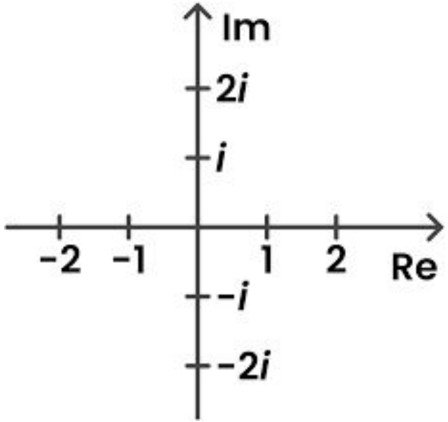
The (Real) People Behind Imaginary Numbers

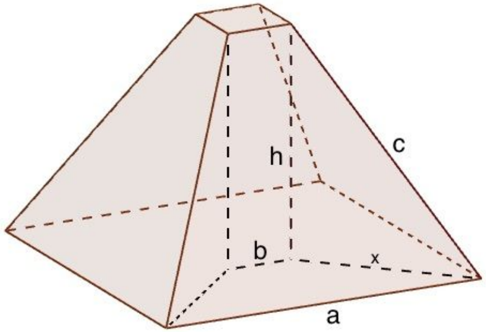
$$i^2 = (\sqrt{-1})^2 = -1$$

By Kyle Gomez



Introduction

- How were complex numbers developed?
- Who were the people developing and popularizing the ideas about complex numbers?
- How did their usefulness evolve over time?



A Brush with the Imaginary

- **Heron of Alexandria** (60 AD) — Greco-Roman mathematician and engineer
    - Steam-powered device
- *Stereometria*: formula for height of frustum of pyramid:

$$h = \sqrt{c^2 - \frac{(a-b)^2}{2}}.$$

- a=28, b=4, c=15 ⇒ h=$(-63)^{½}$
- He wrote it as $(63)^{½}$ instead, **avoiding imaginary numbers** completely

$$x^3 + px = q,$$
$$x^3 = px + q.$$
$$\sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}.$$

Discovery and Disregarding

- **Gerolamo Cardano** (1501–1576) — Italian mathematician and scientist
  - Binomial theorem
- *Ars Magna* (1545): first published solutions for cubic and quartic equations (del Ferro/Tartaglia)
- Find two numbers that add to 10 and multiply to 40:
  - Solution: $5+(-15)^{½}$ and $5-(-15)^{½}$
- **“As subtle as it is useless”**

1 x i = i	-1 x i = -i
1 x -i = -i	-1 x -i = i
i x i = -1	i x -i = 1
-i x i = 1	-i x -i =-1

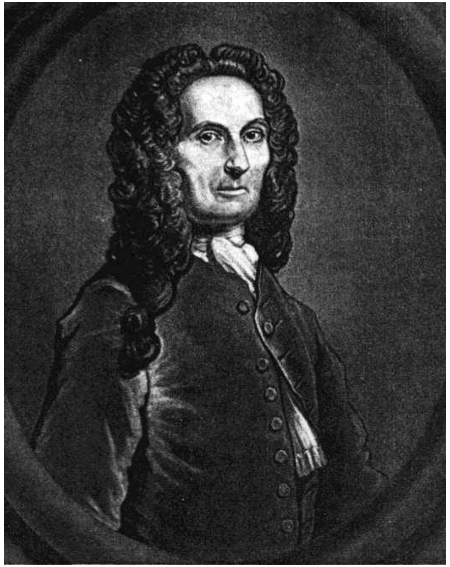
Finding the Square Root of -1

- **Rafael Bombelli** (1526–1572) – Italian mathematician
  - Simplified algebra for commoners
- *L'Algebra* (1572): Uses Cardano's formulas to introduce complex numbers with basic **properties and arithmetic**
- *i*: "plus of minus" and -*i*: "minus of minus"

Forever Imaginary

- **René Descartes** (1596–1650) – French mathematician and philosopher
  - Analytic geometry, “I think, therefore I am”
- Still couldn’t find a geometrical approach to complex numbers
- *La Géométrie* (1637): Coined the term **“imaginary”**; more disregard of non-real roots

Building Further



- **Abraham de Moivre** (1667–1754) — French mathematician
  - Probability theory, normal distribution
- “De sectione anguli” (1722): Published **de Moivre’s formula**
- More applications for complex numbers now

$$(\cos x + i \sin x)^n = \cos(nx) + i \sin(nx).$$

Opening the Floodgates

- **Leonhard Euler** (1707–1783) — Swiss mathematician
  - Graph theory, topology, modern math notations, etc.
- Introduced **i** for imaginary unit
- *Intoductio in analysin infinitorum* (1748): derives de Moivre's formula, then uses complex exponentiation to find **Euler's formula**

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

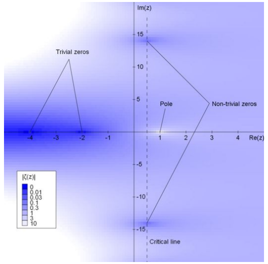
Geometric Interpretation At Last

- **Caspar Wessel** (1745–1818) — Danish-Norwegian mathematician and cartographer
- While surveying, decided to use a real axis and imaginary axis to make coordinates
    - Others before had theorized this without actually implementing it
- *Om directionens analytiske betegning* (1797): outlined this procedure
    - Published only in Danish! No-one read it for years!
- This was discovered in the 1890's

Finalizing the Basics

- **Johann Carl Friedrich Gauss** (1777–1855) — German mathematician
  - Proved fundamental theorem of algebra, etc.
- More principles of complex numbers, termed “complex numbers”
- “Theoria residuorum biquadraticorum. Commentatio secunda” (1831): popularized the complex plane

Complex Analysis



- **Augustin-Louis Cauchy** (1789–1857) — French mathematician
  - Rigorous proofs of theorems of calculus, etc.
- **Bernhard Riemann** (1826–1866) —German mathematician
  - Riemann integral, Riemann hypothesis, etc.
- Along with others, really developed complex analysis
- Greatly improved the usefulness of complex numbers

$$\oint_C f(z)dz = 0, \qquad f(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{z-a}dz, \qquad \operatorname*{Res}_{z=a} f(z) = \lim_{z\to a}(z-a)f(z),$$