MATH 400: Network Algorithms

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Abstract: Network optimization techniques provide solutions to complex problems in many different fields and are fundamental to principles in mathematics, computer science, finance, and much more. This paper will discuss the significance, historical context, and applications of several network algorithms using examples of major concepts such as shortest path routing and maximum flow algorithms. We will cover the history behind operations research and focus on the attributes of these optimization methods as well as their challenges.

1. Introduction.

(1.1) During my experience studying mathematics, I've realized that I appreciate the practical applications more than any abstract theories. I enjoy areas where math feels more tangible. This preference led me to explore fields where concepts connect with real-world problems. One field that particularly caught my interest was operations research, a section of mathematics that uses optimization of systems and quantitative techniques to maximize efficiency, improve productivity, and increase overall performance of a given problem or task.

(1.2) While in college, I had the opportunity to take Deterministic models, which enhanced my interest in operations research. This was my first exposure to how business, organizations…etc might use mathematical models and algorithms for everyday purposes. It might be maximizing profit in business operations, improving decision-making processes in logistics, or optimizing resource allocation in supply chains. Whatever it was that we studied, these kinds of optimization problems really piqued my curiosity to dive deeper into this field. Furthermore, I decided to pursue roles after graduation as an operations research analyst, and after college I landed a role working with a cost estimating team for the Navy. With my future heading in this direction, naturally I've been even more attuned to the world of operations research. I've tried to take any opportunities to learn more about optimization techniques, and one concept that I've enjoyed learning about is the array of different network algorithms, which can be utilized to optimize routing, flow, and connectivity in all different kinds of systems. I've seen online videos about the maximum flow problem in regards to airport scheduling, but I didn't have any knowledge of how the math worked nor the significance of how networks are utilized within operations research. As such, I decided to do my first talk on this topic and to continue with it for my second presentation.

2. Historical Context and Development

(2.1) The roots of operations research (OR) and optimization date back to the early 20th century, with advancements in mathematical theory and the ongoing industrialization. During World War II, the Allied forces faced complex logistical challenges which demanded unique strategic planning and resource allocation. In response, military planners and mathematicians turned to new methodologies to optimize supply chains, troop movements, and tactical decision-making. The British are most notably attributing the rights to using such methodologies with the royal air force to better handle radar technology. Albert Rowe, a 20th century British physicist, helped conceive the idea for an early-warning radar system, which took the code name “Chain Home." Rowe analyzed the radar equipment, its communication networks, and how it was operated by personnel. Using optimization techniques, Rowe was able to identify limitations and ways the radar system could be improved. The following depiction (Figure 1) shows some of the radar systems built by the Royal Air Force during World War 2.

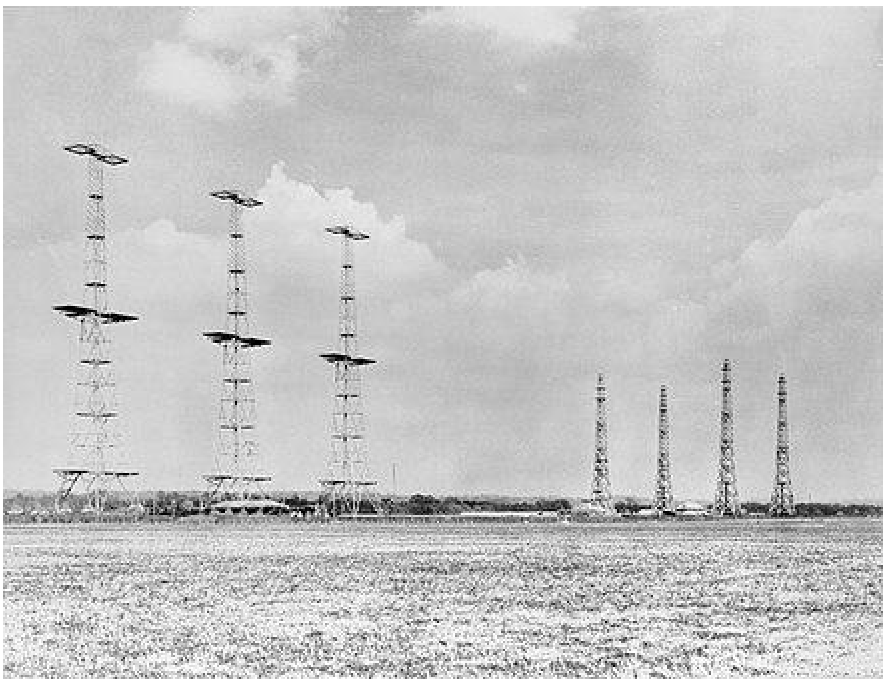

Figure 1 (“Chain Home” radar system)

