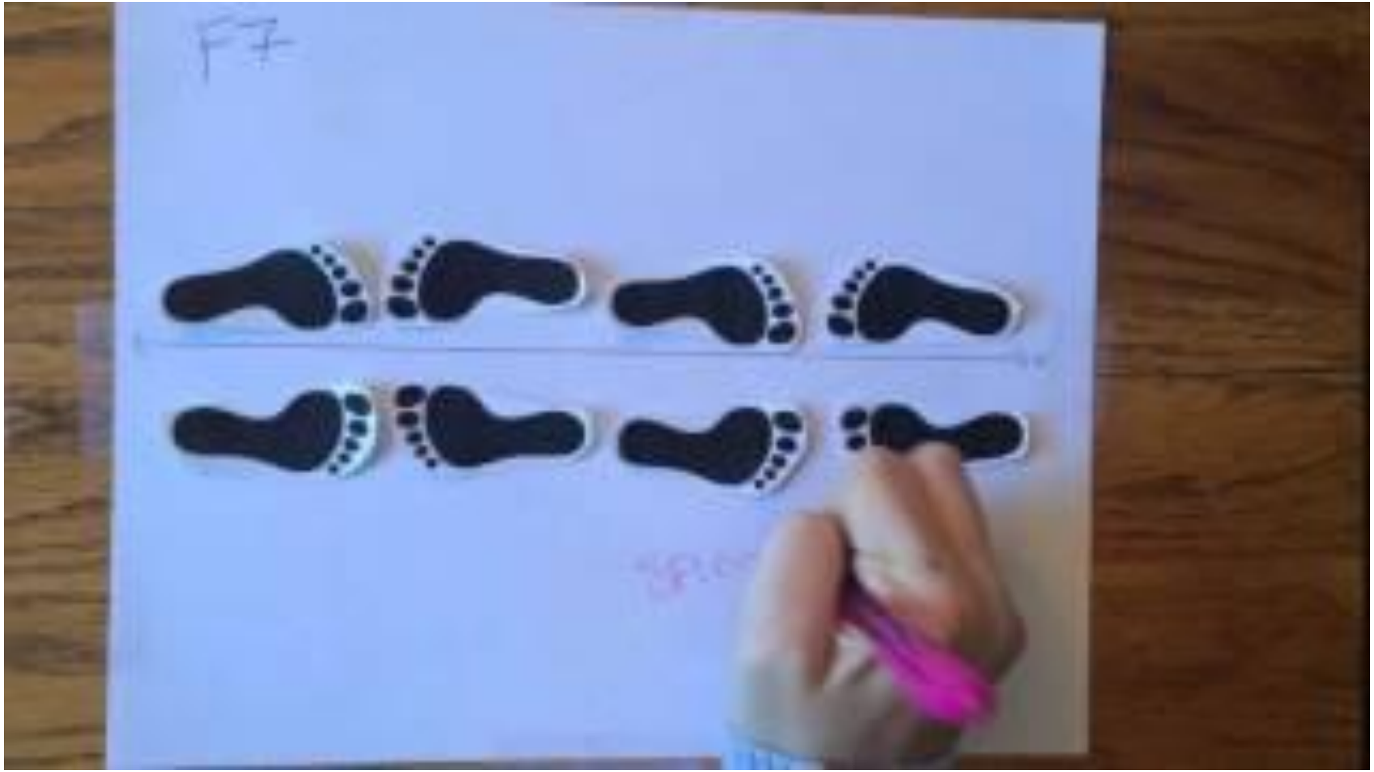
Art and Group Theory

Frieze Groups



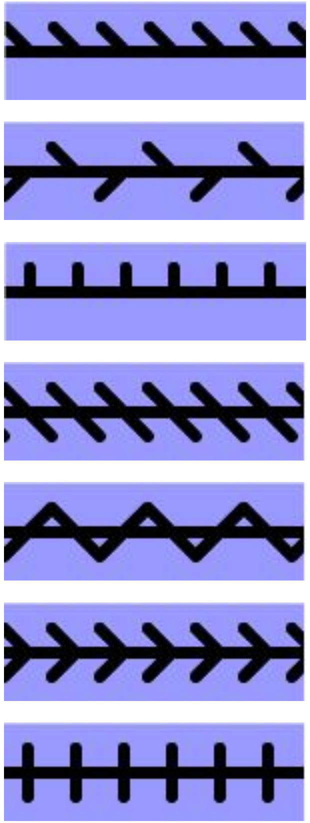
Combining the types of symmetries (reflection, rotation, translation, and glide), you can only get 7 possible groups of frieze patterns.

