The Minimum Toll Booth Problem

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Abstract

The Minimum Toll Booth Problem (MINTB) analyzes the least number of toll booth locations required to create an efficient transportation network from an origin location to a destination location where optimization is found at a Tolled User Equilibrium with the fewest number of toll booths. We will explore how the equilibrium can be determined through a Genetic Algorithm, as well as identify other heuristic solutions and expand on the applications of this problem.

1 Introduction

With the extensive road networks that have been built throughout the US and the world, traffic congestion has risen as a major issue globally. With millions of travelers taking the same roads, it's natural to experience traffic delays and backups simply due to the volume of people in limited space. Traffic congestion cost the US \$126 billion in 2013 with an expected rise to \$186 billion by 2030 [1], as well as 4.2 billion hours of travel delay in 2007 and 2.9 billion gallons of wasted fuel in 2005 [2]. The great cost of such congestion in monetary value, time, and resources as sparked the need for traffic control and management.

While there are many approaches to managing our road networks to minimize cost, one popular method does so through the utilization of toll roads. The theory is that placing tolls on congested roads will make certain routes less appealing to some travelers and therefore divert demand and reduce congestion. I performed a simple thought experiment in our class to help us understand why this theory works. First, consider two paths from point A to point B. They are entirely similar in every aspect except the time to reach point B from point A is 1 hour on path 1 and only 40 minutes on path 2. When asked which path they would rather take, the class unanimously voted for path 2, the least costly, thus demonstrating the idea that travelers choose paths based on whichever path is best for them personally. However, with all 15 students following the same path, we are likely to experience traffic congestion. We next considered if a toll had been placed on path 2 for \$10. Now, the class was asked to select between path 1, which costs 1 hour of time and \$0, and path 2, which costs 40 minutes of time and \$10. We now saw the class split their choices, as some saw those 20 minutes as worth more than the monetary cost of \$10 whereas other students preferred the longer yet free of monetary cost path. Each student chose the path they saw as less costly for themselves, yet now this decision allowed demand to be diverted and therefore congestion reduced. This is the underlying theory of traffic control with the use of tolls. But in practice, where road networks are less straightforward, the question remains of which roads should have tolls and at what price in order to optimize the travel network for all travelers.

One optimal solution to this question is considered the Minimum Toll Booth Problem (MINTB). As demonstrated, the choice of a traveler that is best for himself is not necessarily the choice that is best for the system as a whole. Thus, the MINTB solution aims to insert tolls that will ensure an optimal system of travel when travelers choose the least costly path for themselves. However, if this solution requires an unrealistic number of toll booths, it is not an optimal solution for the system. Toll booths are costly in money, since they need to be manned by employees and physically upkept, and in time, as they require travelers to slow down in order to pay the toll. Thus, the MINTB has two overarching objectives: 1) to encourage travelers, who are driven only by their personal travel costs, to choose routes that benefit all travelers for the efficient use of transportation systems, and 2) determine tolls such that the system requires the least number of toll booths.

With this background in mind, this paper will explore the MINTB setup and solutions. However, the MINTB solution is far from the perfect solution to the problem of traffic congestion, as it is difficult to solve and largely unexplored for extensive travel networks. We will explore other traffic control methodologies, weighing the pros and cons of each, and then consider the greater application of research into the MINTB. I will conclude my paper with a reflection on my class presentation and responses to feedback provided by my peers.

2 Defining the MINTB

Mathematically, a transportation network is best represented by a web of arcs and nodes, representing roads and intersections, respectively. We define an O-D pair as a set consisting of an origin and a destination point and we will consider all paths that could be taken from the origin to the destination. Figure 1 gives an example of a 4 node, 5 arc transportation system with the given cost (c_{ij}) and flow (x^{ij}) for each path. If we were to consider the O-D pair 1-4, for example, we see there are two possible paths: 1-4 and 1-3-4.

