

Mathematical Education

2022 Spring Math 400 group

Abstract

Mathematical education is a vast and important topic. We share different views and suggest some improvements of the current systems.

1 Introduction

Mathematical education is a vast and important topic. We share different views and suggest some improvements of the current systems. The stakeholders include students, teachers, parents, educators, policy makers, employers, etc. It is challenging to design a perfect system to fit the needs, and produce the best results for different parties, especially in view of the rapidly changing society. We will share some experience and make some suggestions for possible improvement of the systems with taking into the account of the limit resource and diverse need of people to have a good mathematical training. The discussion will focus on the following topics.

- 1) Sharing some good and bad experiences of mathematics study in the US or elsewhere. Suggestions of changes of the teaching or systems that might make differences. (This is also related to problem 4 below.)
- 2) Sharing knowledge about previous or current mathematical reforms if you are aware.
- 3) How to address questions such as why study mathematics which one will not use, does advanced study of mathematics just deal with more complicated equations, abstract theory, etc.”
- 4) How to improve mathematics education at the grade school level, college level, especially, taking into account that resources are limited, and there are diverse needs and interests of students.
- 5) Additional comments.

2 Different views

- 1) I wouldn't characterize any of my experience studying math as strictly good or bad mainly because I do not enjoy the subject as much to give a fair assessment on my experience. I would say though that I don't think mathematical education, and education in general, should

be placing so much emphasis on grades, although I must also admit that I do not know of a better way to objectively assess how well students are absorbing the materials. I feel that education is ultimately about learning, and I think students should own that responsibility. To have a third party, an instructor, say, assign a number that supposedly reflect student learning outcome is, I think, a betrayal of the value of education.

2) There are some countries, Nordic states in particular, that incorporate very little, if any, standardized tests into their education. I do not know how well that approach pan out, but I recall seeing many positive reports on Finland's educational system, and the Finns seem to perform better than their peers in the United States that experience a test-based education on academic metrics such as the PISA.

3) I think people study math not necessarily because of its real-world applicability, but mostly because they find beauty in it. I personally don't find studying advanced mathematics to be inherently useful, but I can see how it appeals to a select group of individuals who are able to perceive beauty and dedicate their careers to it. "Beauty lies in the eyes of the beholder," they say.

4) I feel that a good mathematics educator motivates students to seek and discover knowledge as opposed to impart them. I am no expert of pedagogy, but I feel that mathematics education, at least the way I experienced it, is mostly centered on learning to apply algorithms to specific problems. I think if the students are given greater liberty to steer their own course of study instead of following a rigid curriculum, they would find the experience much more rewarding and end up owning the knowledge they acquired. It is by no means clear how this can be realized, but a good place to start, I feel, is to have the instructor's responsibility switched from imparting knowledge to asking the right questions and have the students take on the adventure to discover the answer.

- 1) I think I am fortunate in having relatively few bad experiences when studying math during my grade school, high school, and college career. Of the few bad experiences, I think they can mostly be attributed to a poor teaching style, rather than the material being difficult. Certainly, challenging material can't be fully understood by the whole class, but the manner in which it is taught, or sequentially presented can aid in how much of that material is retained. I think with any new math area, establishing the basics or important recurring ideas are essential; and if those are presumed to be known or well understood before jumping right into new, applied content, I think many students will quickly fall behind. CK mentioned "honors" classes toward the end of lecture, and it reminded me of how all of my high school classes were set up. Classes were ranked from "hardest" to "easiest" by AP, Honors, Accelerated, College prep 1, and College prep (CP). I think that distinguishing classes in this way, perhaps due to nomenclature, might influence students to approach their studies differently if they know what level they're in. I would assume, to an extent, that a student in the CP level would approach their focus on studies differently (either with less intent or not as diligent) as

someone in an honors class because they knew they were in a “lower rigor” class. A change I would propose for teaching systems (should they choose to keep a segmented class level), is to adopt a neutral naming scheme, like A-E for example. I think this way, even if A is the highest and E is lowest, the overall perception of where a student lies relative to other students is significantly reduced; And psychologically, this could lead students to approach studying differently. The gaps between E to D or even E to A, this way, are ambiguous, and could be very small; whereas hearing College Prep vs Honors or AP gives the impression that there is a huge intelligence gap between these two students; And I think that might influence students to look at school and possibly themselves with a “lesser” mindset.

2) Other than the textbook reform in Florida that was brought up in class, I am unaware of any mathematical reforms, current nor previous.

