MATHEMATICS & MARINE NAVIGATION

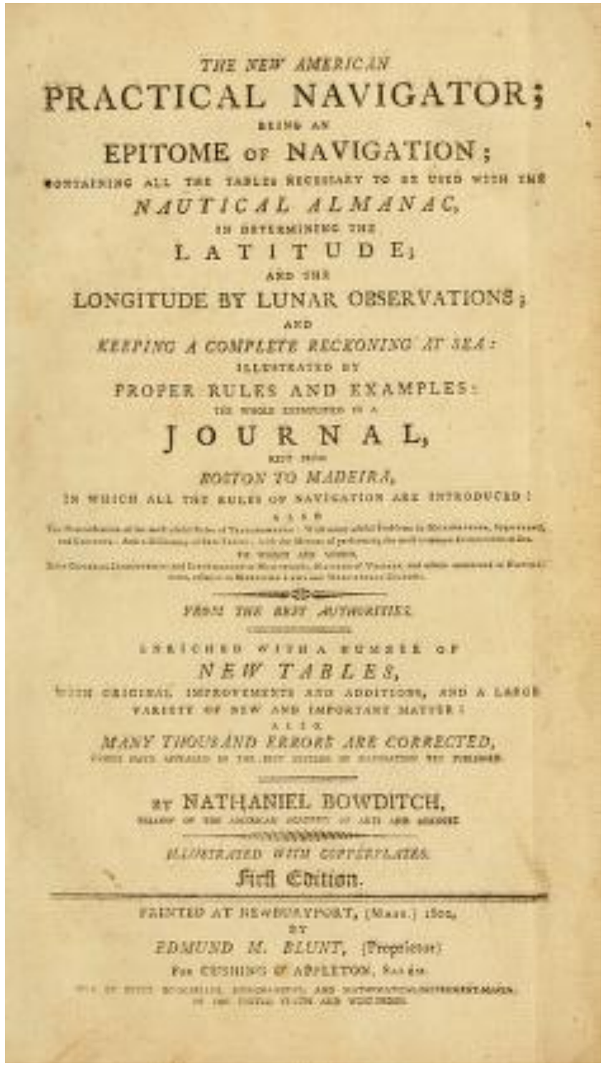
By Chris Kuebler, May 2024

OVERVIEW

I. Significance of Navigation
II. Geographical & Astronomical Concepts
III. Celestial Navigation
    I. The Latitude Problem
    II. The Longitude Problem
IV. Charts & Dead Reckoning

WHERE ARE WE?

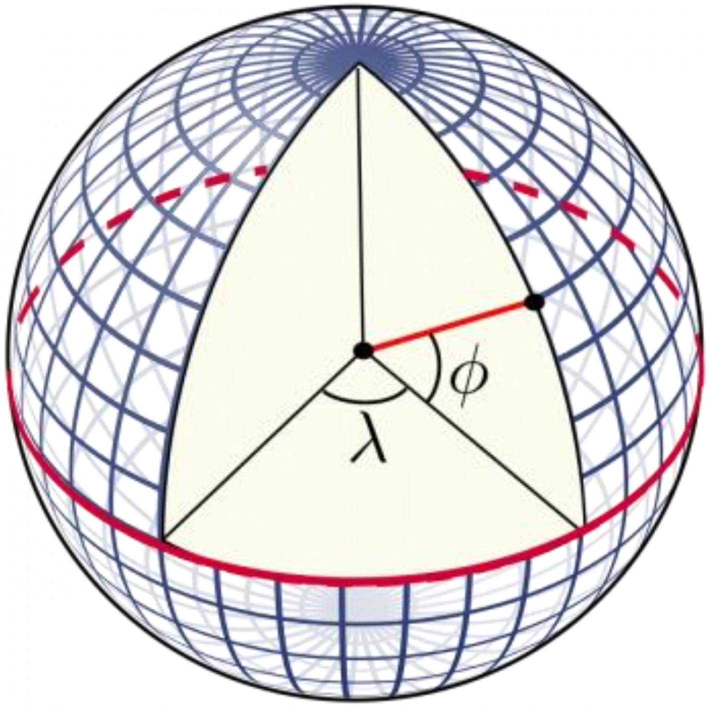
- Navigation a central endeavor for human culture
- Would like to know location accurately and precisely
- Navigation and mapmaking are mathematical arts
    - Trigonometry, spherical geometry, projective geometry, astronomy
    - Many mathematicians involved: Edward Wright, Johann Lambert, Leonhard Euler (of course), Nathaniel Bowditch, Gladys West, and more
- Navigation became of special scientific, economic, and political interest in the Age of Discovery in the 15th through 18th centuries


THE NEW AMERICAN
PRACTICAL NAVIGATOR;
BEING AN
EPITOME OF NAVIGATION;
CONTAINING ALL THE TABLES NECESSARY TO BE USED WITH THE
NAUTICAL ALMANAC,
IN DETERMINING THE
LATITUDE;
AND THE
LONGITUDE BY LUNAR OBSERVATIONS;
AND
KEEPING A COMPLETE RECKONING AT SEA:
ILLUSTRATED BY
PROPER RULES AND EXAMPLES:
THE WHOLE EXEMPLIFIED IN A
JOURNAL,
KEPT FROM
BOSTON TO MADEIRA,
IN WHICH ALL THE RULES OF NAVIGATION ARE INTRODUCED:
ALSO
[illegible]
FROM THE BEST AUTHORITIES.
ENRICHED WITH A NUMBER OF
NEW TABLES,
WITH ORIGINAL IMPROVEMENTS AND ADDITIONS, AND A LARGE
VARIETY OF NEW AND IMPORTANT MATTER:
ALSO
MANY THOUSAND ERRORS ARE CORRECTED,
[illegible]
BY NATHANIEL BOWDITCH,
[illegible]
ILLUSTRATED WITH COPPERPLATES.
First Edition.
PRINTED AT NEWBURYPORT, (Mass.) 1802,
BY
EDMUND M. BLUNT, (Proprietor)
For CUSHING & APPLETON, Salem.
[illegible]

