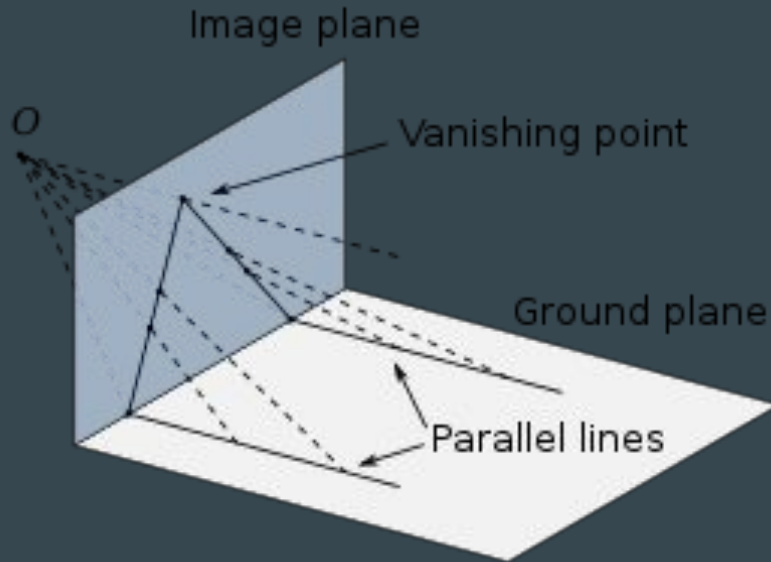


The Genesis of Linear Perspective

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What is linear perspective? Why do we need it?

Linear perspective is the term used for creating depth in space with lines.



Linear perspective is one of many applications of projective geometry, which is a sect of geometry that involves projecting images onto a surface.

It utilizes a **projective plane**

- Assuming parallel lines intersect

The study and axiomatization of projective geometric planes first emerged in the late 14th century

Why then?

Renaissance: the rebirth of classic greek and roman thought

Beginning around the end of the 14th century, a revival of classical thought began that stretched into all facets of life; politics, art, science, economics

- Return to classical antiquity
- Art and science become intertwined
- Study of Euclid, Pythagoras, Archimedes
- In order to create art, we must consider the science
- Humanism and Naturalism emerge
 - Emphasis on anatomy, education, innovation, heightened form, classical christian themes
 - Realism in form, softening of features, 'humanlike' expression
 - The human experience, interaction with nature







Why did we want art to obey the laws of nature once again?

Medieval paintings often focused on secular scenes, and used subjective hierarchical systems, which resulted in a neglect of realistic perspective and form.

As a result of the revival and glamorization of classical antiquity, the populus declared:

Geometry and proportions are of the utmost importance















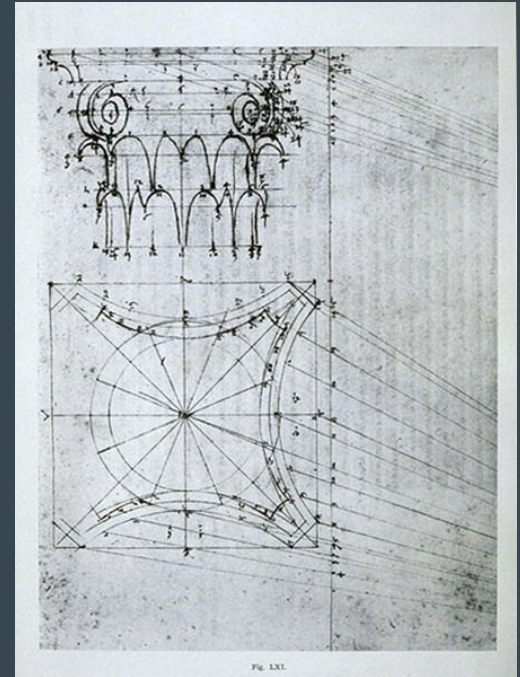


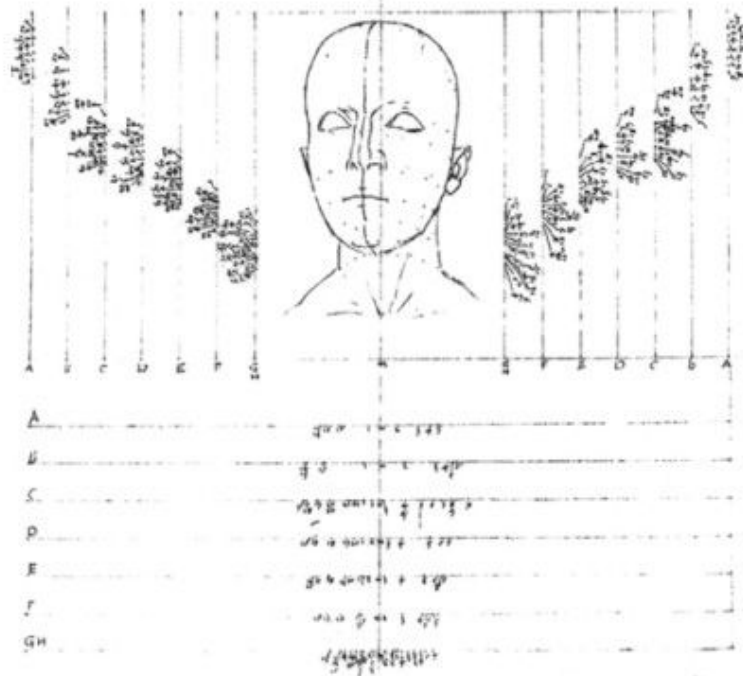
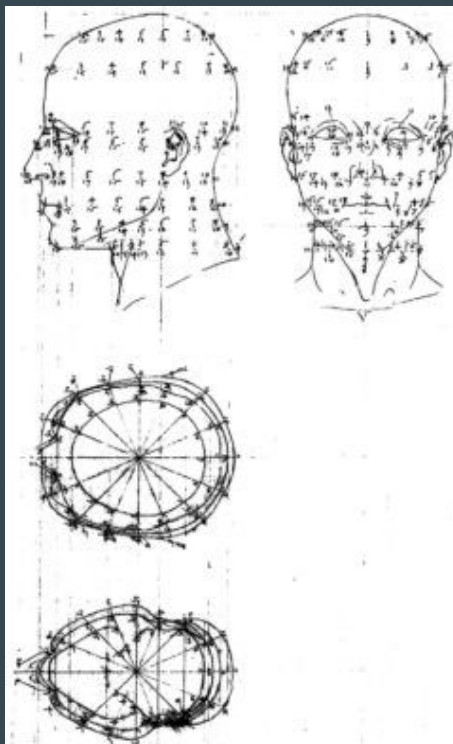
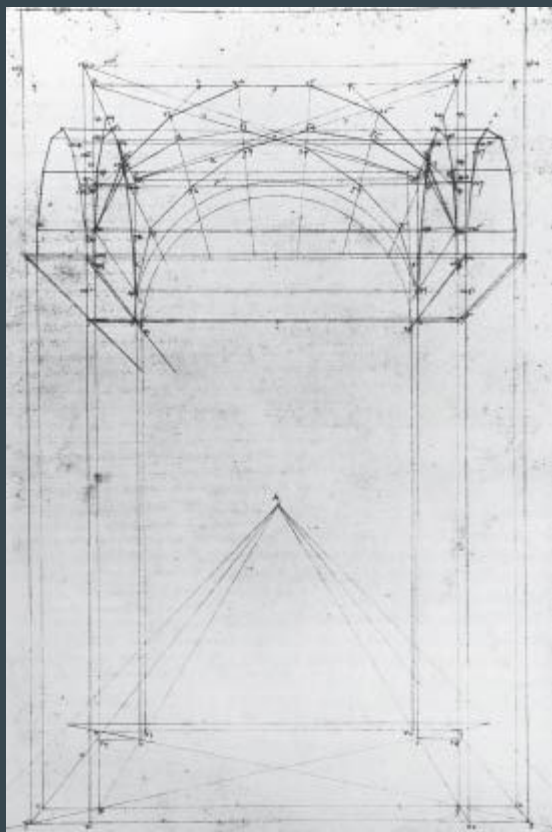
Can we trace the origins of linear perspective in geometry?

Not exactly, but we can trace the first documented proofs of it.

Piero della Francesca is believed to have written the first proof on linear perspective in space.

- Italian painter, active in the 15th century
- Some abacus background, but no formal mathematical training
- Wrote three treatises on geometric proofs, including anatomical proofs
 - Never published, likely due to the era
 - *De Prospectiva Pingendi*
 - Provides several important results, the most significant of which is a simple proof on forced linear perspective, “proving” why it works
- Piero gives a mathematical result that shows *why* forced perspective works





