The Cournot Duopoly Model

Jackson Howell
Introduction

The Cournot Duopoly Model is a strategic game between two firms choosing how much of a particular product to bring to market, taking the other firm’s choice into consideration to maximize their own profit.
Antoine Augustin Cournot

- 1801-1877
- French philosopher and mathematician
- Known for his work in econometrics and early understanding of oligopoly.
Setup

- Firm 1 and Firm 2 simultaneously choose production quantities $Q_1, Q_2$
- Each firm faces the same production cost per unit $MC$
- The market price is a decreasing function: $P(Q_1, Q_2)$
- The market always clears: each firm sells all their product
Profit

- Firm profit is Total Revenue - Total Costs
- Revenue is \((\text{Price of Good})\times(\text{Quantity of Goods Sold})\)
- Cost is given by the \((\text{Cost of Good})\times(\text{Quantity of Goods Sold})\)

Therefore, the Profit function for firm i is:

\[ P(Q_1, Q_2)\times(Q_i) - (MC)\times(Q_i) \]
Simultaneity

Each firm makes their production decision simultaneously without knowledge of or influence over the other firm’s decision.

How does each firm maximize their profits?
Best Response Functions

A firm’s best-response function gives its profit-maximizing quantity given each possible quantity chosen by the other firm.

Perhaps this would be a good place to start?
Specify Functional Forms

\[ P(Q_1, Q_2) = 60 - Q_1 - Q_2 \]

MC = 2
Best-Response Functions

Q_1 = 29 - Q_2/2
Q_2 = 29 - Q_1/2

Nash Equilibrium: the point at which neither firm regrets their choice given the other firm’s choice. (19.33, 19.33)
Equilibrium Price and Profits

- Firm 1 Quantity: 19.33
- Firm 2 Quantity: 19.33
- Price: 21.33
- Firm 1 Profit: 373.65
- Firm 2 Profit: 373.65
Takeaways

- Firms are limited by incomplete information of what their opponent is doing.
- However, there seems to be a steady state condition where both firms are happy to stay where they are as long as the other firm stays where they are.
- Does this happen in practice? Coordination Problem
Coordination under Duopoly

- What if both firms collude with each other to set production quantities? Does this equilibrium change?
- Let both companies maximize their joint profit over $Q_1 + Q_2$, and split the joint profit equally.
- How does the equilibrium change?
New Equilibria

- Quantity: 19.33 -> 14.5
- Price: 21.33 -> 31
- Profit: 373.65 -> 420.5

However, this situation requires cooperation...
Firm 1 Defection

Firm 1 Best Response: 29 - Q_2/2
Firm 1 should bring 21.75 units to market

Profit: 420.5 -> 473.06
Prisoner’s Dilemma

- Cooperation yields the mutually best outcome
- Defection from the Nash equilibrium is better for defector than mutually best outcome
- If both defect, then mutually worst outcome is achieved
Cooperation and Market Economics

- Economic Theory predicts that price gouging through collusion will occur in smaller markets or unregulated markets.
- This leads governments to break up monopolies and restrict the ability for companies to collude with each other.
- As fewer firms start to control a market, there is strong incentive to collude: e.g., OPEC or telecommunications companies.
Perfectly Competitive Market

- So many firms in the market that price is no longer a function of the firm’s choice of quantity
- What does the first-order condition become?
- Zero-profit equilibrium
Real World Duopolies

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BY GIUSEPPE COLANGELO, OSCAR BORGOGNO  April 7, 2022
Discussion/Thoughts

Market Analysis

● Can you think of markets that may be prone to collusion?
● Can you think of ways to deter collusion between companies?

Mathematics and Economics

● Do you think mathematics is a useful tool to analyze questions like this, or does it abstract too much from specifics?
● Would you find this analysis compelling if you were a lawmaker considering antitrust legislation?
Sources/Further Reading


“Cournot’s Duopoly Model,” University of Toronto Department of Economics. https://www.economics.utoronto.ca/osborne/2x3/tutorial/COURNOT.HTM.