NATHANIEL BOWDITCH'S *AMERICAN PRACTICAL NAVIGATOR*

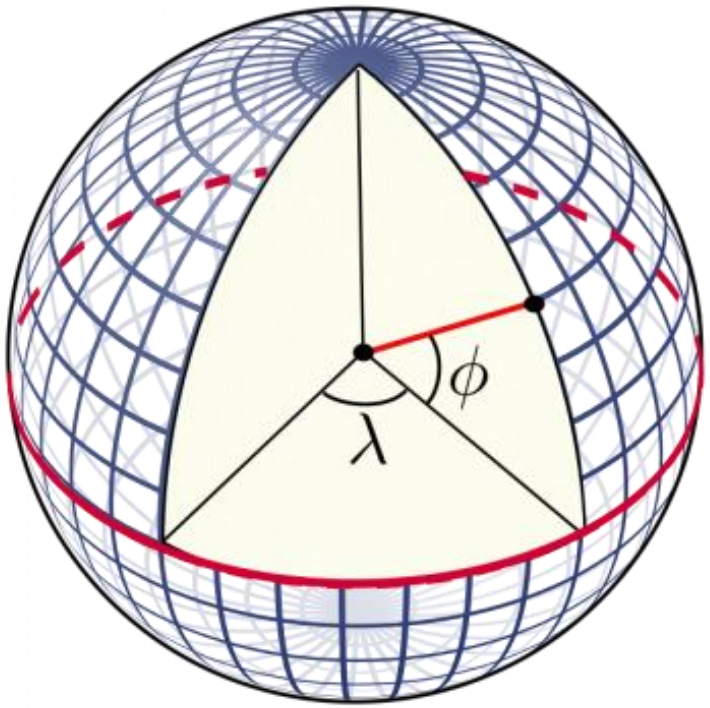
- Has been in continual publication since 1802
- Current edition guides much of my presentation

THE EARTH & SPHERICAL COORDINATES



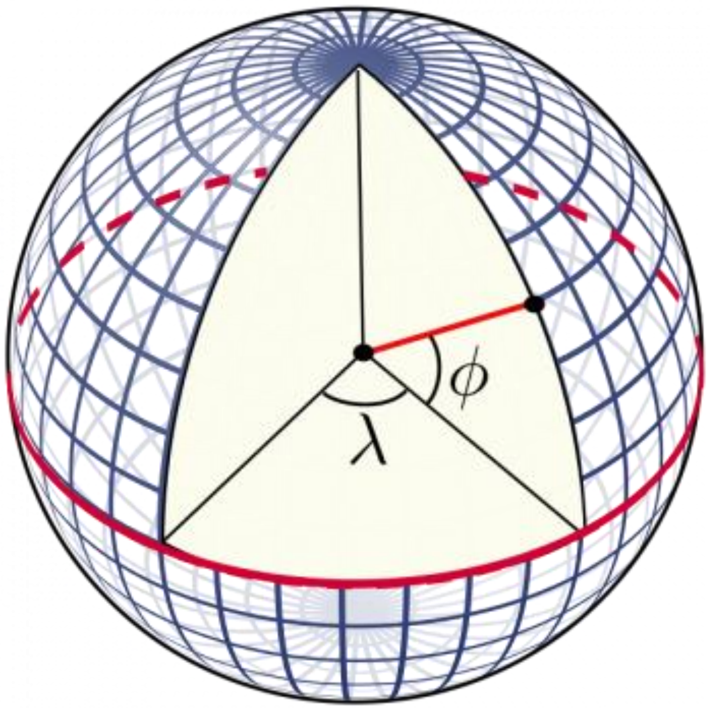
- Earth is an oblate spheroid; idealized as sphere
- Imaginary coordinate grid uniquely determines points on sphere
  - Lines of latitude and longitude first developed by Eratosthenes in 3rd century BC
  - Lines of longitude going 360° around a circle first developed by Hipparchus in 2nd century BC
- **Latitude:** Vertical angle ϕ above equatorial plane; 90°N to 90°S
- **Equator**: Intersection of sphere and **equatorial plane,** the plane that passes through the sphere's center and is perpendicular to the earth's axis of rotation

