Kurt Gödel and the Philosophy of Mathematics

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Math 400: Presentation 2

Kurt Gödel (1906-1978)

First Incompleteness Theorem: Peano arithmetic is incomplete

Second Incompleteness Theorem: it is impossible to prove that Peano arithmetic is consistent using finitary methods

Objective Mathematics: the system of all true mathematical propositions

Subjective Mathematics: the system of all demonstrable mathematical propositions

Kurt Gödel (1906-1978)

"With respect to subjective mathematics, it is not precluded that there should exist a finite rule producing all its evident axioms. However, if such a rule exists, we with our human understanding could certainly never know it to be such [with mathematical certainty; rather, it could be known only with empirical certainty on the basis of finitely many experiments]."

Subjective Mathematics: the system (or set) of all demonstrable mathematical propositions

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He presents two general attacks on 'materialistic philosophy' (Aristotelian idealism) in mathematics:

1. The working of the human mind cannot be reduced to the physical composition of the human brain
2. There exist absolutely undecidable mathematical propositions

"We don't create machines out of nothing, but build them out of some given material. If the situation were similar in mathematics, then this material or basis for our constructions would be something objective and would force some realistic viewpoint upon us"

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He presents three arguments in favor of Platonic realism:

1. Ignorance of our own creation would only occur through a lack of clarity, but clarity has not improved the situation of our ignorance
2. 'The activity of the mathematician shows very little of the freedom a creator should enjoy … the mathematician does not create the validity of his theorems'
3. Sets of integers are required to prove properties of the integers, which contradicts the assumption that they are separate creations

Kurt Gödel (1906-1978)

Gödel turns his attention to the view that mathematics is merely a collection of semantical propositions that can be reduced to a set of equivalent tautologies (such as a=a)

"It seems to me that one ingredient of this wrong theory of mathematical truth is perfectly correct and really discloses the true nature of mathematics. Namely, it is correct that a mathematical proposition says nothing about the physical universe existing in space and time, because it is true already owing to the meaning of the terms occurring in it, irrespectively of the world of real things. What is wrong, however, is that the meaning of the terms is asserted to be something man-made and consisting merely in semantical conventions."

Goal: prove the existence of 'non-tautological relations between the concepts of mathematics' (or, accept that axioms require pre-acceptance before the alleged reduction to a tautological statement)

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Thus, Gödel concludes the existence of such 'non-tautological relations'.

Note (from Gödel): analytical propositions are, in general, far from necessarily being 'void of content' (like a tautological statement)

"Even the assertion of the existence of a concept of 'set' satisfying the axioms of set theory is so far from being empty that it cannot be perceived without again using the concept of 'set' itself."

In the end, Gödel endorses a famous quote from Hermite:

« Il existe, si je ne me trompe, tout un monde qui est l'ensemble des verites mathematiques, dans lequel nous n'avons acces ques par l'intelligence, comme existe le monde des realites physiques; l'un et l'autre independant de nous, tous deux de creation divine »

Bibliography

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