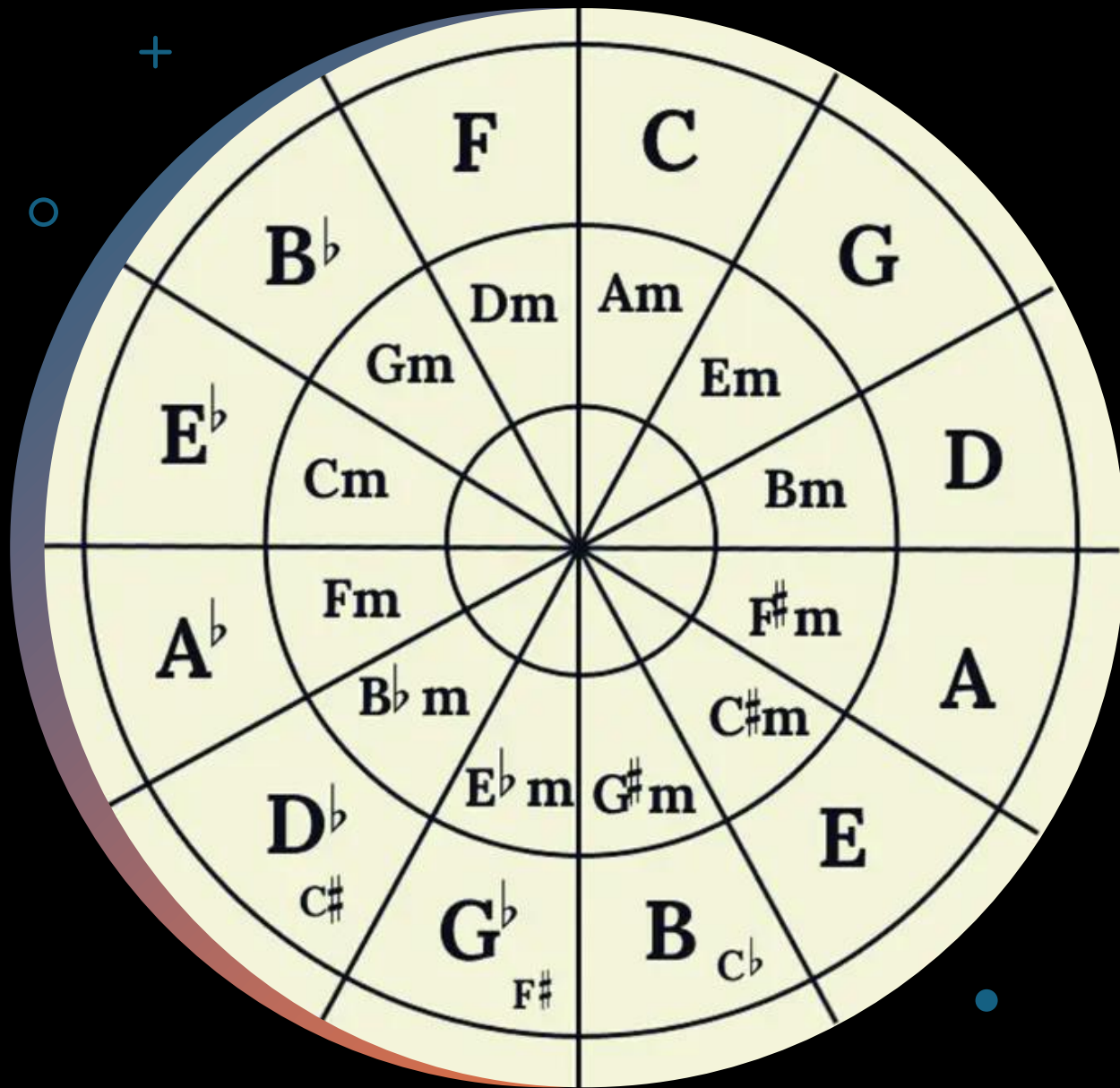


Group Theory & Music

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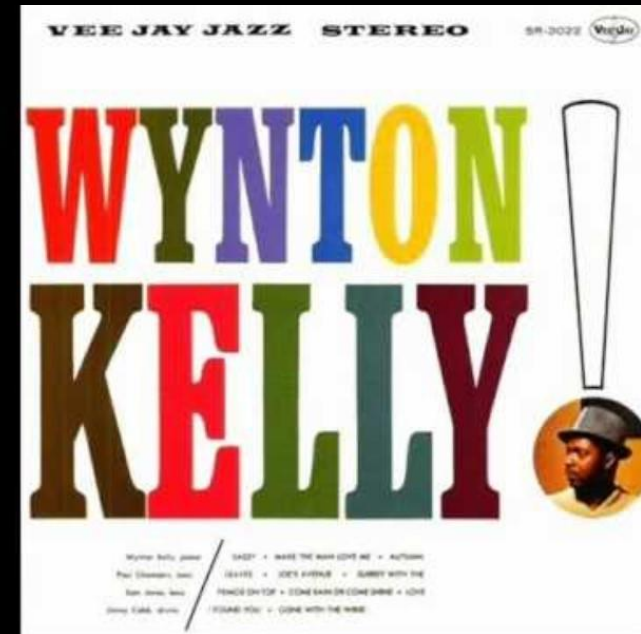
What musician even cares about group theory?



Most of them
(kind of)!

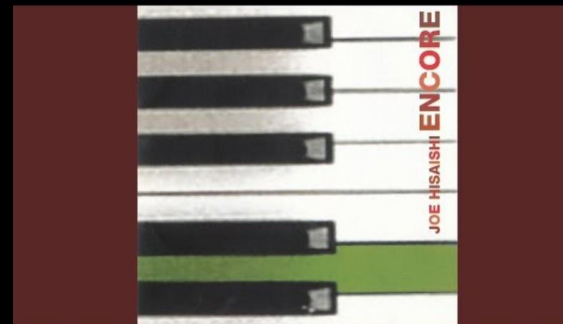
- **Circle of Fifths:** Important for all Western harmonic practice beginning with the Baroque in the 17th and 18th centuries
 - (also everywhere in jazz)

Circle of Fifths examples



Other applications?

- Symmetry of the tritone and the diminished seventh chord allows greater freedom in harmonic progression
 - E.g., can substitute Cdim7 with Ebdim7 with Gbdim7 with Adim7
 - E.g., can substitute a G7 with Db7 as a **tritone substitution**
- **Quintal and quartal harmony** (respectively, repeated perfect fifths and perfect fourths) began to be used extensively in more recent late-19th/early 20th century



Other applications?

- Atonal music!
 - Composers like Arnold Schoenberg, Anton Webern, Alban Berg
 - Very mathematical; essentially uses permutations between ordered 12-tuples whose entries are taken from the integers modulo 12; group theory underlies a lot of it
 - ...an acquired taste



Where else can we explore?

- Other scales & tuning systems
 - Quartertone scale
 - Used in contemporary classical music
 - Modern Arabic scale (developed in 19th century)
 - Is \mathbb{Z}_{24}
 - Five-tone equal temperament scale and others
 - Approximate example: Javanese *slendro* scale (five tone)



Where else can we explore?

- 2-Dimensional Piano??
 - Can regard pitches as ordered pairs (e.g., (x_1, x_2)) from $\mathbb{Z}_{12} \times \mathbb{Z}_{12}$ and the size of ascending intervals as

$$\sum_{i=1}^2 x_i - y_i \pmod{12}$$

where $x_i, y_i \in \mathbb{Z}_{12}$

Maybe??? Generalize to n ?

The End