The mathematical techniques used would later lay the foundation for what would become known as operations research. One of the most iconic early applications of optimization in military operations during World War II was the "diet problem" solved by economist George Stigler. Stigler optimized the nutrient intake for soldiers while minimizing costs using linear programming techniques. George Dantzig, noted as the “father of linear programming”, contributed to the creation of this method in the 1940’s. Linear programming transformed decision-making techniques across many different fields by providing mathematical techniques to maximize and minimize a linear objective function at the same time under given constraints.

Stigler's 1939 Diet

Food	Annual Quantities	Annual Cost
Wheat Flour	370 lb (170 kg)	$13.33
Evaporated Milk	57 cans	$3.84
Cabbage	111 lb (50 kg)	$4.11
Spinach	23 lb (10 kg)	$1.85
Dried Navy Beans	285 lb (129 kg)	$16.80
Total Annual Cost		**$39.93**

Table of nutrients considered in Stigler's diet

Nutrient	Daily Recommended Intake
Calories	3,000 Calories
Protein	70 grams
Calcium	.8 grams
Iron	12 milligrams
Vitamin A	5,000 IU
Thiamine (Vitamin B_1)	1.8 milligrams
Riboflavin (Vitamin B_2)	2.7 milligrams
Niacin	18 milligrams
Ascorbic Acid (Vitamin C)	75 milligrams

Figures 2 and 3 (Stiglers Diet)

In the United States, operations research formally began being studied in the 1940's. The initial focus was on naval warfare, but by the end of World War II, it had ventured into the Air Force and other military branches. After the war, many British operations researchers moved into government and industry work. As a result, this led to the creation of several industrial operations research groups, most evidently in industries like electricity and coal.

(2.2) Operations research also found applications beyond the military including fields such as transportation, manufacturing, finance, and healthcare. The post-war era saw the creation of academic programs dedicated to operations research, as universities noticed the interdisciplinary nature of this field. The surge of computers in the mid to late 20th century further enhanced the development of optimization techniques. One of the biggest problems within network optimization is the maximum flow problem, which essentially aims to find out the maximum amount of flow that can be transferred from a source node to a sink node in a network considering capacity constraints are in place on the given edges. The Ford-Fulkerson algorithm, developed by L. R. Ford Jr. and D. R. Fulkerson in 1956, was one of the first algorithms developed to answer this problem. This algorithm works by iteratively augmenting flow paths along residual networks until there exists no remaining augmenting paths. An augmenting path can be defined as a path from the source node to the sink node where each edge traversed has a current edge capacity at least greater than zero. The maximum flow along a path is typically the minimum capacity of the edges. After this augmentation step, the residual graph is updated and the process repeats until there are no remaining paths. The algorithm is simple but has its drawbacks. One such being that there is no structure or guideline for choosing augmenting paths. Because of this factor, the algorithm does not have a guaranteed time complexity. See the images below to help visualize how this process works.

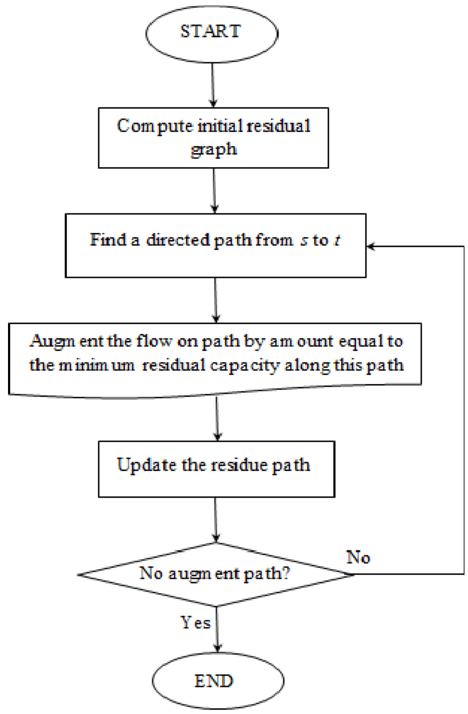

Figure 4 (Ford Fulkerson Basis Steps)