3) I think I would answer the “why study this if one won’t use it” question with, “because it is important to know, or at least understand, the relationship/interaction between things in this world.” Would you advocate for not teaching basic arithmetic since computers and phones have calculators, so there’s no need to use a pencil and paper to write out the calculation? Even if it does seem unnecessary, teaching the concept ensures that the student will know how numeric properties work and how they interact with one another. For topics in algebra like polynomials, sure you won’t be pulling out the quadratic equation on your free time but knowing that these sorts of relationships exist in real life and that there’s a way to explain things, I think, is the reason why one should study math topics even if they won’t be used. Additionally, I would say advanced study in mathematics, does involve more complicated equations, etc., but it does the same as early math (as discussed above): explaining and modeling relationships between things in the world. I think they seem more complicated and abstract to most people because they are so far removed from these relationships. Basic arithmetic is not complicated since it is commonly used day to day; whereas something like linear algebra seems complex and abstract because most people are not computer scientists, for example, and do not need to readily understand how matrix operations work.

4) I think one of the biggest improvements that could be made with mathematics education is ensuring no student is left behind. On the basics and fundamentals of mathematics (like arithmetic and most algebra/geometry), I think it serves no help to the student if they don’t fully understand a concept that will build on itself continuously later in their career. The chances of them coming to dislike math because they “don’t get it”, I think are very high if they are not given the time to fully grasp the concept. Another improvement that could be made, mainly at the grade school and most of high school level, is to teach and test the key parts of the subject/area being taught. I think most students tend to dislike math because what they are being taught is the same things that were being taught decades ago. Not that these things aren’t important or shouldn’t be taught altogether, but I think for a general math education, hammering in how to solve different logarithmic equations, side angle side,

or congruent angle theorems, for example, aren't the most important things growing math students should be memorizing. As another student in class mentioned, I would agree that things like logic or proof techniques would be more useful in early math classes

- 1) I think that, at the grade school level, mathematics in particular is a subject that is undeserved. It was common to have math teachers that had little to no background in mathematics, for the sake of having a teacher. This made it difficult to learn higher level topics, and hindered the ability for students to receive alternate explanations and fundamental principles, as the teachers often did not know these themselves. However, the times I did have teachers who had a mathematics background, I found the lessons far more enjoyable and easier to digest. As someone who has always done well in mathematics, I did not suffer so much from the differing backgrounds, but having under qualified teachers definitely impacted my interest in the subject for many years until I reached the college level, and rediscovered my interest.
- 2) At present, there is an impressive politicization of mathematics happening in the US. The study of mathematics has come under great scrutiny for hardly a reason beyond political alliances. In particular, the common core curriculum is being flagged and used as a talking point of the Republican party. This is all despite the effort being supported for many years, and actually developed by the very same political party. The turn on the common core initiative came with the election of Barack Obama, and his implementation of the standard. As a leftover push from the Bush administration, it was altered minimally, but the sheer fact of implementation by a Democratic president made it a sudden source of anger that has persisted to this day and metamorphosed into the argument that CC framework now pushes students into 'being indoctrinated', as executive orders pop up in states that effectively ban CC curriculum.
- 3) I think the best answer to 'why study mathematics' is the honest one, which is that it will make other facets of life simpler. Having an understanding of logic and analysis is useful in nearly every facet of life, from reading and writing to coding. Being able to dissect information and break down logical arguments using the critical thinking and analysis skills from a study of mathematics is invaluable.
- 4) There is a common notion that schools should not impose a sense of 'otherness' in the US. That is, it is frowned upon to separate students based upon skill level. This practice, in my experience, results in upset parents and students who feel they've been left out. However, I think such a system is wholly necessary in areas such as mathematics, since people learn at different rates, and understand material in different ways. This sort of system is more developed in upper grades and college, when the student and advisor can choose the courses taken. In these upper grade cases, it's not often seen as 'othering'. However at the elementary age, there is no such option. This leaves for a sort of three-bucket system in mathematics, where those who struggle the absolute most, and those who excel the very best, are the only

ones taken out of their standard course and placed into specially-accommodating courses. This is not enough to improve the abilities of students at this age, as it neglects those who struggle, but make it through, or those who simply fake it until they make it, so to speak. Having more options for placement for students who are identified in those categories could greatly improve the development of mathematical skills at an early age, in my opinion.