The basic construction, layman's terms

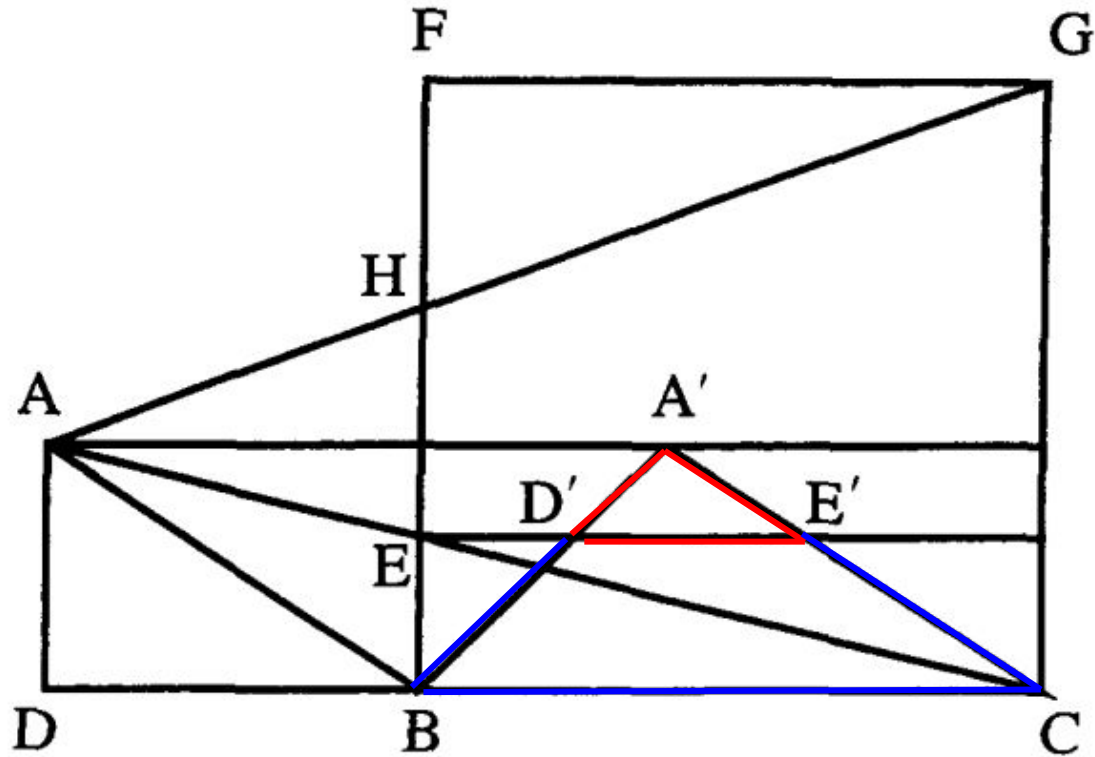
- Draw a line DC, with some point B between D and C.
- From C, draw a perpendicular CG with length equal to BC.
- From G, draw a parallel GF to BC, where F is at a point perpendicular to B.
- Construct a point A perpendicular to D, such that $AD < BF$.
- Draw from A a line AC, and a line AG, where AG intersects FB at H, and AC intersects FB at E.
- Draw a line parallel to BC from point A.
- Divide BC into equal parts at point I, and construct a perpendicular from I.
- Let this perpendicular cut the line parallel to BC from A at point A'.
- Draw a line parallel to BC from point E. Let the intersection of this line with CG be point K.
- Draw a line from A' to B, which divides EK at point D', and a line from A' to C, which divides EK at point E'

CLAIM: BCD'E' IS A SQUARE WITH THE SAME AREA AS BCFG

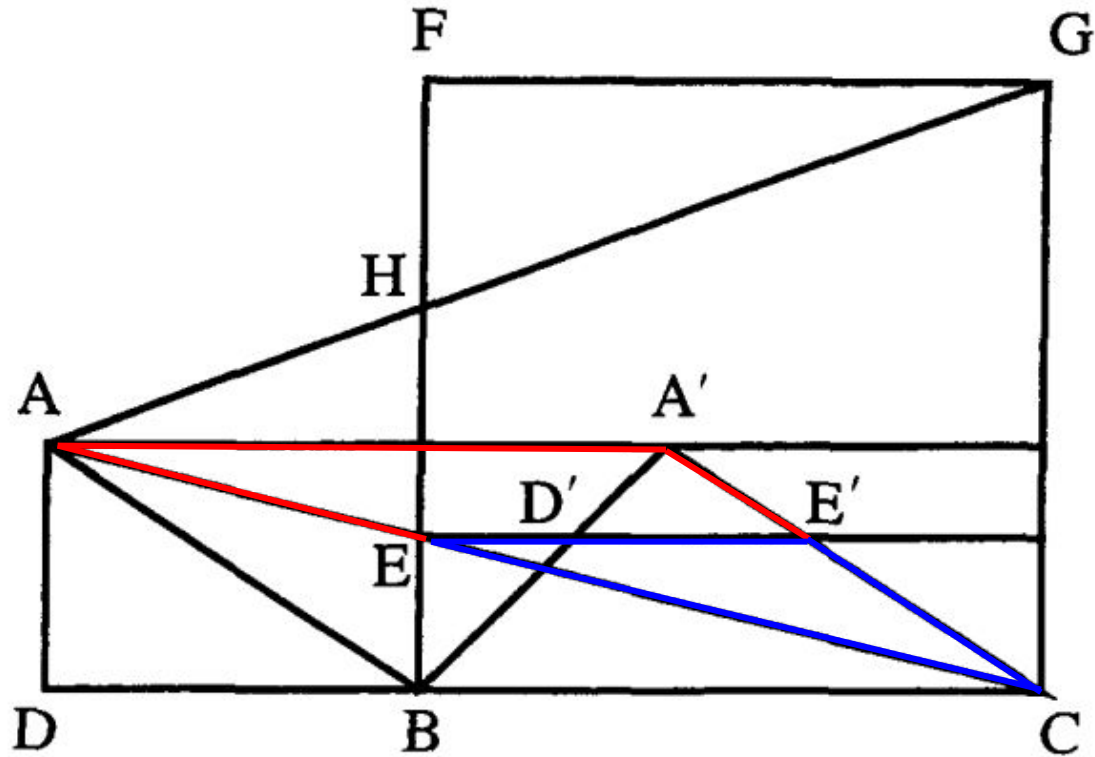
Why can he make this claim?