THE EARTH & SPHERICAL COORDINATES



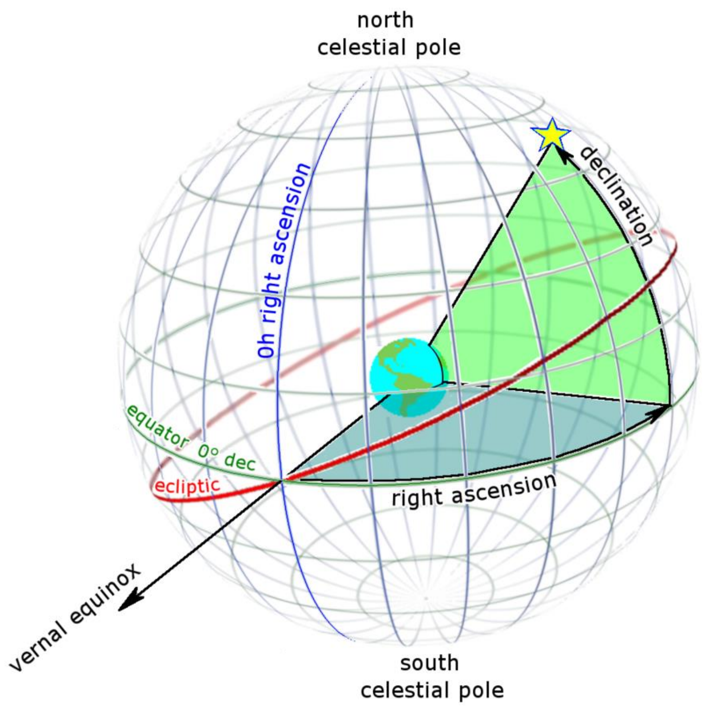
- **Lines of latitude** (parallels): Intersection of sphere and planes that are parallel to the equatorial plane (within Euclidean space!)
- Not straight lines in spherical geometry
  - This is why parallels are usually not the shortest path between two points on the globe!
- **Great circle**: Intersection of sphere and plane through sphere's center; **is** a line in spherical geometry

THE EARTH & SPHERICAL COORDINATES



- **Lines of Longitude** (Meridians): Halves of great circles that pass through axis of rotation
- **Prime meridian**: Meridian through Greenwich, England arbitrarily chosen to be 0° E/W
- **Longitude**: Horizontal angle λ from plane of prime meridian; 180°E to 180°W

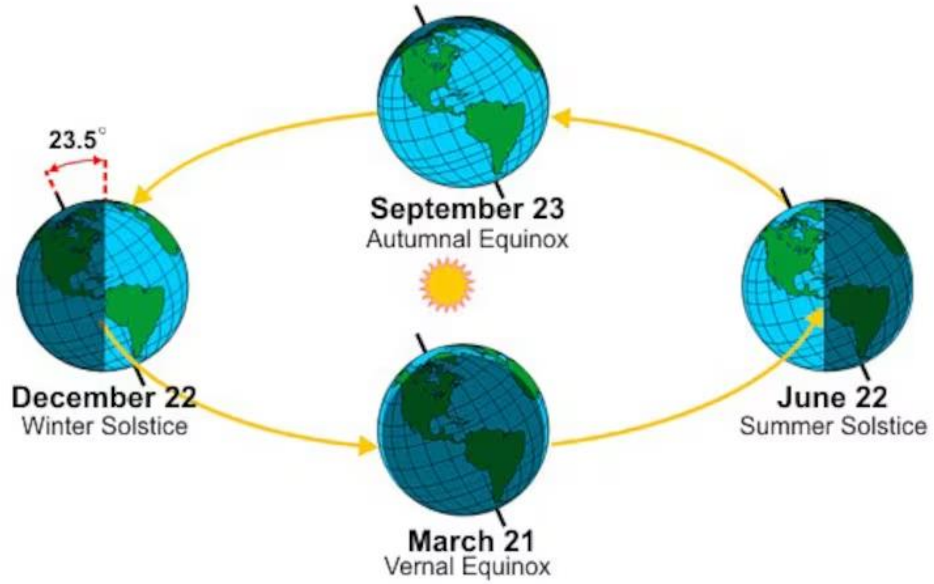
SOME ASTRONOMICAL DEFINITIONS



- **Celestial sphere:** Imaginary sphere that can be centered at earth's center with an infinite radius; celestial bodies are projected onto this sphere
  - Aligned through the earth's poles
- **Celestial Equator:** intersection of equatorial plane and celestial sphere at infinity
- **Declination:** Angular distance north or south of a heavenly body from the celestial equator
- **Altitude:** Angular distance of heavenly body above the horizon **at the place of observation**

AXIAL TILT & DECLINATION OF THE SUN

- With respect to the earth's orbital plane, the earth's axis of rotation is tilted approx. 23.5°, which causes the sun's declination to vary between 23.5°N and 23.5°S
- Example: the Equator at Noon
  - On autumnal and vernal equinoxes, sun is directly overhead; altitude of sun is 90°
  - On summer solstice, altitude of sun is 66.5°; toward the north
  - On winter solstice, altitude of sun is 66.5°; toward the south
- If above the Tropics: In N. Hemisphere, the sun is always in the south; in S. Hemisphere, the sun is always in the north

