


# The Cournot Duopoly Model

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## Introduction

The Cournot Duopoly Model is a strategic game between two firms choosing how much of a particular product to bring to market, taking the other firm's choice into consideration to maximize their own profit.




# Antoine Augustin Cournot

- 1801-1877
- French philosopher and mathematician
- Known for his work in econometrics and early understanding of oligopoly.



# Setup

- Firm 1 and Firm 2 simultaneously choose production quantities  $Q_1, Q_2$
  - Each firm faces the same production cost per unit  $MC$
  - The market price is a decreasing function:  $P(Q_1, Q_2)$
  - The market always clears: each firm sells all their product
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# Profit

- Firm profit is Total Revenue - Total Costs
- Revenue is (Price of Good)\*(Quantity of Goods Sold)
- Cost is given by the (Cost of Good)\*(Quantity of Goods Sold)

Therefore, the Profit function for firm  $i$  is:

$$P(Q_1, Q_2) * (Q_i) - (MC) * (Q_i)$$



# Simultaneity

Each firm makes their production decision simultaneously without knowledge of or influence over the other firm's decision.

How does each firm maximize their profits?



# Best Response Functions

A firm's best-response function gives its profit-maximizing quantity given each possible quantity chosen by the other firm.

Perhaps this would be a good place to start?



## Specify Functional Forms

$$P(Q_1, Q_2) = 60 - Q_1 - Q_2$$

$$MC = 2$$



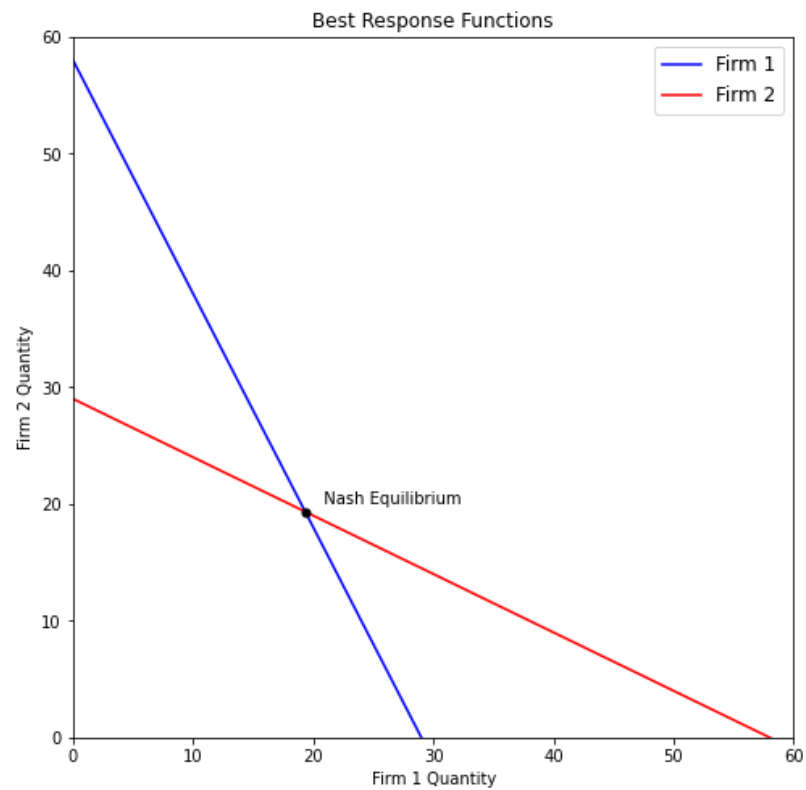


# Best-Response Functions

$$Q_1 = 29 - Q_2/2$$

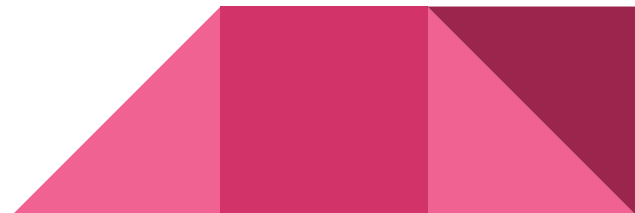
$$Q_2 = 29 - Q_1/2$$

Nash Equilibrium: the point at which neither firm regrets their choice given the other firm's choice. (19.33, 19.33)



## Equilibrium Price and Profits

- Firm 1 Quantity: 19.33
- Firm 2 Quantity: 19.33
- Price: 21.33
- Firm 1 Profit: 373.65
- Firm 2 Profit: 373.65