The underappreciated properties of similar triangles, mainly.

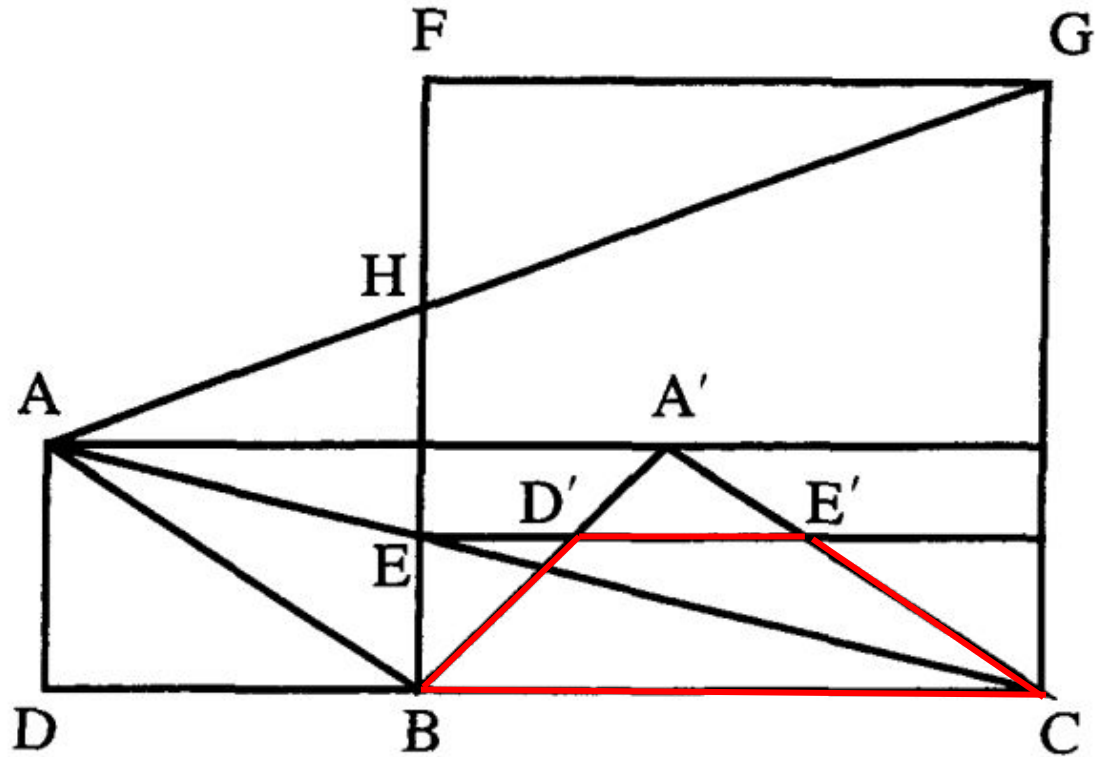
- To proceed, we make the critical assumption that the initial degradation of the plane BCFG is ECHG, found by applying similar triangle properties to $AFG \sim HFG$, and $CDA \sim CBE$, Euclidean Optics results, and another of Piero's results. By this, **we are assuming that ECHG is a square that appears equal in area to the square BCFG**
- With that assumption, **BCD'E' is a square equal in area to the square BCFG** due to equivalencies in three similar triangles that let $D'E' = EH$.