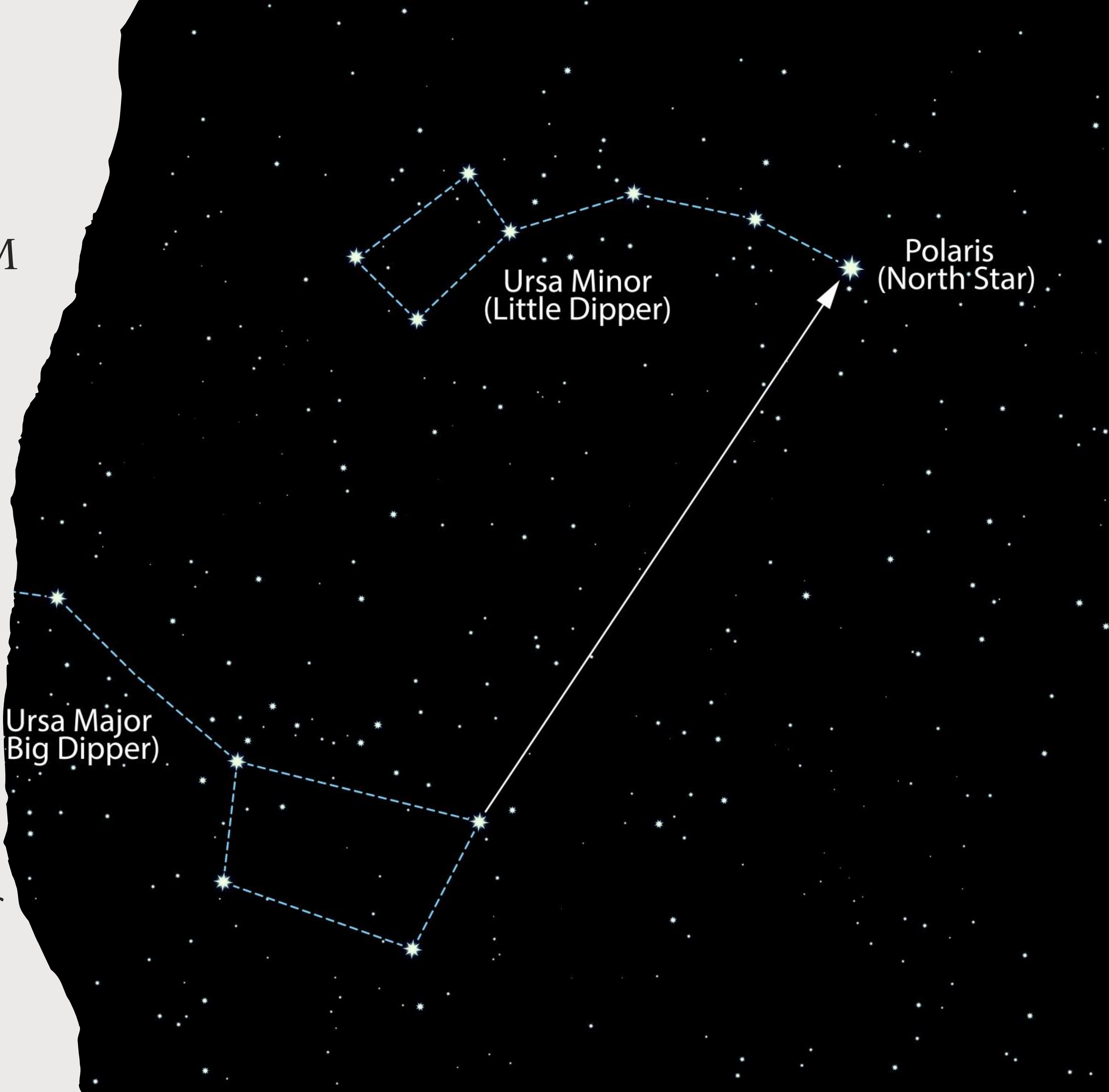


CELESTIAL NAVIGATION

What is our latitude?
What is our longitude?