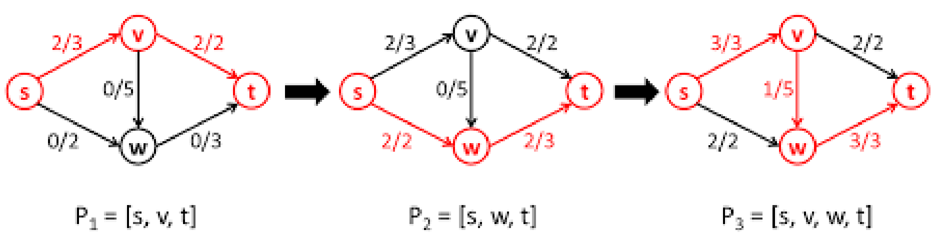

Figure 5 (Ford Fulkerson Visual)

(2.3) Since Ford Fulkerson doesn't have a set rule for choosing a desired path, many mathematicians reference it as an incomplete algorithm. This weakness led to the development and advancement of more efficient variations, including the Edmonds-Karp algorithm, which was proposed by Jack Edmonds and Richard Karp in 1972. The algorithm is identical to Ford-Fulkerson in terms of steps except for one factor; It details how to select and choose an augmenting path. The algorithm uses a breadth-first search to find augmenting paths and chooses the one with the shortest distance from the source to the sink node. This enhances the efficiency of maximum flow computations and guarantees the algorithm a polynomial-time complexity of O(V * E^2), where V is the number of vertices and E is the number of edges.

(2.4) Another major network algorithm is the A* algorithm, developed in the late 1960s by computer scientists Peter Hart, Nils Nilsson, and Bertram Raphael. A* is a heuristic search algorithm that finds the shortest path in a network-graph, taking into account both the cost incurred and an estimated cost to reach the goal. The estimated cost is something we can denote as a “heuristic”. This is a measure or technique used to find an approximate solution of a problem. For A*, the heuristic is an estimated cost from the current node to the goal node, and we can calculate this in a few ways. Euclidean distance is most commonly used. This method takes the straight line distance between two points in Euclidean space. Assuming a location has some (x,y) coordinate, this can be calculated using the pythagorean theorem. Since we're taking the diagonal, this line represents the absolute shortest path between any two locations. As such this approximate will always be less than or equal to whatever the actual distance may be. This method is employed when one’s movement is unrestricted and has the ability to move in any direction, including the diagonal. As such, it is suitable for situations where nodes are positioned in continuous space, such as navigating terrain or calculating distances in geographic maps. The Manhattan distance is another approximating distance for a heuristic. Given its name as an attribute to the big apple, the Manhattan distance technique is also commonly referred to as the “taxi-cab distance” or “city block distance”. This technique takes the distance between two points along axis-aligned paths at right angles and it’s used when paths can be traversed in the 4 cardinal directions (where moving in a diagonal is not possible). As such the grid-like pattern is what gives this method its name, as movement can only be done on horizontal or vertical paths. Manhattan Distance is often utilized in pathfinding problems with city layouts, mazes, or some kind of game grid. Again assuming locations have some (x,y) coordinate, this method can be calculated as the sum of the absolute difference in their coordinates.

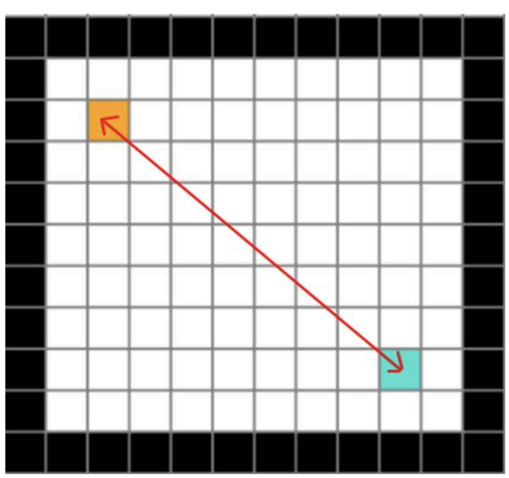

Figure 6 (Euclidean Distance)