Frieze Groups



Frieze Groups

- Seven groups
  - F1: (T) only translation
  - F2: (TG) translation and glide reflection
  - F3: (TV) translation and vertical reflection
  - F4: (TR) translation and 180 degree rotation
  - F5: (TRVG) translation, glide reflection, and vertical reflection
  - F6: (TH) translation and horizontal reflection
  - F7: (TVH) translation, vertical reflection, and horizontal reflection



Wallpaper Groups

Similarly to friezes, combining the types of symmetries gets you only 17 possible groups of wallpaper patterns.

- List here
- Interactive drawing tool here

Group Theory and Art

Hyperbolic Geometry

Euclidean geometry

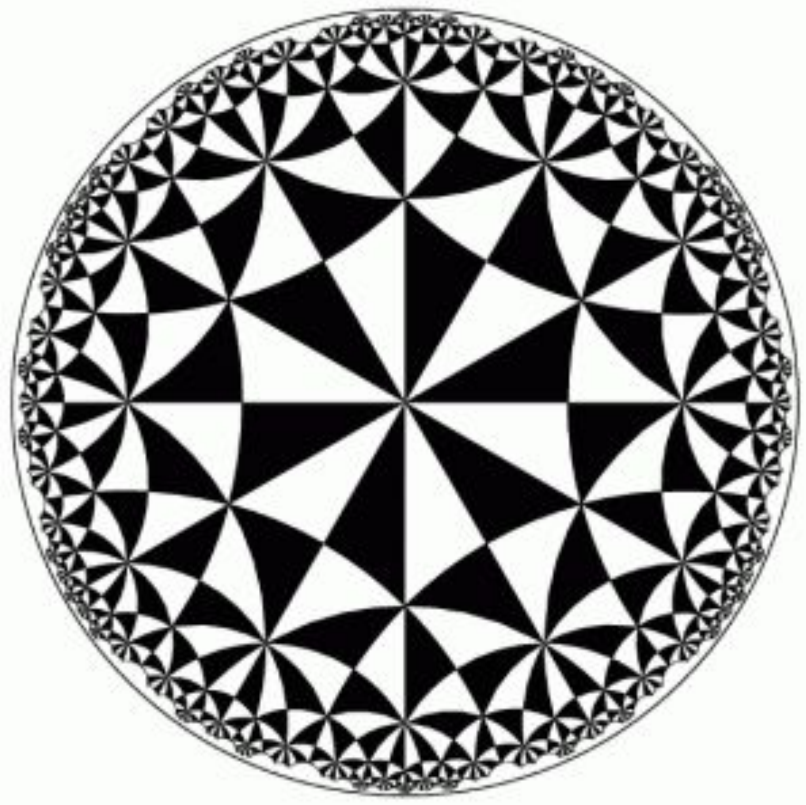
- Assumes Euclid's fifth postulate:
  - Let *l* be a line and *P* be a point not on *l*. There exists a unique line *m* through *P* such that $l \parallel m$.
- Zero curvature

Hyperbolic geometry

- Negation of Euclid's fifth postulate holds:
  - There exists a line *l* and a point *P* not on *l* such that at least two distinct lines, *m* and *n*, go through *P* such that $m \parallel l$ and $n \parallel l$.
- Negative curvature

Poincaré Disk Model

Represents straight lines in hyperbolic space as either an arc within the disk or as the diameter of the disk.



