The Doomsday Algorithm and its Applications

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Volunteer?

- Need some date mm/dd/yyyy

Origins

- Popularized by John Conway 1973
- Extended Lewis Carroll's ideas
- Multiple variations to solve
  - Zeller's Congruence (suitable for computer)
  - Carroll (mental)
  - Odd + 11 method
- Main goal to determine Day of Week (DoW) of any given date

Some calendar information

- Gregorian 400-year cycle
- 365.25 days in a year?
- More precisely, 365.2422 days (365 days , 5 hr, 48 min 46 sec),
  - By rounding to .25 every year, it extends the calendar leap cycle by ~44 min
  - After 100 leap cycles, would be ~3 days ahead
  - So only include leap days on centuries divisible by 4 and 400 (2000,1600, etc.)

“Doomsday”??

- Days that will always fall on the same DoW of a given year
  - 4/4, 6/6, 8/8, etc.

Month	Month/Day	Mnemonic[8]
January	1/3 **OR** 1/4	the 3rd **3** years in 4 and the 4th in the **4**th
February	2/28 **OR** 2/29	last day of February
March	3/0 or 3/14	"Pi Day" 3/14
April	4/4	**4/4**, 6/6, 8/8, 10/10, 12/12
May	5/9	**9-to-5** at 7-11
June	6/6	4/4, **6/6**, 8/8, 10/10, 12/12
July	7/11	9-to-5 at **7-11**
August	8/8	4/4, 6/6, **8/8**, 10/10, 12/12
September	9/5	**9-to-5** at 7-11
October	10/10	4/4, 6/6, 8/8, **10/10**, 12/12
November	11/7	9-to-5 at **7-11**
December	12/12	4/4, 6/6, 8/8, 10/10, **12/12**

How it works

- 3 main steps
  - Century anchor day
  - Doomsday for that year
  - Nearest day

Notation

- Easier to work with DoWs as numbers
  - Sunday (0)
  - Monday (1)
  - …
  - Saturday (6)

Century Anchor Day

- Remember the 400-year cycle?

Two ways to calculate anchor:

[1] $(5 * (c \bmod 4)) \bmod 7 + 2$

$c = \lfloor year/100 \rfloor$

[2] $r = c \bmod 4$

r=0 ➔ 2

r=1 ➔ 0

r=2 ➔ 5

r=3 ➔ 3

- Gives us our starting count

What is that Year's doomsday?

- $365 \equiv 1 \bmod 7$
  - Days shift by 1 on regular years
  - By 2 on leap years
- Year DD = Anchor + #days shifted from reg years and leap years
- Conway's method :

$$\left(\left\lfloor \frac{y}{12} \right\rfloor + y \bmod 12 + \left\lfloor \frac{y \bmod 12}{4} \right\rfloor\right) \bmod 7 + \text{anchor} = \text{Doomsday}$$

  - Where *y* is the last two digits of a year
- Where's this 12 coming from?

“I have to do all that in my head??”

Not quite!

- Conway's method works well for larger years, but a lot to manage
- Simpler version:

$$\left(y + \left\lfloor \frac{y}{4} \right\rfloor\right) \mod 7.$$

- Downside of dealing with large years (like XX89)

Nearest Day to Doomsday

- Almost there!
- Find the difference between (given date) and (that month's DD)
- Add that difference to the DD date for that year
  - Relate that number back to Z_7
- Convert back to a day

Example

- Nov 5 2014
- DD = 2 (century) + (14 + 3) mod 7 = 5
- November DD: 11/7 (fri) ➔
  - Nov 7 – Nov 5 = 2 days
  - 5-2 = 3 ==> wed

Some more examples

- Jun 12 1939
- July 4 1776
- Dec 5 2143

Computer Implementation: Zeller's Congruence

$$h = \left(q + \left\lfloor \frac{13(m+1)}{5} \right\rfloor + K + \left\lfloor \frac{K}{4} \right\rfloor + \left\lfloor \frac{J}{4} \right\rfloor - 2J\right) \bmod 7,$$

- Where,
  - q = Day of month
  - m = month (jan =13, feb =14)
  - K = year mod 100
  - J = [year/100]

Alternative method: Odd + 11 method

- Another method suitable for mental calculations
- Let *Y* be last two digits of a year
  - If *Y* odd ➔ +11
- *Y*= *Y*/2
  - If *Y* odd ➔ +11
- *Y* = 7- (*Y* mod7)
  - Output achieves the same as step 2
  - Still add century anchor after

$$7 - \left[\frac{y + 11(y \mod 2)}{2} + 11\left(\frac{y + 11(y \mod 2)}{2} \mod 2\right)\right] \mod 7.$$

Other applications

- Moon
  - Phase cycle is 29.53 days
- Tides and energy sources; animal life
  - Use moon phase and its cyclic nature to make predictions about dependent processes
- Financial analysis / business planning
- **Discussion:** Where else do you think this could be used?

Questions

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