- 1) I think that my favorite way of learning mathematics is by seeing the real world applications of it, which is why I like statistics so much. In statistics, most problems that we do have very obvious real world applications and the problems presented allow us to think about what we know and choose the best technique to solve a given problem. Most of my negative experiences has to do with memorization of things. An example of this is math 311. For the exams, we had to memorize 15-20 proofs and then write down about 10 of them on the exams.
- 2) I'm not sure about any sort of mathematical reforms that are going on now. I think that finding a way/courses that might provide without an interest in mathematics something useful that is related to mathematics would be helpful in people having a less negative view on mathematics.
- 3) Mathematics is used in nearly every subject to some extent, including the humanities. Understanding how the mathematics works is vital to understanding how other things in a particular discipline work because there is likely some sort of mathematical connection. Also, mathematics is more than just learning equations, it also provides problem solving skills, especially in high school, that will be useful later in life. The advanced study of mathematics has lots of implication for important things relating to technology and other things that are rapidly changing in our world. For example, the things that I am learning in Math 452 this semester forms some of the basis and theory behind machine learning, which has become more and more prominent in our society.
- 4) I think that one of the biggest improvements in math education at the grade school level is introducing some theory or proofs in a highschool level. I think exposing people to this early on would help show that there is more to math and we're not just memorizing or learning things for no reason. I wish that I would have known about this in highschool, because when I got to math 214 I had no clue what was going, or what a proof even was. I also think that providing math courses for people who aren't really math people is important, so that way they still have the exposure to it, but in a way that is interesting to them.
- 1) I myself have had pretty good experiences with math. However, many of my friends have not. There seems to be a widespread belief that some people are good at math and some people are bad at math. I'm not sure if there is any quantitative evidence of such a division, but I think that this is pretty much accepted as fact through personal or anecdotal evidence. The math educational system seems to compound this understanding by splitting students into advanced/non-advanced groups, and kids learn pretty quickly which group they are in.

2) The best example of failed math reform in this country is surely New Math (https://en.wikipedia.org/wiki/New_Math), which was instituted following the Soviet success in the Space Race to try to get American children ahead of the Soviet children in terms of mathematical education. From Wikipedia: "Topics introduced in the New Math include set theory, modular arithmetic, algebraic inequalities, bases other than 10, matrices, symbolic logic, Boolean algebra, and abstract algebra." These topics were unknown to most parents, likely too abstract/complicated for children, and New Math was a very unpopular failure that was quickly abandoned. It is a cautionary tale in starting abstract math too early, without parental consultation.

3) Most subjects taught in school are meant to teach kids processes rather than specifics. I think teachers should be upfront that most students will not use any math in their daily lives beyond basic arithmetic. However, the truly important things they are learning is systematic logic, learning a method and applying it, and simple discipline. This is perhaps even more true for abstract math, although I would argue that the basic logic inherent in advanced mathematics is perhaps the most useful lesson mathematics can teach.

4) As I said in class, I think the best reform to college-level math would be to offer a basic, simple class on introductory logic in the math department that humanities majors could take for their NQR credit. These people have already had 12 years of computational math exposure from elementary school to high school, and they have probably already discovered whether they like it or not and gotten as much as they are going to get out of it. Introductory logic would be a change of pace for them and it would directly help them as they make arguments in their own classes. An English major may get an enjoyment and utility out of learning what the basic proof method is than they would ever get out of Calculus II, in my opinion. If they want a more computational class, they are free to take Calc II, but there should be another option for them.

- 1) I guess most of my bad math experiences have been with memorization and timed exams. I have never been good at timed tests, since elementary school, and still in college the exams I have done worst on are the ones that I've been incapable of finishing, even though I do well on the parts I do finish. My good experiences have been kind of obvious, like ultimately doing well in a course I struggled with or being encouraged by professors or teachers.

2) My mom is a math teacher in middle school, and I know that the school system in my county in Virginia (Hanover County) is significantly changing the standard track for students' long-term math education. I do not know specifics off the top of my head, but I know that they are trying to make alternatives to the 'calculus' track. So usually students take two years of algebra, one year of geometry, and whenever they are done with that there are almost only calculus and precalculus courses left (the exception being AP Statistics). Many students, however, who will not need calculus, find it extremely difficult, or are not interested in it then find themselves taking very easy, unpopular math courses to fulfill requirements or keep

math on their schedules for college. So, they are trying to add alternatives for students with these objections to the current options.