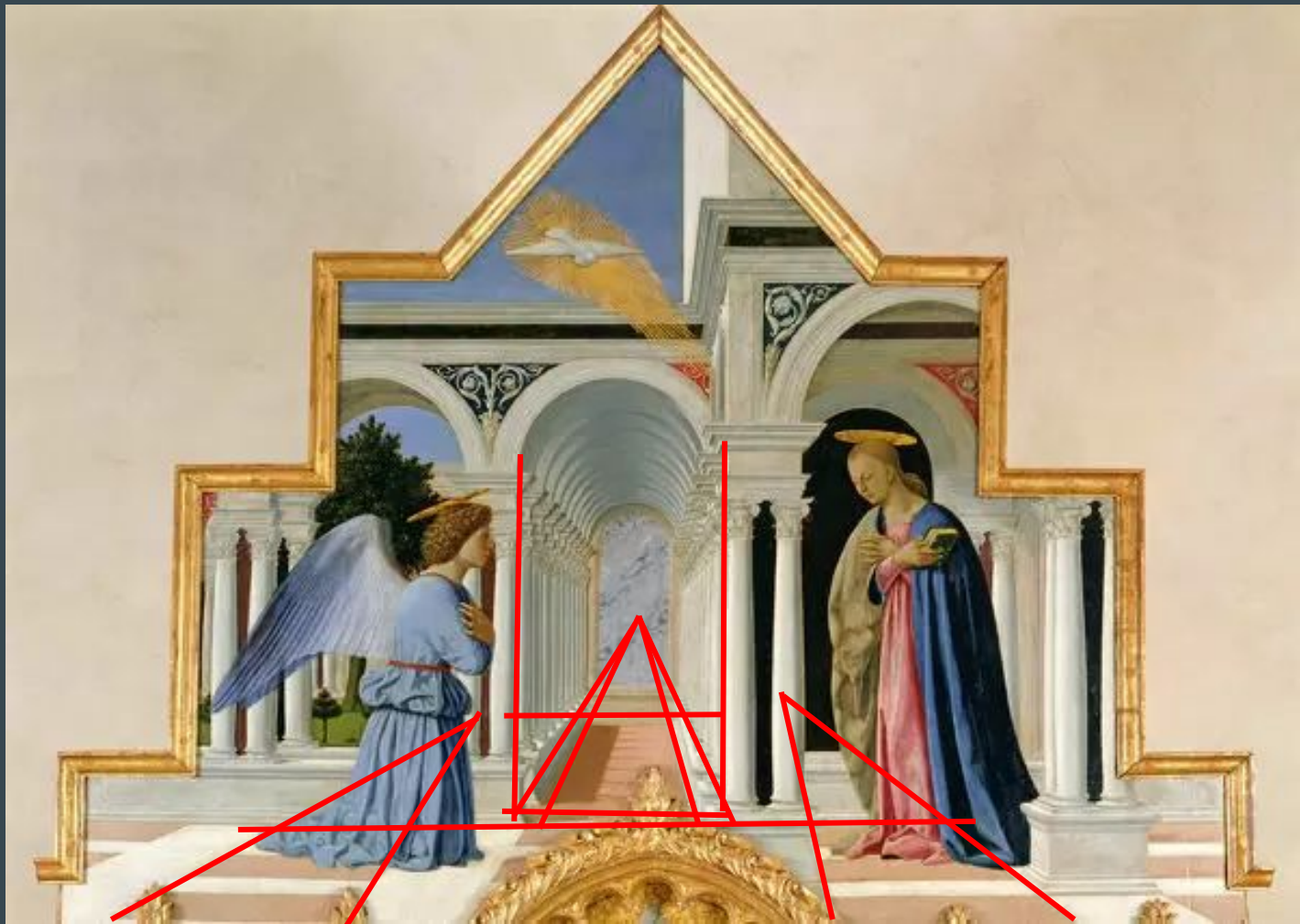
Similar triangles $A'D'E'$ and $A'BC$

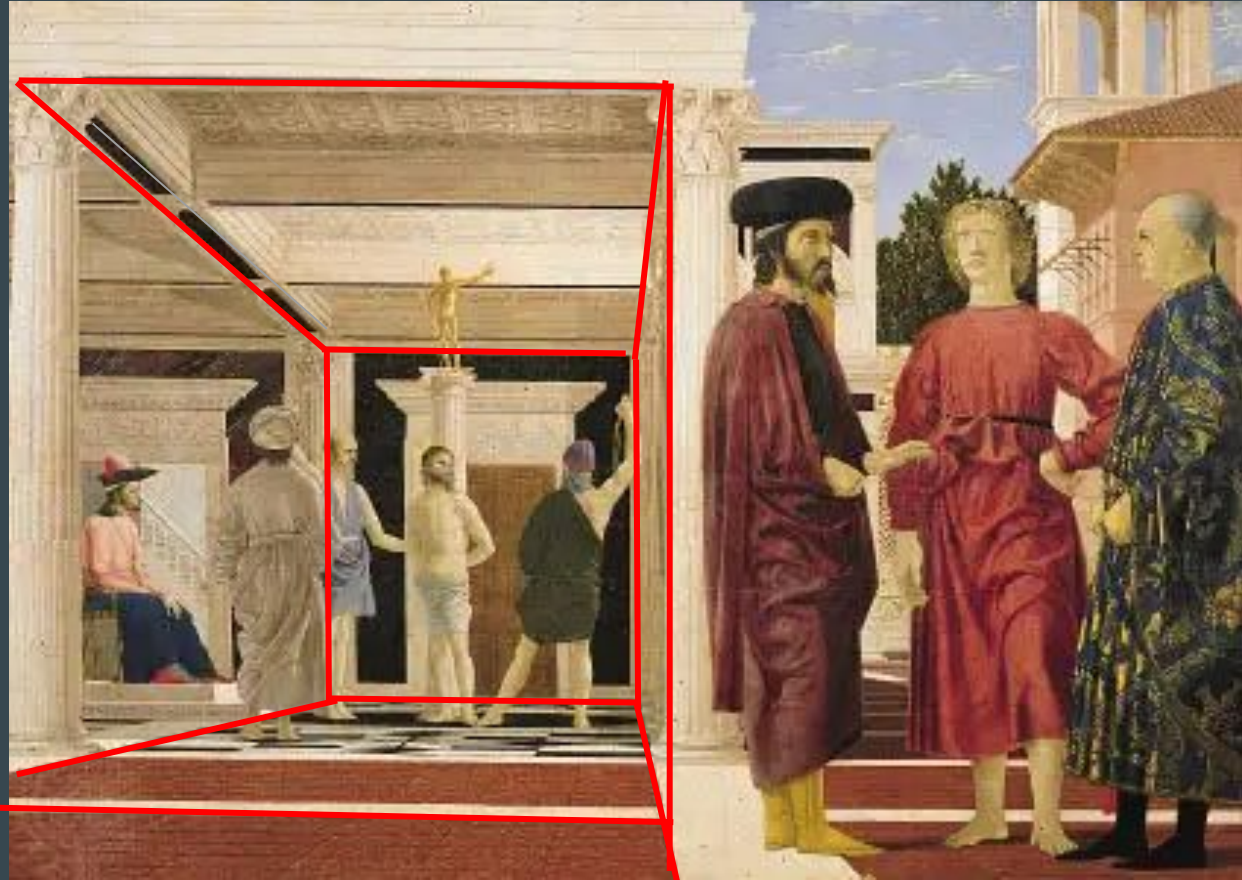


Similar triangles $CE'E$ and CAA'



Second degradation of the surface, $BCD'E'$





**Modern mathematicians hate this proof
(if it can even be called that)**

Piero's first construction of the plane to be degraded, BCFG:

“Let a perpendicular line be drawn up from C equal to BC which is to be CG and from the point G let there be drawn a line parallel to BC which is to be GF such I say will be a square with equal sides BC, CG, GF, FB.”

Piero's first construction of the plane to be degraded, BCFG:

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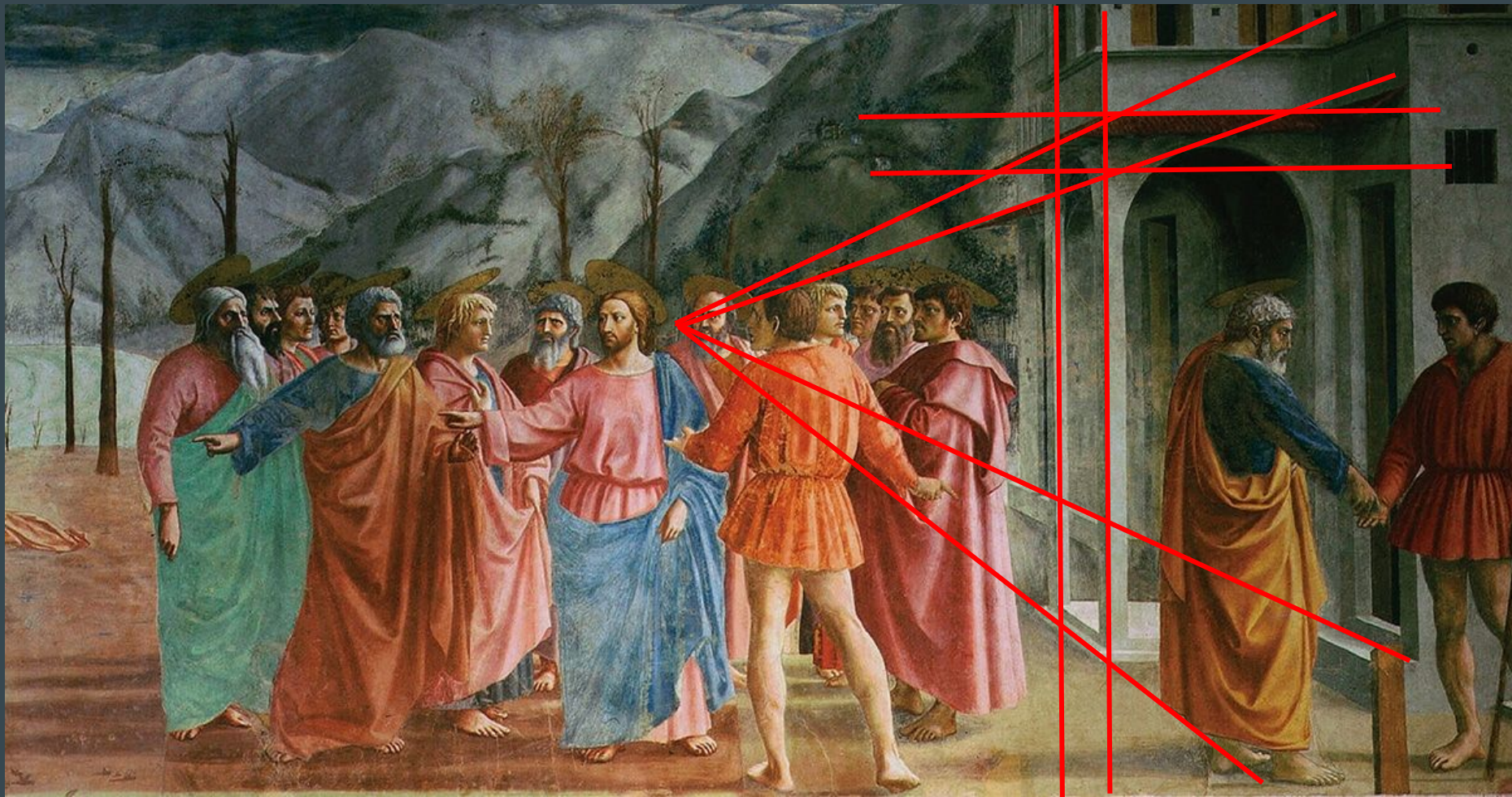
Piero's justification of the initial degradation, EHCG:

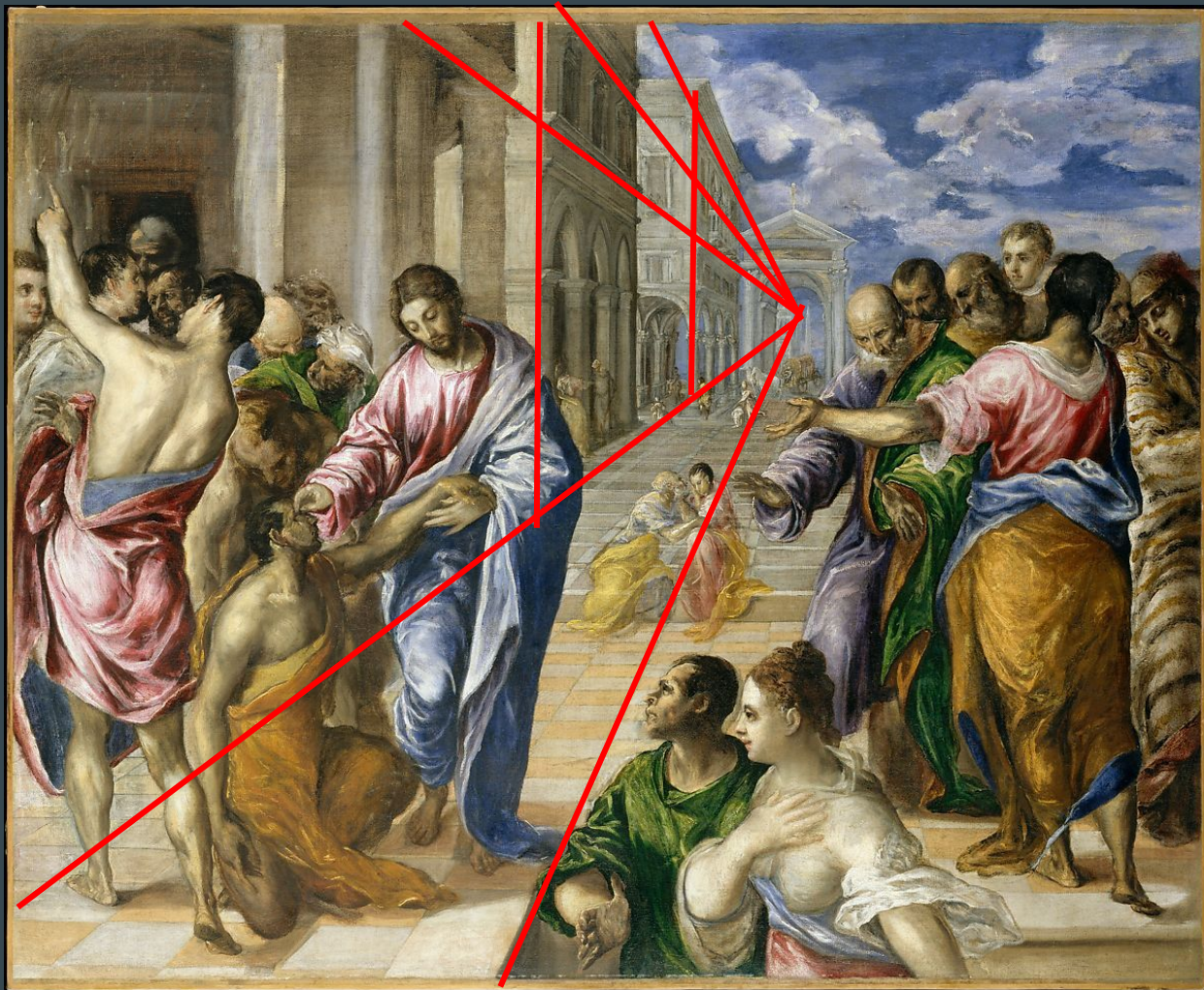
“And when the distances and the things are in the same proportion as the height of the eye and the thing as degraded, it is clear that the degradation is correct. So I shall say that [the rectangle] EHCG is the surface of [that is, the same area as] BE made into a square...”