CELESTIAL NAVIGATION: THE LATITUDE PROBLEM

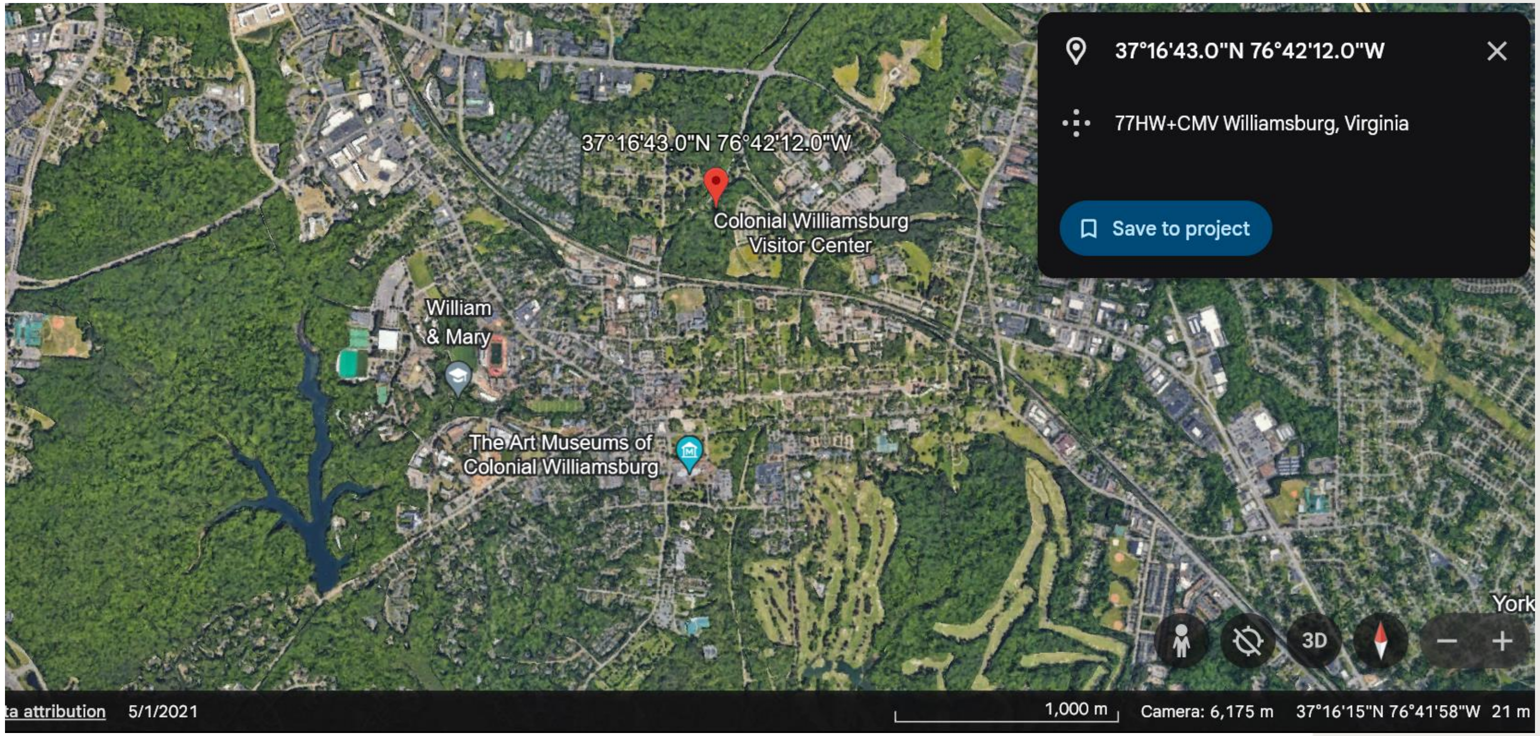
- Finding latitude is far simpler than finding longitude
- Finding cardinal directions relies on astronomical reference points like **Polaris** in N. Hemisphere or the **Southern Cross** in the S. Hemisphere; or magnetic compass
- Using Polaris or Southern Cross, observer only needs altitude of respective star/constellation
- Using the sun, observer needs a cardinal direction, the day of year, the altitude of the sun, and a table of solar declinations



EXAMPLE: USING THE SUN TO DETERMINE LATITUDE

- Find latitude of Williamsburg, VA given the solar declination on May 2, 2024, is 15.7°N.

1. Use a *gnomon* (orientated toward North) to determine noon.
2. Measure altitude of sun at noon.
3. Calculate latitude.

