Understanding Symmetries and Geometry through Dance

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Based off research by Karl Schaffer

Abstract

When you think back to elementary school, which class did you enjoy more? Math class or P.E.? For most people, this question prompts a vivid memory that leads the person to know the answer within milliseconds. If you were a student that enjoyed physical activity and going outside, you will choose P.E. If you were a student that resented any form of physical activity that required you to do something more than walk, you may choose math. Most people would argue that they enjoyed P.E. more since mathematics is generally viewed as difficult and the cause of frustration amongst many students. It may appear as if mathematics and physical activity, specifically dance, have no common ground, but I would like to investigate how mathematical concepts such as symmetry are present in dance, how mathematics can be taught through dance, and the physics behind a dancers movements.

Introduction

Every dance choreography incorporates the use of angles, geometric shapes, parallels, patterns, and symmetry in some way. The use of symmetry and geometry in dance provides a visually appealing display that improves the dancers' performance. For this paper, let us think of dance as an artistic endeavor and look at how deep mathematical concepts can be viewed through dance. I will be referencing *Dancing Mathematics and the Mathematics of Dance* by Sarah Marie Belcastro and Karl Schaffer to elucidate the interplay of mathematical concepts and dance.

Superficial Links

When one thinks of the rudimentary connections between mathematics and dance, you might consider how music is divided into counts and how dancers use counts to mark the times at which they will execute their movements. Dancers utilize the rhythm of a count to influence the speed of a dance and may change the tempo throughout a performance for artistic effect. Also, dancers create geometric shapes and angles with their own individual bodies and with other dancers in a group performance. These are what we will note as superficial links between mathematics and dance.

Teaching Math Through Dance

Erik Stern and Karl Schaffer, founders of the Dr. Schaffer and Mr. Stern Dance Ensemble, gave a Tedx Talk in 2012 on the connections between mathematics and dance in the classroom. They began their talk by simply moving to greet each other on stage then proceeded to incorporate dance movements as they attempted many ways to shake hands. Stern and Schaffer cleverly found an interactive way to teach their students vaudevillian handshake permutations. Allowing student to incorporate movement while learning mathematical concepts helps the brain remember the lesson kinetically and mentally. Also, students are able to completely immerse their full body in the lesson which provides a break from normal desk-learning.

Corinne Wolfe uses dance movements to teach her students the connections between different shapes. In the article, "Dancing Triangular Squares—The Process of Creating a Mathematical Dance," Wolfe was able to convey how two consecutive triangular numbers make a square number to her students and the audience of the performance.

Symmetry and Rhythm

Symmetry is present throughout all of nature. We can see symmetry in microorganisms, plants, animals, and even humans. Humans have reflection symmetry that allows us to make symmetric shapes and movements by themselves or with a group but maintaining symmetry throughout a piece is not an easy task. Humans lack rotational symmetry so it's impossible to for a dancer to continue a turn or spin forever since they will lose their balance.

Another type of symmetry that is present in dance is pattern symmetry throughout the choreography. Types of pattern symmetry include reflection, rotation, translation and glide. We can now take a closer look at how symmetries are present in dance. In the following image, we can see that a mirror reflection and a 180-degree rotation of the top dancers yields the same position as the bottom dancer who simply did a glide reflection.

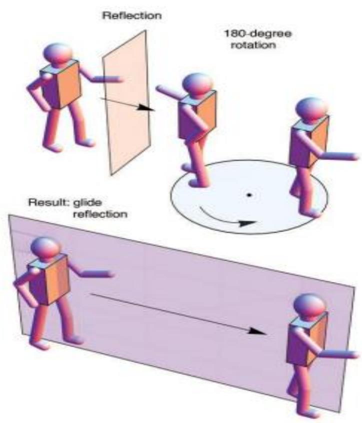


Figure 2. Composing a mirror reflection with a rotation yields a glide reflection.

