The Doomsday Algorithm and its Applications

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Introduction

Have you ever wondered on what day you were born? Or maybe what day of the week is a future event, like a milestone birthday or an anniversary? You might look up the day online or scroll to the specific date your phone's calendar app and find out that you were born on a Friday and your 21^{st} birthday, for example, falls on a Wednesday. However, if we did not have these resources available, how would we find out this information? It would be quite tedious and cumbersome to manually count forwards or backwards the number of days, but still possible. Luckily, there exists a method, devised by John Conway, to calculate the day of the week of any given day, known as The Doomsday Algorithm. With a little memorization and some practice, you can calculate an answer without needing a phone or an online calendar. This paper will examine its origins, how it works, and some potential applications.

Origins

This problem of coming up with a method to determine the day of the week of any given date was primarily initiated and worked on by Lewis Carroll in the late nineteenth century; And although effective, one would have to memorize more steps and compute more mental calculations than needed with his method. Conway sought out to create an algorithm that was relatively easy and simple enough for people to learn and use on their own. His method was popularized nearly a century later in 1973.

The Algorithm

The Doomsday Algorithm takes advantage of the fact that our current calendar system, the Gregorian Calendar, repeats every 400 years. In general, it takes the earth 365.25 days to complete a rotation around the sun, hence an extra leap day every 4 years to keep the days aligned with the seasons and the calendar. However, 365.25 is a rounded estimate of the actual 365.2422 days it takes to complete one rotation, making our years roughly 11 minutes longer than they should be. Albeit a small difference, this rounding error will add up over time. By the end of a 400-year period, our calendars would be roughly 3 days ahead of the earth's actual position if we were to add a leap day every 4 years. For this reason, the Gregorian calendar adds an additional rule for leap years: Every 4 years is a leap year (2004, 1992, etc.), and a century is a leap year if it is also divisible by 400. Thus 2000 and 1600 are leap years whereas 1900 and 1800 are not.

Conway noticed that each year, there were certain dates that always fell on the same day of the week, which he called "Doomsdays" (DDs. For example, in 2021, October 10th, November 7th, and December 12th, all fell on a Sunday (full list on Table 1). Conway chose these days since they are simple to remember, like 4/4, 6/6, 8/8, etc., and for the other months, there is a mnemonic to help remember its doomsday. Additionally, we are going to introduce some notation numbering the days of the week to make mental computations easier (Sunday = 0, Monday =1, …, Saturday = 6). The next section will explain how to calculate the day of the week (DoW) for a date.

Month	***Doomsday***	*Mnemonic*
January	1/3 or 1/4 (on leap years)	The **3rd** for 3 years and the **4th** on the fourth year
February	2/28 or 2/29 (on leap years)	Last day of February
March	3/14	Pi Day
April	4/4	
May	5/9	"I work **9** to **5** at the **Seven Eleven"**
June	6/6	
July	7/11	(See May)
August	8/8	
September	9/5	(See May)
October	10/10	
November	11/7	(See May)
December	12/12	

Table 1. Doomsdays for Each Month

Calculating the Day of the Week

There are three main steps to this algorithm. We begin by examining the century component of the year (i.e., "19" in 19XX) and finding the DD for that century; This will be our starting count. From there, we will examine the year component ("42" in XX42) to see how many days the doomsday has shifted since the beginning of the century and add that to our count. And lastly, we will find the difference between the day we are given and the closest DD from Table 1 and add that to the running count; This count is converted back to a DoW using modulo 7, and we get our answer. To see these steps in depth, let's use the date May 24, 1942, as our example.

Since our calendar repeats every 400 years, the doomsday at the start of the century will also follow that pattern. For example, the DD in the year 2000 was a 2 (Tuesday); and 2100's DD will be a 0 (Sunday); 2200 a 5 (Friday); and 2300 a 3 (Wednesday), and so on. We can use the following formula to determine which century corresponds to which doomsday:

$$r = c \bmod 4, \text{ where } c = \lfloor year/100 \rfloor$$

$r = 0$	Tuesday (2)
$r = 1$	Sunday (0)
$r = 2$	Friday (5)
$r = 3$	Wednesday (3)

Table 2. Century Doomsday

In our example we look at 1942 first; $c = \lfloor 1924/100 \rfloor = 19$; $r = 19 \equiv 3 \bmod (4)$. So, our starting count is 3.

Since a regular calendar year consists of 365 days with 7 days a week, we will always have 1 day that does not fit evenly into a full week. Thus, each day will shift by one after each year (If January 1 is a Monday one year it will be a Tuesday the next year). Similarly, a leap year will shift some day by 2 days the next year. When examining the year component, we add to our count the year number itself, and the number of leap years that happened in that period, to see how many days the century DD shifted. The following formula illustrates this part of the algorithm:

$$Days\, shifted = \left(y + \lfloor \frac{y}{4} \rfloor \right) \bmod 7, \text{ where } y \text{ is the year component}$$

Continuing the example looking at 1942, doomsday in 1900 was a 3 (Wednesday). And we see that DD shifted (42 + $\lfloor 42/4 \rfloor$¿ = (42 + 10) = 52 total days since 1900; Or in terms of weekdays, $52 \equiv 3 \bmod 7$, so DD moved 3 days ahead in the week. Since in the example our DD started on a 3, we add the 3 days shifted we just calculated to give the new position of DD in 1942; So, our count is now 6, which means that in 1942, doomsday is on a 6 (Saturday).

Lastly, we find the number of days between our given date and the nearest DD from Table 1. For the month of May, doomsday is May 9th; And in context of the year 1942, it follows that May 9th is a 6. We see that there are 24-9 = 15 days between our example date (5/24) and May's DD (5/9); Thus, we want to know what DoW 15 days after Saturday (6) is. We add these 15 days to our count giving us 6 + 15 = 21 ≡ 0 *mod* 7. That is, May 24th, 1942, is a 0, or a Sunday.

It is worth noting here the significance of numbering the days of the week. Without numbering, it can be difficult to figure out mentally what day is 12 days after Wednesday, for example. We would easily subtract 7 from 12, leaving us still on Wednesday, but then it might be tough to figure out 5 days after Wednesday in our heads, eventually ending up on Monday. Whereas numeric notation allows us to take 3 + 12 = 15 ≡ 1 *mod* 7, which is also a Monday.

Applications

Algorithms and formulas for calculating the day of the week are implemented in various computer programs and websites where this information is needed. The methods used in these cases can be repeated by hand, though they are not suitable for mental calculations like Conway's method. Most notably, Microsoft Excel uses an equation that takes the month, day, and year as input parameters and makes some calculations based on those values and returns a DoW. The following is not the exact formula used by Excel, but one commonly used in other computer programs, known as Zeller's Congruence:

$$h=\left(q+\frac{\lfloor 13(m+1)\rfloor}{5}+K+\lfloor\frac{K}{4}\rfloor+\lfloor\frac{J}{4}\rfloor-2J\right) mod\ 7$$

Another process that follows a cyclic pattern like the calendar, is the moon. Its phases follow a cycle of roughly 29.5 days long. This predictability of the phases can be used to model the timing of high and low tides as well as the strength of them, potentially useful information for improving the usage of tidal energy as a renewable source.

Conclusion

With some practice, this algorithm and the mental math needed can be completed in a few minutes or less. And although it is a simple trick, the underpinnings of the doomsday algorithm can be applied to other cyclic processes that influence our current and future lives. We often see repeating patterns in life and the more we understand how and why they work, the more we can use them to get a more accurate prediction of what will happen in the future.

Sources

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