3) A good friend of mine told me amusedly that, when he was beginning undergrad, he was skeptical of the math major because and said he thought, "What are you going to do for four years? Just more and more complicated equations?" It was only after a prospective PhD student he knew explained what we do in math at length that he understood that a math major is much different than what is done. I think he specifically was the kind of person that would have really enjoyed theoretical math but didn't like the kind of procedural math usually taught in high school. I think the solution to that is just to introduce at least some theoretical math to students at some point in high school, something usually never done (in a meaningful way) in at least Virginia. As far as the "I'll never use it" objection goes, I think it is just important to point out to students (especially at the grade school level), that when they object like this, they are often just quite wrong. Sure, I rarely use algebra in my daily life, and to the extent I do it's probably because I'm a math major. But many students want to do engineering or a science-or math-heavy career, and I think it's more than enough to point out that, let alone algebra, but calculus and even more complex kinds of math are necessary in many (if not most) of those careers. Objecting that you'll never use them just presupposes you've closed off those options to yourself.

4) I don't have too many ideas about how to improve math education, but I think memorization and procedural learning should be reduced as much as possible in favor of concept-based learning. Many of the tricks I learned to solve simple equations or math problems I have just totally forgotten, and largely because they are actually nearly useless (even to a math major). Aside from that, I don't have any decisive ideas.

5) I think another way to improve math education in a college setting is to provide courses which would give humanities students who need to take STEM courses good options that would be useful or interesting to them, not just independently but in connection to their interests. The easiest two examples I can think of are to do a logic and proof techniques course or a history of mathematics course. I actually had a friend who is now a PhD student and never took a math course in college say he would be quite excited to take a history of math course, and my impression is that this, at least for history of science in general, is a common interest among humanities students; it combines a kind of historical study these students are more comfortable with with learning about a discipline often referenced or relevant to their own and which they can learn about without getting lost in details they would not be able to master easily.

- 1) A good experience I have from the study of mathematics is how wide the range of the study is. Being in the major for 4 years shows how many different areas of study there really are with all of the classes. I have been on an applied track for these 4 years and have met a lot of people doing the same thing. However, in this class I have seen how others have taken

the major and it has showed me that math is more about the computations and analyztics but also about how math is created and the system that is put in place in order for us to use math in such an applied way.

2) I am not aware of any reforms current of past that have taken place in the classroom as I have always seen math being taught the same way.

3) While I am not the biggest advocate for logic based mathematics classes, I do think that they do teach important skills and can be used outside of the classroom. Different forms of proofs such as induction or contradiction are great skills that can be applied in other fields and do not have to involve complicated numbers or equations.

4) While it is tough to teach certain subjects to younger developing kids, I do think it is important that kids are exposed to more fields of math other than geometry and algebra as those are heavily based in the computations and not the applications. As many people say, computational mathematics is not used often outside of the classroom, but if the spectrum of math subjects were more available to the younger audience, I think that it could be a way to improve interest in the subject. And with better interest, kids apply themselves more and learn better for themselves.

- 1) We mentioned in class the negative attitudes that surround mathematics in the United States, particularly in grade school. Unfortunately, I held these attitudes until I stopped taking grade school classes and switched to community college in my junior year of high school, where I found the mathematics classes to be much more engaging. I think these negative attitudes represent the main issue that should be targeted by math reform. As we have seen, pedagogical orthodoxy has not always considered ‘teaching mathematics’ to be principally important; many educators tend to consider the process of learning mathematics to be more important than mathematics itself because it presents opportunities to teach generic learning skills (so-called SEL). I think negative attitudes towards mathematics develop mostly in early grade school and are usually formed by high school, and that they are probably the result of mathematics being taught without respect to the field of mathematics itself, i.e. due to basic mathematical principles not being taught or referenced at these earlier stages (there is usually no reference to the existence of sets, for example, and I think most students finish grade school with little to no understanding of what mathematicians actually do or of the intellectual structure of mathematics is a field). The case of a student not understanding mathematical material but not knowing exactly why seems to be fairly common, and I think that this is a result of certain principles not being taught. In the end, the question of whether educators want to seriously teach mathematics in grade school seems to be a less trivial dilemma than many might assume, for example with reference to the statement ‘if you asked 100 C.E.O.s what skills they want in a new hire, the top five skills are going to be about social-emotional learning — not algebra’. I think an ambivalence on the part of many educators towards whether mathematics is worth teaching is probably a contributing factor

in prevailing negative attitudes towards mathematics among grade school students.