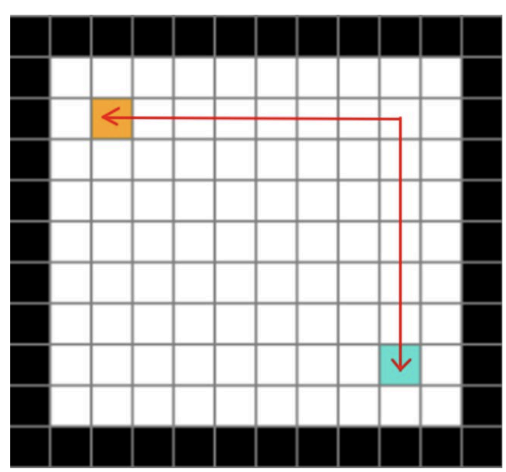

Figure 7 (Manhattan Distance)

(2.5) Regardless of the heuristic technique used, this approximate estimate is what makes A* algorithm much more applicable than Dijkstra's algorithm for finding the shortest route. When choosing what node to visit next, an estimate serves as a guide in the right direction. And when considering complex networks, where there may be thousands of potential neighbor nodes to visit, having the ability to disregard certain parts of a network and only fixate on the directionally optimal nodes will save valuable time and space. This is a huge example of where network algorithms come into play and where operations research principles are executed to efficiently solve problems. Operations Research continues to advance in the digital age, attributed by developments in computational methods, an increased emphasis in data analytics, and artificial intelligence. Optimization techniques are used in all kinds of industries today influencing how organizations interpret data/information, allocate resources, and optimize processes in dynamic and ever changing environments.

3. Applications in Real World and Challenges

(3.1) Operations Research and network optimization algorithms have practical applications across all different kinds of real-world industries, from transportation and logistics to telecommunications, healthcare, finance, and much more. By utilizing methodologies like those discussed above, organizations can improve efficiency, reduce costs, and enhance decision-making capabilities in complex systems.

(3.2) For my presentation, the main topic I studied benefiting from network optimization was transportation and logistics. In this context, algorithms such as the maximum flow and minimum cut algorithms are often used when optimizing transportation networks, routing vehicles, and managing supply chains. In terms of vehicle routing, algorithms like A* and edmonds-karp can be used to determine the most efficient routes for vehicles, minimizing travel time, and reducing fuel consumption. Similar techniques are utilized for all networks in general. Algorithms such as A* and Dijkstra's are used for routing in computer networks, finding the shortest paths between network nodes while minimizing congestion. With traffic engineering, operations research techniques are used to optimize traffic flow, reduce congestion, and increase overall road safety in urban transportation networks. By modeling traffic patterns, analyzing data, and using this information in smart transportation systems, cities have created efficient ways of optimizing flow. Some key features of these smart systems include adaptive traffic signal control, dynamic route guidance, and strategies to reduce traffic volume.

(3.3) Two examples I looked into were SCOOT and SCATS. Sydney Coordinated Adaptive Traffic System (SCATS) was developed in Australia during the 1970s. It was one of the first of its kind, and as such paved the way for a lot of other management systems. SCATS uses a central control center. The system divides the road network into zones, where each zone is operated and regulated by a local controller. These controllers adjust signal timings accordingly based on current traffic conditions using real time data from sensors, CCTV cameras, and traffic detectors along roads. Scoot helps to minimize delays, optimize traffic flow, and reduce travel time. Local Controllers regularly report data back to the central controller. This feature allows for constant monitoring, giving engineers the ability to analyze performance at intersections, check for bottlenecks in the network system, and make more data backed decisions with how the system could improve in the future. The image below depicts SCAT in work on a sample intersection.