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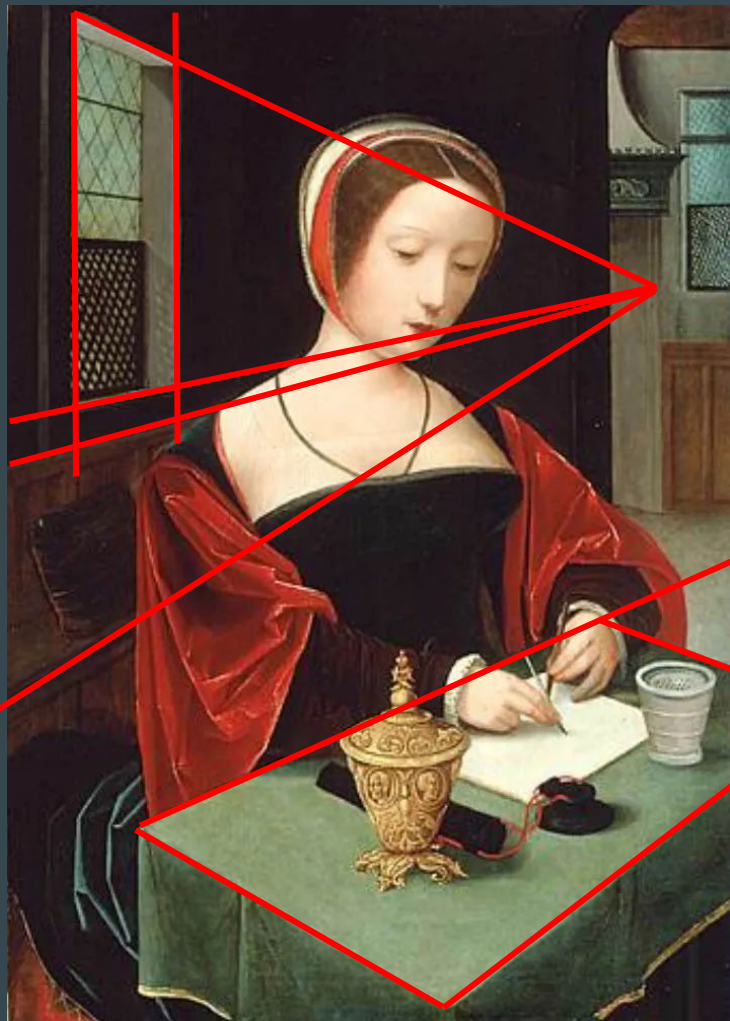
“And when the distances and the things are in the same proportion as the height of the eye and the thing as degraded, it is clear (this is not clear) that the degradation is correct. So I shall say that [the rectangle] EHCG is the surface of [that is, the same area as] BE made into a square...”

But now, we must remember, this proof was not for us









“But if you were to say: why do you put the eye in the middle? Because it seems to me more convenient for seeing the work; all the same, it can be put wherever one wants, provided you do not go beyond the limits that will be shown in the final figure [of this book], and, wherever you put it, it will see the same proportion.”