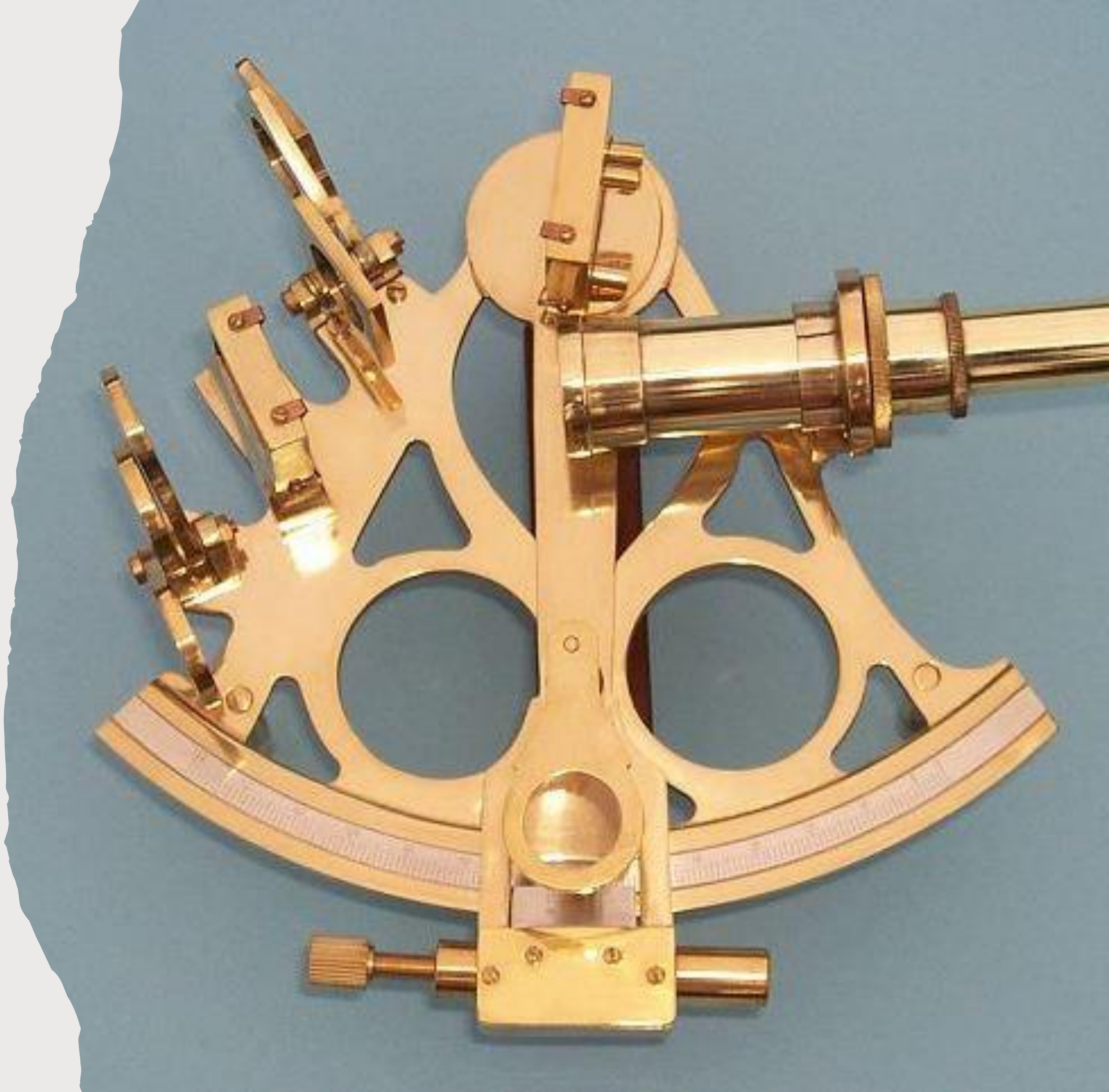


TOOLS TO FIND ALTITUDE

- **Kamal:** Wooden board attached to string with knots
- **Jacob's staff:** Staff with vertical beam of fixed length, whose horizontal component can be shortened and lengthened
- Problem: Imprecise; also often inaccurate due to rocking on ship

TOOLS TO FIND ALTITUDE

- Led to invention of the **sextant** in 18th century
  - Far more precise
  - Still in use today by mariners as a back-up!



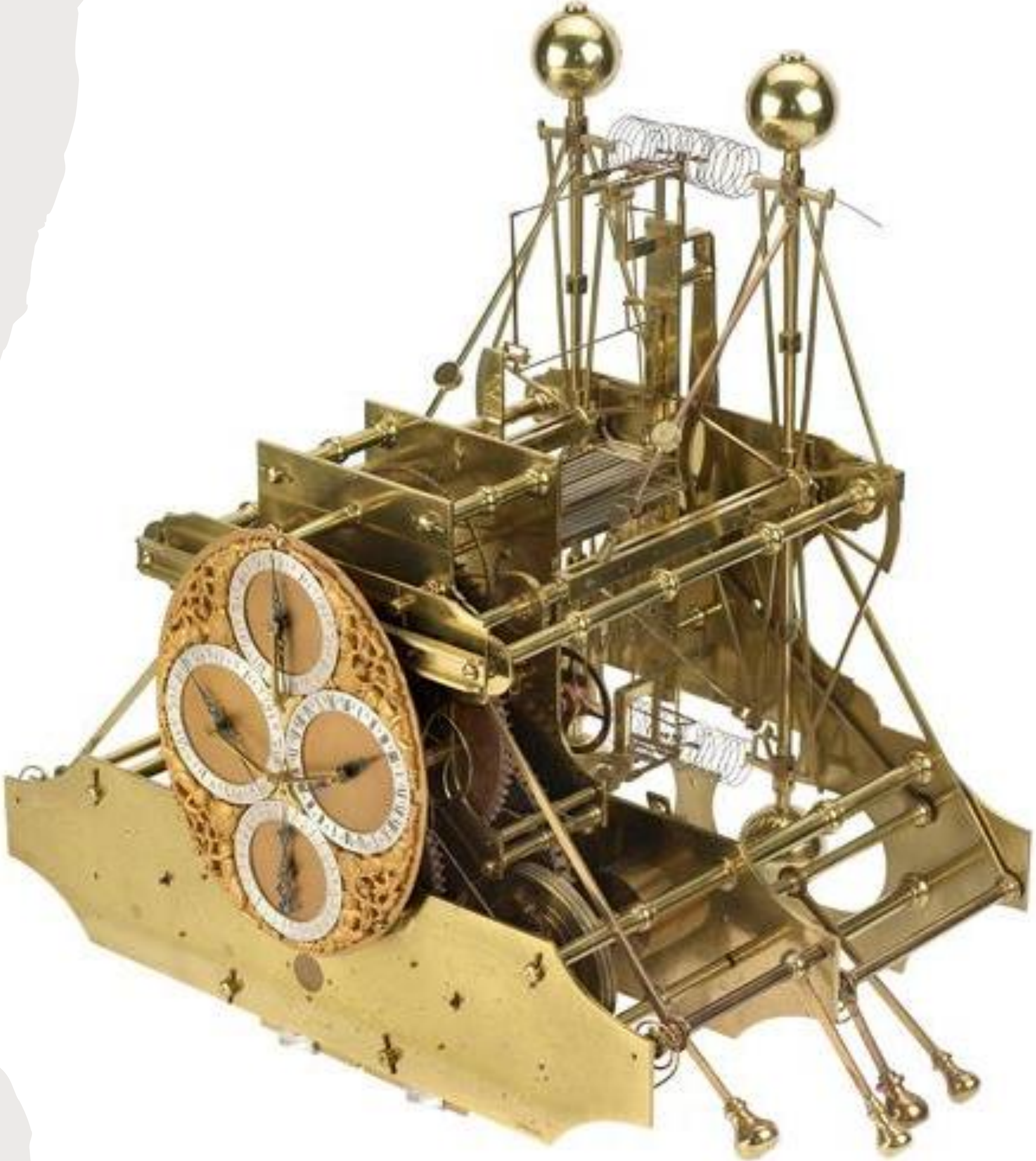
CELESTIAL NAVIGATION: THE LONGITUDE PROBLEM