Here, we can observe the connection between the Klein four group and dance movement if we are to consider combining translation, mirror reflection, 180-degree rotation, and glide reflection pairwise. The Klein four group is the smallest non-cyclic group and the only dihedral group that is abelian. Geometrically, the four elements of the Klein four group include the identity, the vertical reflection, the horizontal reflection, and a 180-degree rotation. Let us now consider the translation movement (T) as the identity, mirror reflection movement (M) as the vertical reflection, glide reflection (G) as the horizontal reflection, and clearly the 180-degree rotation as itself (R). We then find that we have made the Cayley table of the Klein four group with operation multiplication.

	T	**G**	**R**	**M**
T	*T*	*G*	*R*	*M*
G	*G*	*T*	*M*	*R*
R	*R*	*M*	*T*	*G*
M	*M*	*R*	*G*	*T*

There are also symmetries in time and space such as unison, canon, inversion, and retrograde that dancers use and perform. A unison is a set of movements performed at the same time for each dancer. A canon is a set of movements offset in time from each other. For example, if there are two dancers completing the movements on an eight count, one dancer may begin a set of movements on the first beat and complete them on the eighth beat, then the second dancer will begin the same set of movements on the fourth beat and complete them on the second fourth beat. In this case, there is a symmetry of movement at a delayed time in space. Inversion is movements performed in reverse sequence and retrograde is when each movement is reversed, in addition to the sequence being reversed.

Dance forms such as tap dance and clogging rely heavily on the use of sound and rhythm. Schaffer and Belcastro found that the pattern of connecting the vertices of a seven-pointed star with Schläfli symbol {7 / 5} where you start with one vertex then count counterclockwise to the fifth vertex around the outside until you end at the vertex you began with can be demonstrated by having two people clap a five-beat and seven-beat rhythm, respectively. The two people must start at the same time and clap with the same tempo. Similarly, an eight-pointed star with Schläfli symbol {8 / 3} pattern can be demonstrated by having two people clap as eight-beat and three-beat rhythm, respectively.

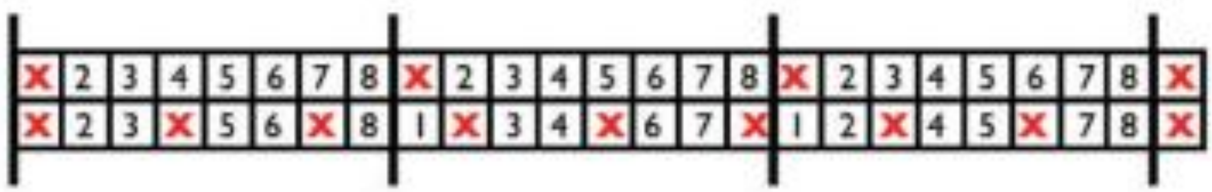
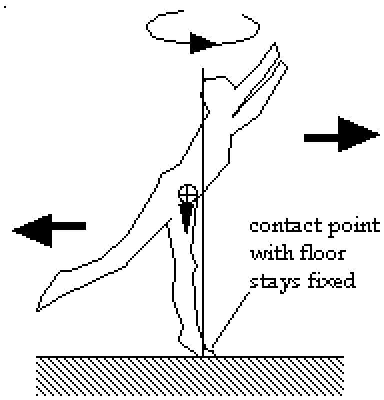


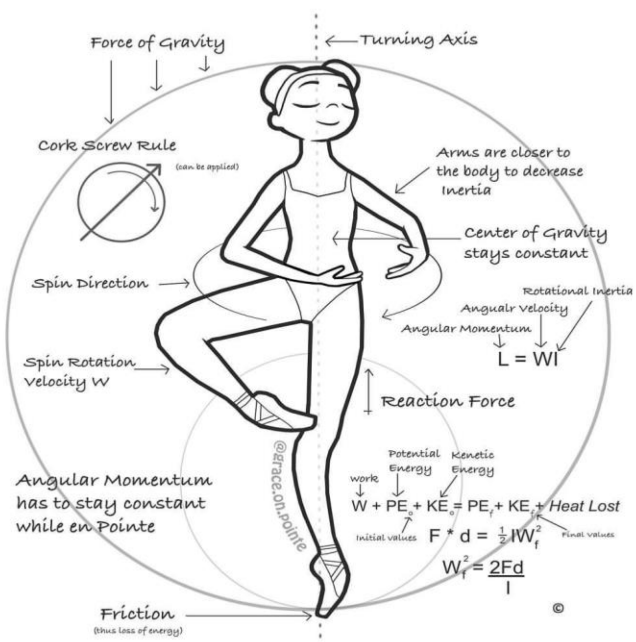
Figure 9. Clapping on every third beat against an eight-beat count produces a single clap on each of the eight beats before repeating.