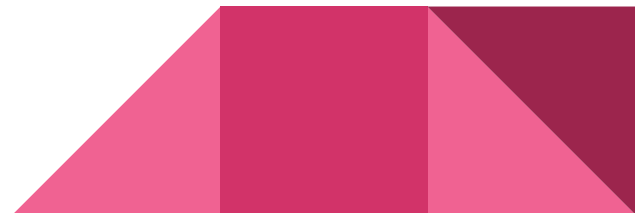
# Takeaways

- Firms are limited by incomplete information of what their opponent is doing.
- However, there seems to be a steady state condition where both firms are happy to stay where they are as long as the other firm stays where they are.
- Does this happen in practice? Coordination Problem



# Coordination under Duopoly

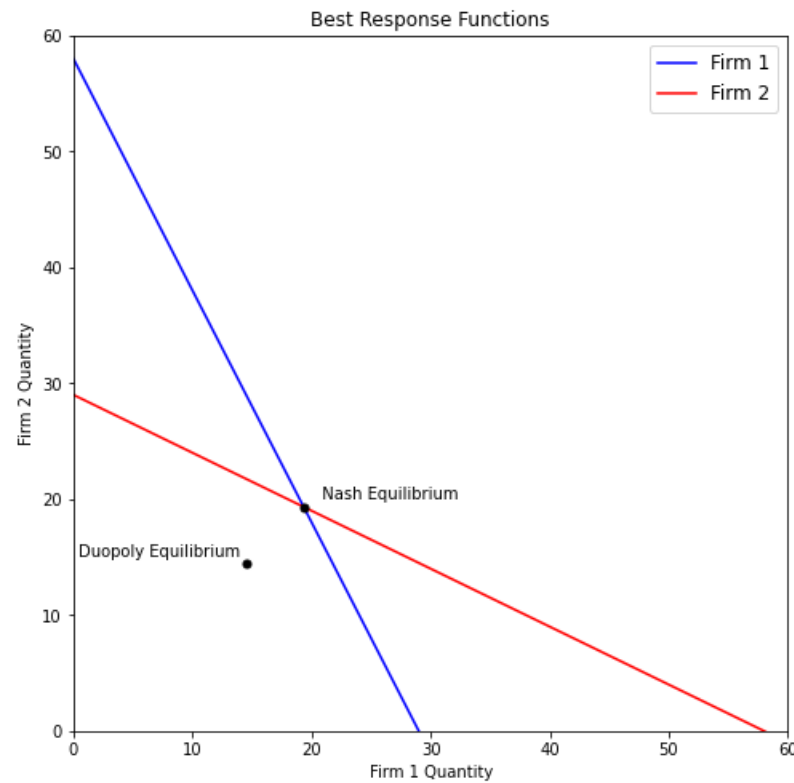
- What if both firms collude with each other to set production quantities? Does this equilibrium change?
- Let both companies maximize their joint profit over  $Q_1 + Q_2$ , and split the joint profit equally.
- How does the equilibrium change?



# New Equilibria

- Quantity: 19.33 -> 14.5
- Price: 21.33 -> 31
- Profit: 373.65 -> 420.5

However, this situation requires cooperation...