- Far more difficult than determining latitude from celestial bodies alone
- Main idea:
  - Earth rotates 360° every 24 hours, or 15° every hour, meaning 15° longitude = 1 hour
  - Compare the time of observation of an astronomical event at an unknown longitude with the time of observation of that same event at a known longitude
- Impossible to keep accurate time on ships prior to 18th century
- British Parliament passed Longitude Act of 1714 that established a £20,000 reward for anyone who could determine a practical method of finding longitude at sea accurate within half a degree
  - Reward equivalent to $3.7 million in today's money

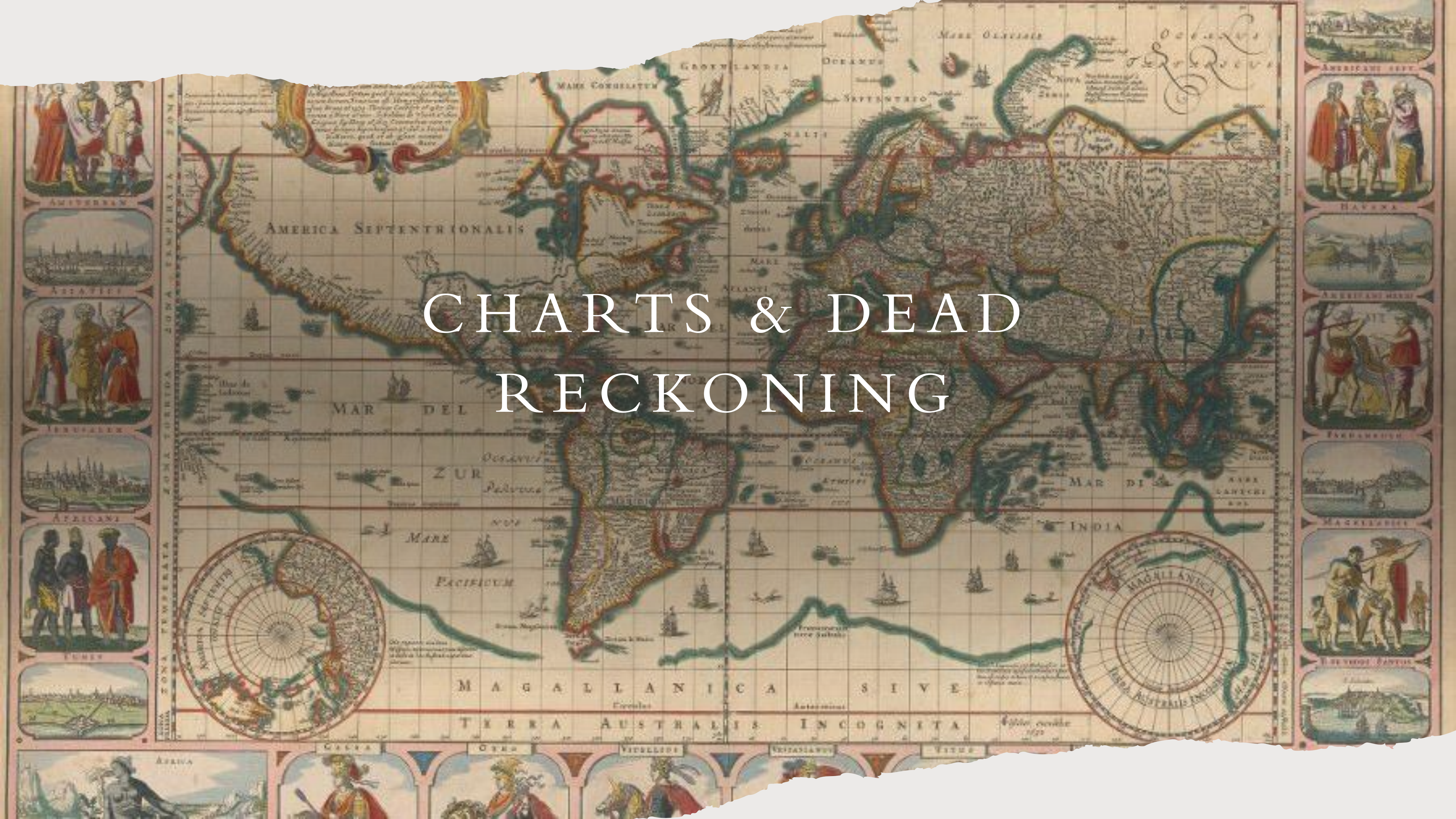
CELESTIAL NAVIGATION: THE LONGITUDE PROBLEM

- Solved by the invention of the accurate **marine chronometer** by John Harrison in 1761

- Now sailors could know their location on the globe with only the stars, a tool to measure altitude, and a chronometer