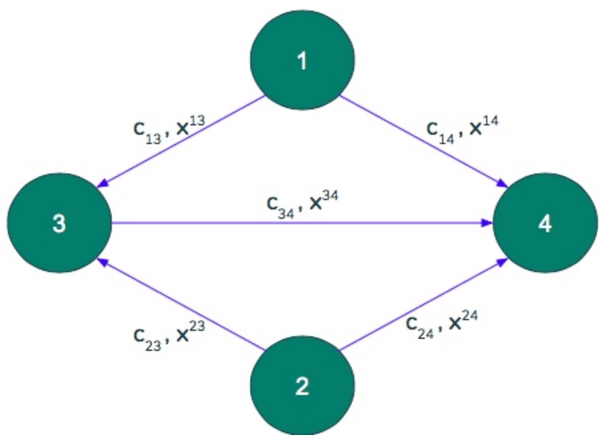

Figure 1: 4 node, 5 arc system example

Let's circle back to the two objectives of the MINTB. The first goal of our solution is to encourage travelers, who are driven only by their personal travel costs, to choose routes that benefit all travelers. When this optimization is found using toll roads, we call the solution the tolled user equilibrium (TUE). The TUE is essentially a compromise between two other types of system optimizations that we could consider: user equilibrium (UE) and system optimal (SO). User equilibrium describes the behavior of users when every user chooses the personally least costly path available, whereas system optimal behavior minimizes the overall travel cost per user. The difference between UE and SO, as their names suggest, lies in who receives the primary benefit, the individual or the system as a whole. The goal of TUE is to allow users to operate as they would in user equilibrium while maintaining an overall efficient travel network for users collectively. The TUE should operate with the same flow, or number of travelers, per route as SO, but has additional tolls placed such that users may end up taking a more costly route than they would in UE without these tolls.

Remember the thought experiment from the introduction? The first situation, in which there was no toll, was an example of UE. Everyone in the class selected the least costly route and none selected the more expensive route for that O-D pair. Since user equilibrium assumes each user selects the least costly route for themselves personally, we find that all users moving from point A to point B will select the same route. This idea can be expanded to a larger system such as that in Figure 1. In this case, we could consider the O-D pairs 1-4 and 2-4, determine the cost of each route for each O-D pair, and find the number of people (out of a given total number of travelers per O-D pair) that will utilize each route in user equilibrium.

We could then compare this flow of travelers to the system optimal result, which is simply a minimization of average cost per user for each O-D pair. Although in some situations the UE and SO solutions may align, in most cases you will find these solutions are often in conflict with each other. We find TUE by adding a toll to one (or more) of the arcs in our system and determining the cost of that toll such that when the flow of travelers is that of SO, our conditions for UE are also satisfied.

3 Heuristic Solution: Genetic Algorithm

With the first objective of the MINTB solution satisfied by TUE, we must now consider the second objective: to determine tolls such that the system requires the least number of toll booths. With the addition of this second condition, our solution becomes much more difficult to solve by standard mathematical methods. The resulting mixed integer linear algebra problem is better solved using heuristic, or trial-and-error, methods.

The idea of the genetic algorithm is to utilize the concepts of natural selection to filter through random vector solutions for the MINTB to find the optimal solution. The basic algorithm includes the following five steps: initialization, evaluation, selection, alteration and termination [2], which would be coded into a computer program and run to produce a result. The first step generates a given number initial chromosomes (or vectors) each with length that is equal to the number of arcs in the network system. Each index will be assigned a 0 or 1 by a random number generator to indicate whether or not there is a toll on the road that corresponds to that index. In the next step, each chromosome is ranked based on whether or not the vector satisfies the overall system of linear equations, known as feasibility, and on the number of toll booths that the solution requires (the number of times 1 appears in the vector). Ranked highest are the chromosomes that are feasible and require the fewest toll booths. A probability is then assigned and used to select which chromosomes will be the parents for the next generation of solutions. After selection, the next generation is created through the process of alteration. The biological process of crossover is mimicked by selecting shared genes of the two parents, such that if both parents have a toll on the i^{th} row then the child will as well, and randomly selecting traits, 0 or 1, where the parents disagree. The steps of evaluation, selection, and alteration are then repeated for a desired number of repetitions until the process is terminated and our optimal solution is determined.

The genetic algorithm has proved effective for small networks and shown to converge to an optimal solution. However, improvements can be made in order to ensure the optimal solution is determined efficiently. One point of improvement is to introduce the concept of immigration. In this scenario, immigration would mean that only a fraction of the new generation is produced by alteration of the parent chromosomes and the rest are randomly generated as in the initialization step. With this concept, we are able to introduce diversity of results and reduce the risk of converging to a local, rather than overall, minimal and optimal solution. Additionally, the use of a weighted, rather than uniform, probability function for selecting the parent chromosomes allows for the higher ranked chromosomes to have a greater influence on the next generation. Finally, it is predicted that more accurate solutions can be found when increasing the number of generations, as opposed to increasing the population size (or number of total initial chromosomes).

Discussions with the class sparked additional points of improvement for the genetic algorithm.

Many other presentations have introduced machine learning and how this can improve the efficiency of problem solving. The genetic algorithm utilized in this example is a type of local search optimization technique that is often solved using a machine learning model. Although the researchers of [2] did not utilize a machine learning model, such an algorithm can be found in SAS [3] and would likely be more efficient and be able to handle larger, more realistic networks.