Physics

The forces that act on a dancer are gravity, support from the floor and friction from the floor. When a dancer begins to move across the floor, velocity, mass, momentum, and force act upon them. Throughout a performance, balance is key for dancers whether they must balance on two feet in grand plié or on one foot in an arabesque on relevé. Balance requires no total force and no total torque. For a dancer to stay balanced, their center of gravity must remain directly above the area of contact with the floor. When we include turns and spins, we must consider the angular velocity, rotational inertia, and angular momentum acting upon the dancer.



When a dancer begins to spin or turn, they do not think of the fact that angular velocity, rotational inertia, angular momentum, and torque are acting upon them. Rather, a dancer tries to keep control of the turn/spin and avoid becoming dizzy by choosing a point of focus before they begin to turn and rotating their neck to this fixed point as they maintain their center of gravity. This technique called “spotting” and allows for dancers to continue spinning as long as possible before inevitable losing their balance. Humans do not have rotational symmetry like a ball of sphere so we are unable to spin continually.



Links to Computer Science and Pedagogical Applications

Several programs have come about that are encouraging mathematical thought through movement. In an effort to attract more girls to a STEM field of study, Virtual Environment Interactions (VEnvl) software allows girls top program 3-D characters to perform dance moves just by moving their own bodies. The girls are then encouraged to develop new computing strategies to improve their choreography. Math in Your Feet is a program created by Malke

Rosenfeld geared towards primary school students. Students are able to take workshops which teach "fractions by learning the difference between a quarter- and a half-turn, congruence by dancing in unison, and reflection symmetry by taking opposite but equal steps." The goal is to increase the confidence of students in math at a young age. Maths Dance is a program in the United Kingdom founded by Panorea Baka that offers workshops at various schools to teach rations, shapes, probability, combinations and permutations, and number patterns through choreography and movement. Multiple programs have been founded to promote math through dance and find innovative ways to increase promote the interest in math for young students.

References

Scaffer, Karl. “Dancing Mathematics and the Mathematics of Dance”, https://www.jstor.org/stable/pdf/10.4169/194762111x12954578042939.pdf?refreqid=excelsior%3Adb17025fbc4fa70d7cedac450cc2e68c

Schaffer, K., Stern, E. and Kim, S. “Math Dance” MoveSpeakSpin, Santa Cruz 2001.

TEDxTalks. “Math Dance: Erik Stern and Karl Schaffer at TEDxManhattanBeach.” YouTube. YouTube, 18 Nov. 2012. Web. 06 Apr. 2016. https://www.youtube.com/watch?v=Ws2y-cGoWqQ

Kuznetsova, Natalia. “Physics of Dance”, https://ed.fnal.gov/trc_new/demos/present/physofballet.pdf

Corinne Wolfe. “MATHS DANCE: TRIANGULAR SQUARES by Corinne Wolfe.” YouTube. YouTube, 21 Feb. 2014. Web. 06 Apr. 2016.https://www.youtube.com/watch?v=Om4j8Nt2JWQ

Wolfe, Corinne. “Dancing Triangular Squares – The Process of Creating A Mathematical Dance.” Mathematics Teaching 242 (2014): 11-13. Academic Search Complete. Web. 6 Apr. 2016.

Whitney, A.K. “Taking the Leap: Teaching Math Through”, Dancehttps://www.noodle.com/articles/these-3-programs-are-integrating-math-and-dance-in-school165

Gollin, George. “Physics and Dance”, http://web.hep.uiuc.edu/home/g-gollin/dance/dance_physics.html