Mathematician Donald Coxeter

- Well known for his work in group theory and symmetry
- Tiling hyperbolic geometry
- Coxeter groups

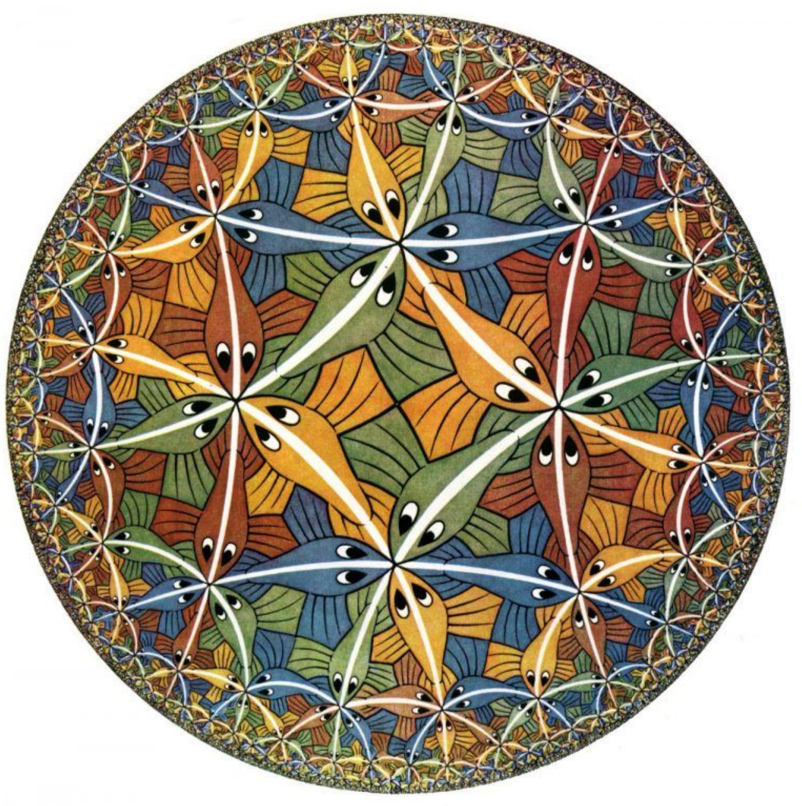
M.C. Escher

Escher was fascinated with symmeries throughout his career

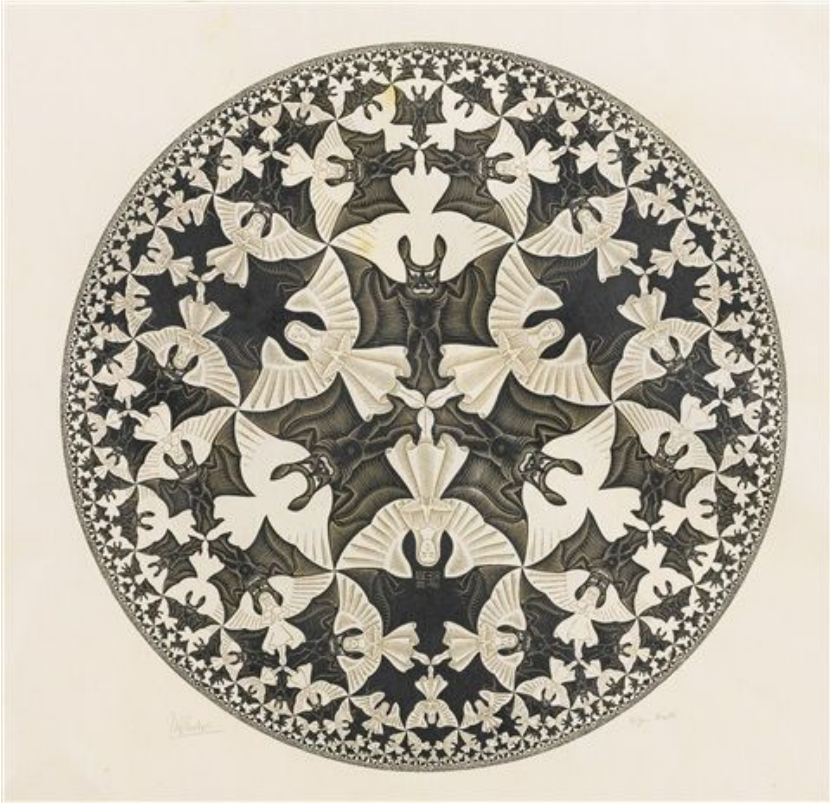
- Visited and make work based on the Alhambra de Granada