2) The developments in Florida are extremely recent, and in fact we do not yet have a full picture of what exactly the government is trying to achieve. Here is the NYT article: <http://nytimes.com/2022/04/22/us/florida-rejected-textbooks.html>

3) I think one could direct this question to many educators in addition to students, and to that extent I think it is too broad to be answered concisely (by ‘mathematics which one will not use’, are we talking about pure math? because many students think that calculus is not useful, when in fact it is necessary to perform any meaningful statistical operation. So, the answer to this question when directed towards pure math will be very different than the answer to this question when directed towards applied math). It seems to me that the most useful mathematical concepts for most high school students relate to arithmetic and statistics. So, it is easy to see that those concepts should be taught. This question, however, has nothing necessarily to do with the debate over whether mathematics as a discipline should be taught in grade school, since many people assume that it is possible to teach arithmetic and statistics without approaching the principles of mathematics as a field at all (basic set theory, etc.), which is essentially what high school math represents for most students, even though they do not usually learn any meaningful statistics (we used to call AP Statistics ‘AP Calculator’ – in fact, it is necessary to introduce calculus and operations of set theory to learn any meaningful statistics, for example in MATH 351; perhaps the operations of set theory could be introduced earlier).

4) I think that most problems in American math education begin early in grade school, not in high school or college.

- 1. Most of my experiences with math have been good ones. Actually, I enjoy learning by reading books better than from lectures. I found the lectures that professors discussed things in addition to textbook more intriguing than those that focused only on what is in the book. See problem 4 for more opinions on improving mathematics education!
- 2. I know kids in Beijing had to compete solely on mathematics to get into a good middle school (which was insane). Now they are evaluating students from a more well-rounded standard (this happened within the past decade).
- 3. I feel that advanced study of math boosts one’s thinking. I personally associate advanced math with general laws in nature or society, or gain inspiration for philosophy from time to time. I want to emphasize that mathematics is NOT just about knowledge that is content based. It helps to build the mindset. If one enjoys dealing with math, it is beneficial to go deeper into this subject.
- 4. For primary school, I believe one of the main obstacles for better math education is the limited profession of teachers. Given that they are often not so good at math, it might be helpful for them to encourage students to obtain resources outside of the classroom—from books

or the internet, if students show interests to certain topics not discussed thoroughly in the textbook. From my personal experience, for college level, I want a course to introduce me to the branches of advanced mathematics when I decided to take math seriously. When I claimed my major as applied math focusing on statistics, the truth is that statistics is one of the very few things I know about math. I have no idea what people studies for graph theory or topology or so many terms that are so cool yet I never studied. The reality is that I could not afford the time of an entire 3-credit class for an area that I'm only curious about what is it about (I'm curious about so many areas in math and want a slight touch on each one of them). I believe most of the students would find some areas of math attractive if they are introduced to the tremendous world of mathematics.

- 1) I remember when I was in elementary school, there is a question like: "a snail is climbing up a rod of 10 meters. It climbs 5 meters in the day, and falls 3 meters in the night. Then how many days it needs to climb up top?" Actually the answer is not $10/(5-3)=5$. Instead, the snail climbs 6 meters in the first three days, and on the fourth day it can complete the rest 4 meters. I think these questions requires little math knowledge and a huge part of it is designed to trick students, and create a difference in scores within a test.
- 2) When I was in elementary school, I need to take olympic math classes where there are many tricky questions like I mentioned in part 1, and I hear that now this kind of education is not encouraged.
- 3) I think learning math really helps to improve my ways of thinking more flexibly. I might not remember most contents from number theory now, but at least it really helps in my logical thinking.
- 4) I think grade school level math can focus more on logic training, and set concrete foundations of math. I think many contents from math 214 can be introduced at an earlier stage.
- 1) And 4) When I was in elementary school, I learned a problem called Chickens and Rabbits in the same cage. There are different ways to approach this problem. In my opinions, the most interesting one is that we assume that the chickens and rabbits can understand our instructions and let them raise one foot first and then raise another. Young children would not enjoy reciting all the concepts and theorems, but they love some interesting problems where some funny scenarios happen such as chickens raise both of their two feet in the cage. As the saying goes, interest is the best teacher.
- 2) As an international student, I am familiar with the pattern and reforms in American grade school. However, in China, mathematics is required through all grade levels. In The National College Entrance Examination (NCEE), commonly known as the gaokao, mathematics accounts for 150 points out of the total 750 points and has some extremely tricky problems.
- 3) From my perspective, we study what we may not use since it helps us understand the world better. Besides, , mathematics develops our logical reasonings so that we could see practical

things and solve the real-world problems in a more logical way. Besides, in my opinions, advanced study of mathematics does not just deal with more complicated equations, abstract theory, etc. On the other hand, it simplifies everything and aims to study the fundamentals but in a more philosophical way, which could be seen in course like real analysis and abstract algebra

3 Conclusion

References

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