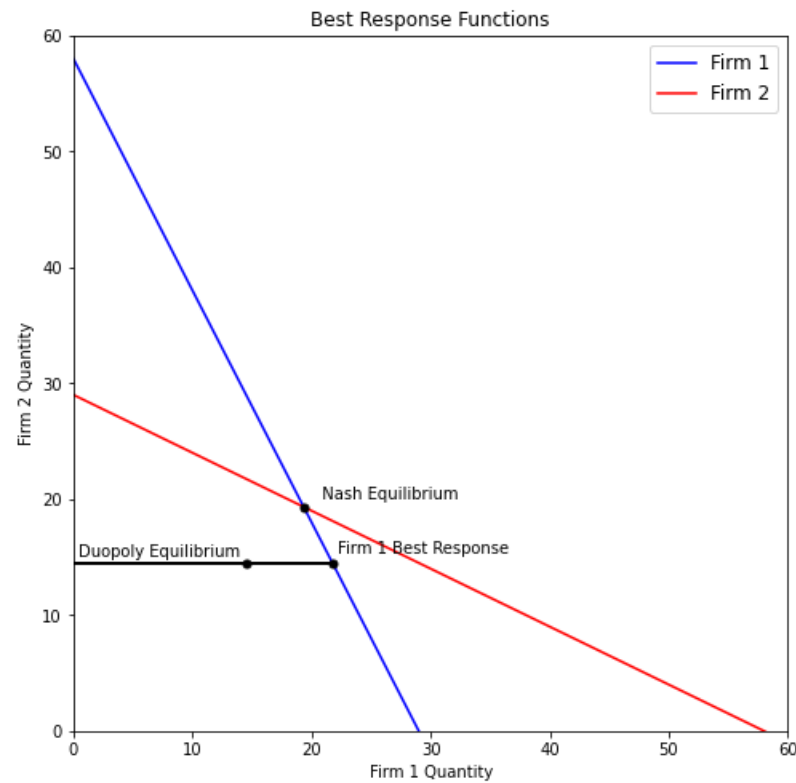


# Firm 1 Defection

Firm 1 Best Response:  $29 - Q_2/2$

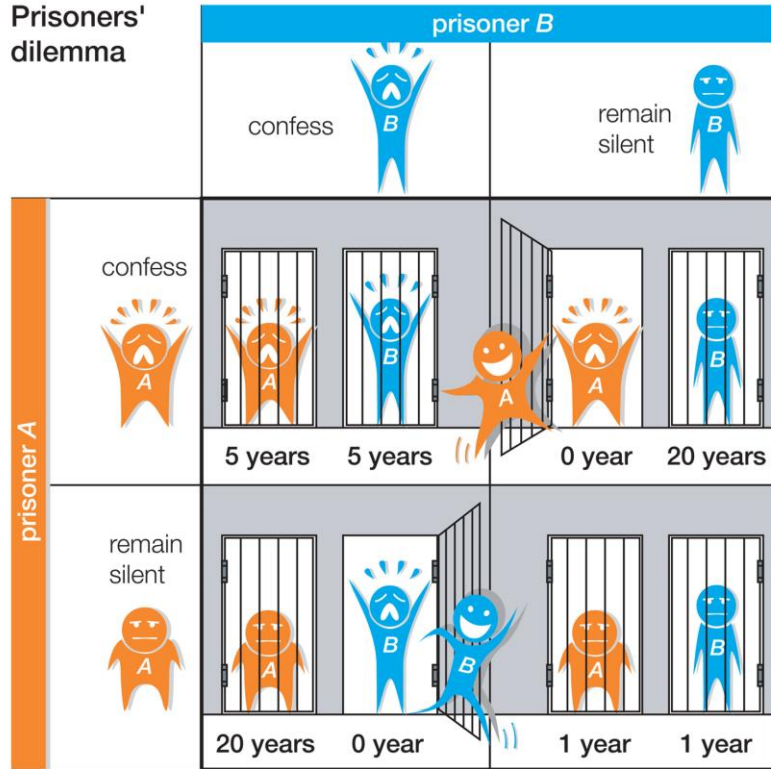
Firm 1 should bring 21.75 units to market

Profit: 420.5  $\rightarrow$  473.06



# Prisoner's Dilemma

Prisoners' dilemma



- Cooperation yields the mutually best outcome
- Defection from the Nash equilibrium is better for defector than mutually best outcome
- If both defect, then mutually worst outcome is achieved

# Cooperation and Market Economics

- Economic Theory predicts that price gouging through collusion will occur in smaller markets or unregulated markets
- This leads governments to break up monopolies and restrict the ability for companies to collude with each other.
- As fewer firms start to control a market, there is strong incentive to collude: e.g., OPEC or telecommunications companies





# Perfectly Competitive Market

- So many firms in the market that price is no longer a function of the firm's choice of quantity
- What does the first-order condition become?
- Zero-profit equilibrium



# Real World Duopolies

BUSINESS & LOBBYING

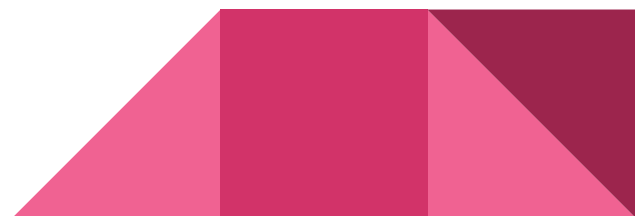
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BY GIUSEPPE COLANGELO, OSCAR BORGOGNO *April 7, 2022*




# Discussion/Thoughts

## Market Analysis

- Can you think of markets that may be prone to collusion?
- Can you think of ways to deter collusion between companies?

## Mathematics and Economics

- Do you think mathematics is a useful tool to analyze questions like this, or does it abstract too much from specifics?
  - Would you find this analysis compelling if you were a lawmaker considering antitrust legislation?
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# Sources/Further Reading

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Choi, Joseph. “Treasury eyes beer industry ‘duopoly’ in urging more competition.” *The Hill*, February 10, 2022.

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