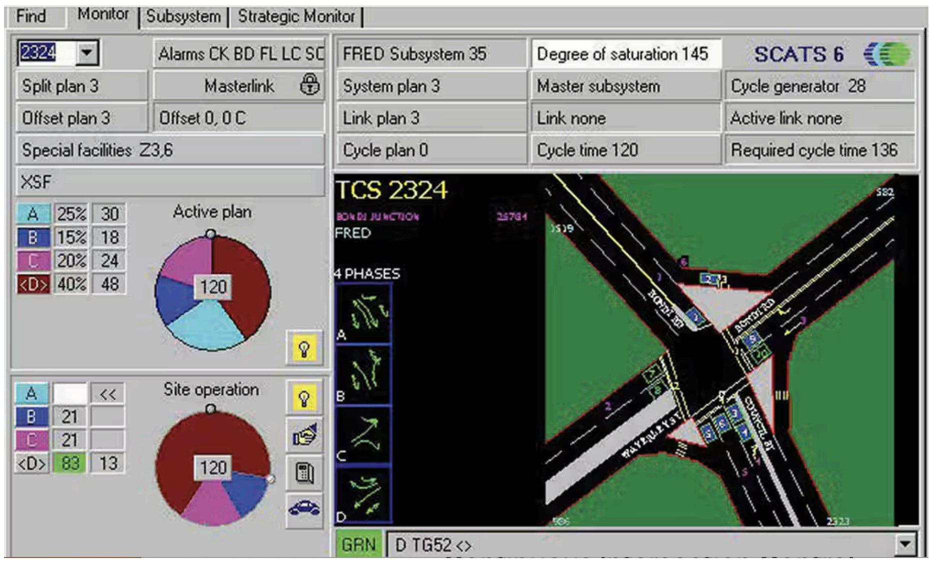

Figure 8 (SCATS)

Split Cycle Offset Optimization Technique (SCOOT) was developed in the UK in the 1980s, and it's currently one of the most prominent traffic management systems. It uses an adaptive traffic control system to manage traffic signal timings in urban areas. This system keeps track of real time traffic conditions and adjusts signal timings, cycle lengths, and offsets based on current data using measures like street detectors, cameras, and other sensors. This nature makes SCOOT ideal for minimizing delays and queues when traffic flow changes.

(3.4) There have been a lot of advancements to operations research including developments of many network algorithms. But it should be noted this field still has many challenges in place. As optimization problems tend to become larger and more complex, scalability becomes more strenuous. As a result, computational efficiency can slow down and struggle to effectively solve problems. Whether it's millions of potential nodes in a network, or millions of variables and constraints, new scalable algorithms and complex frameworks have to consider a lot of information to take into account which traditional algorithms might struggle with. Optimization problems often intersect with other fields as well, be it mathematics, engineering, computer science, finance..etc. Interdisciplinary collaboration is thus important when considering how complex real world systems are. Communication barriers and flaws in different domains can influence the efficiency and success of a network algorithm. Another challenge in optimization is the need for decisions to be made in real time with real time data. Combining optimization algorithms with real-time systems, such as sensor networks, IoT devices, and cloud computing platforms, can be tricky with important factors like reliability not always guaranteed. Similarly missing data, data quality issues, and data biases can negatively influence algorithm and model performance when doing an optimization problem. With organizations seemingly having an increasing reliance on data, the importance of high quality data can pose challenges if not guaranteed.

4. Conclusion

(4.1) In conclusion, operations research methods are applied to a wide range of industries and domains, using network algorithms as a frequent tool. The historical context of network optimization traces back centuries, with roots in graph theory, mathematical modeling, and simple techniques. Over time, algorithms such as Dijkstra's, A* algorithm, Edmonds-Karp, and Ford-Fulkerson have been significant milestones that contributed to the development of many modern optimization methods. Through these methodologies, organizations are enhancing their efficiency, reducing costs, and improving service. I've enjoyed doing research on this topic and continuing off my first presentation. Having the ability to learn from my classmates and benefit from how they communicate their findings was instrumental in how I prepared and decided to research my topic. I tried to incorporate feedback from my classmates into my presentation style, putting an emphasis on being more thorough and clear when doing any step by step mathematical examples. Similarly, there were suggestions about the history and military affiliation behind operations research after my second talk, so I decided to dive deeper on this topic for my paper. Network optimization faces both opportunities and challenges in the future. Innovations in machine learning and algorithms including dynamic optimization techniques can potentially increase optimization performance. However, factors like scalability, trouble integrating with real-time systems, and data reliability still pose challenges or areas where potential problems can occur. Regardless, the use of network algorithms to optimize problems will remain prevalent in our society and will continue to impact complex systems everywhere.

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