4 Other Methods, Applications and Further Research

Researchers have explored varying methods of congestion control using tolls only to find pros and cons to each solution. One popular model amongst transportation economists is Marginal Social Cost Pricing (MSCP), a cost analysis of transportation that considers external, non-market or unpriced costs as well as private or market costs [4]. This analysis considers not only personal monetary and time costs to the individual traveler, but also factors in costs associated with traffic accidents, air pollution, oil dependency, noise, highway maintenance, parking, and so on [4]. The sheer number of factors that would need to be considered in a MSCP to be accurate and realistic is incredible, not to mention extremely difficult considering the nuances to attempting to price the cost to a system of some of these factors. Other methods, such as the MINREV and MINMAX, aim to minimize only the overall toll revenue or the maximum toll on a network, respectively [2]. While these solutions may be easier to solve, they provide less optimal solutions for the overall system given they only consider a singular factor. Thus, the MINTB solution falls somewhere in the middle in terms of complexity and feasibility, yet it is still imperfect.

A classmate questioned how efficiency is impacted by the fact that travelers need to slow down in order to pass through toll booths. The MINTB solution somewhat takes this time into account, aiming to reduce the overall number of toll booths and therefore reduce the total number of times a traveler would need to slow down in order to pay a toll. Yet our solution still doesn't factor in this slow down time entirely. The solution would call for a toll on road x, but what if road x is a 2-lane road? Is there only 1 toll booth? Should there be 2? Or 3? Or 6? This moves into the concept of queueing, which can cause delays when travelers are arriving at the toll booths more quickly than they can be processed through them. This is a classic problem analyzed in computer science courses. Some researchers have taken this queueing delay into account using a bi-level programming formulation that considers user choice under conditions of queueing and congestion as well as optimization in response to alternative tolls [5]. This problem represents a variation on the MINTB that factors in the time it takes to go through a toll rather than number of toll booths, which may perhaps be an improvement on this model. Some classmates also questioned how the MINTB problem changes with electronic tolls, rather than toll booths, as we are seeing gain in popularity. This may, in fact, improve the accuracy of the MINTB solution versus that of the problem that accounts for queueing as the need to queue at a physical toll booth is eliminated and cars are able to simply continue driving in their lane and pay their toll by mail later.

Traffic congestion is largely dependent on the time of day that travelers are on the roads. On popular travel days, such as the day before Thanksgiving, or popular times of day, such as 5pm when people are leaving work, traffic congestion can increase exponentially. A downfall of the MINTB problem is that it largely assumes consistent cost of travel. Travel costs can be determined by functions of route flow, however it can be difficult to determine the correct equation for travel cost on a road when it changes so dramatically day by day and hour by hour. An incorrect cost equation can greatly affect the accuracy of our overall solution. Since toll booths cannot be added and removed depending on when they are needed and when they are not, it is necessary to find a solution that will be optimal at all times and, unfortunately, the MINTB solution does not take these varying factors into consideration. Further research is required to determine the best way to factor in such variations.

Although the MINTB solution is clearly applicable to our road networks, it is natural to wonder if this method can be applied to any other situations. In the most general terms, the MINTB is an optimization problem where we attempt to maximize benefit to the overall system while minimizing the means by which we encourage optimization. Theres an aspect of punishment to minimize demand, which makes me wonder if a similar problem could relate to marketing. Consider a popular musical group that is going on tour. They want to maximize profit but cannot overextend themselves and have too many performances. To determine how many times they should perform, the cost of attending the performance, and where they perform, you could think of this as a MINTB problem where we are placing a price on the tickets to divert congestion at the performances, but you want to minimize the number of performances since it would be unreasonable to have a large number of shows. We could similarly apply parts of the MINTB optimization to a company selling a new product that will be very popular, but they have a limited supply that cannot meet demand. The idea of the toll to divert traffic can be related to setting high prices to divert demand of one product or setting low prices of other products to make them more favorable. One classmate even suggested expanding the toll problem as a reward system, instead of a punishment, in which users are paid for taking less favorable routes instead of charged for taking popular routes. While this likely isnt reasonable in the scenario of toll roads, it is interesting to note the MINTB problem could be solved in such a way instead.

5 Presentation Reflection

Overall, presenting on the MINTB was an extremely beneficial experience. Not only did it give me the opportunity to learn about a new topic and explore a mathematical application to the real world, but it forced me to really dig into the material and think about it beyond what I read online. To prepare my presentation, I had to first have a solid grasp of the topic and then determine which parts of the paper were most important and interesting for the class to learn about. I decided to mainly focus on the optimization routes, as opposed to the process of the genetic algorithm, because it felt more interactive, easily accessible and understandable, and more interesting given I do not have experience creating the genetic algorithm myself. Overall, the comments from the class were favorable and none seemed to be wanting a deeper discussion of the genetic algorithm, which I was worried I didnt quite have enough time to cover in detail.

I do, however, agree with many of the class comments that suggested further applications of the MINTB and alternative methods of solving the problem that are more realistic. I chose to focus on a small example for the purposes of the presentation but should have expanded to talk a bit more about how the problem would look in a larger transportation network. I have taken some of their suggestions and expanded upon them in this paper, however I recognize this further research and extended applications portion was lacking my original presentation. These suggestions will help guide me towards a more thorough presentation in the future and have allowed me the chance to reflect on my presentation and my topic.

References

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