He wrote to Coxeter to get more information on how to model hyperbolic infinity

- *Circle Limit I*, M.C. Escher (1958)



Circle Limit III, M.C. Escher (1959)



Circle Limit IV (Heaven and Hell), M.C. Escher (1960)

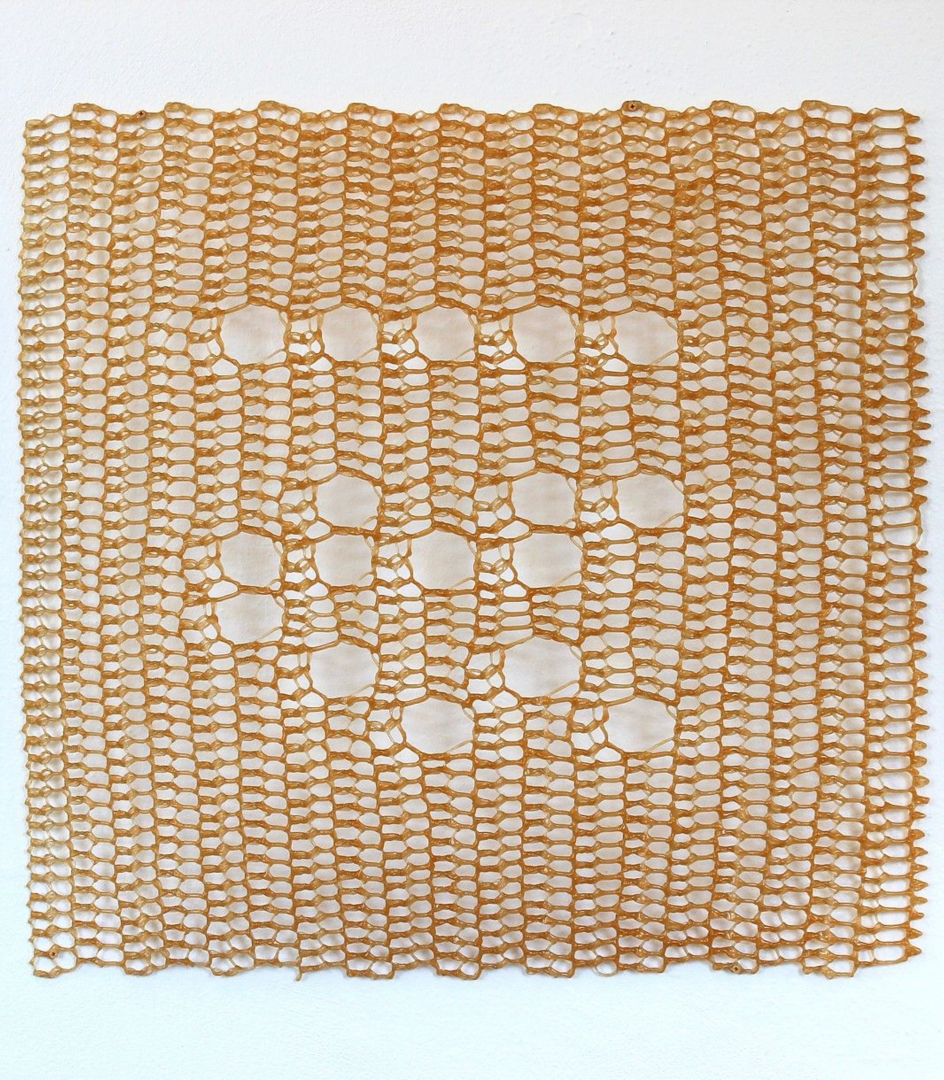
More Math Art

Yvette Kaiser Smith

"I am an artist who uses numbers. I create geometric abstractions by using sequences derived from the numbers pi and e, prime numbers, Pascal's Triangle, grids, and repetition of simple geometric shapes as tools in devising systems for mapping and visualizing numerical values by which I can create new and uncommon patterns."

Pi Etude...3238462643383, Yvette Kaiser Smith, 2012

Lifesaver Movement in e, Yvette Kaiser Smith, 2014



The number 7

Identity Sequence e 4, Yvette Kaiser Smith, 2007

2
.
7
1
8
2
8
1
8
2
8
4
5
9
0
4
…