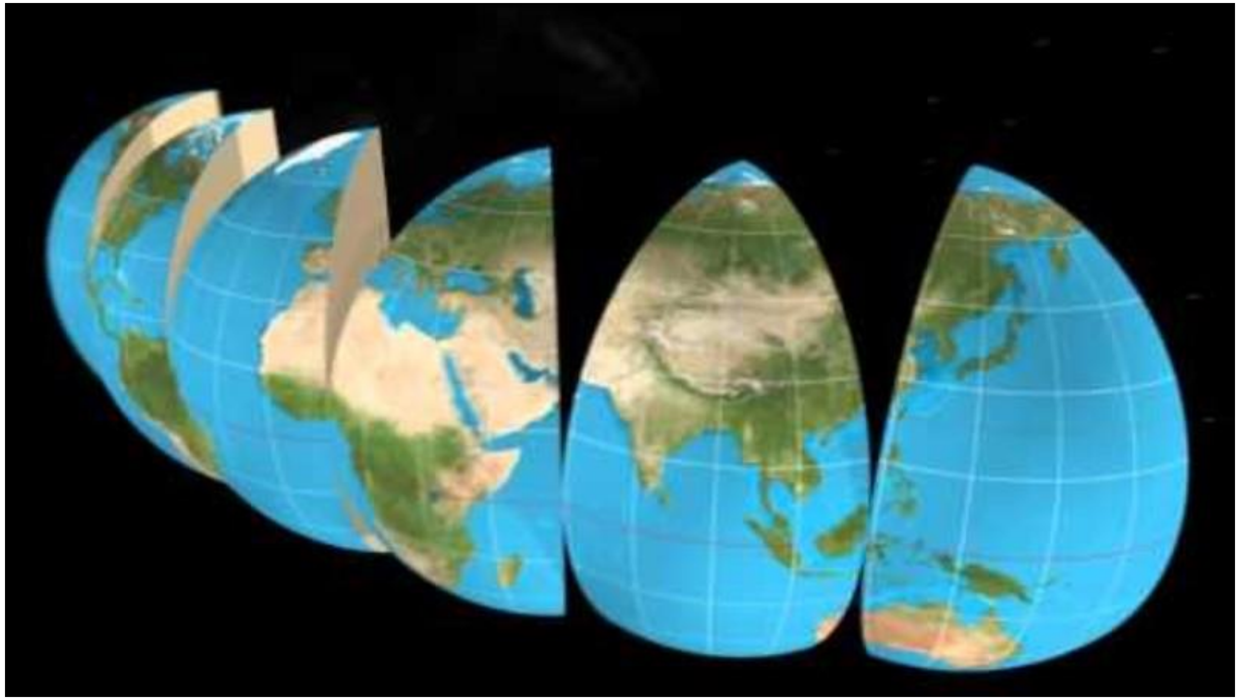


CHARTS & DEAD RECKONING

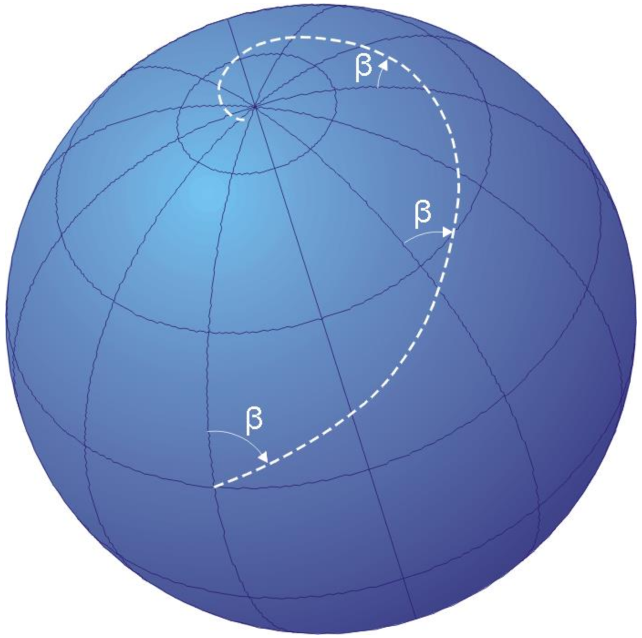


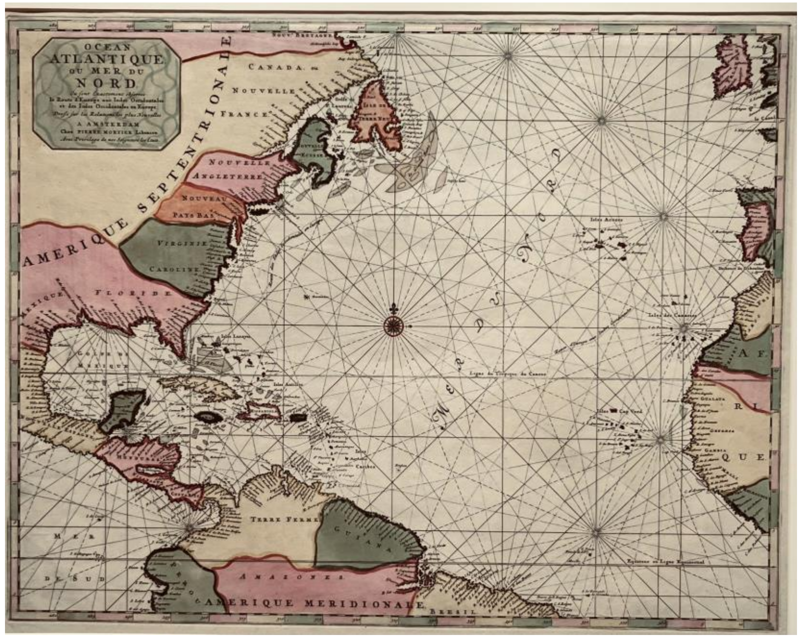
CHARTS, MAP PROJECTIONS

- **Chart:** "graphic representation of areas of the Earth … for use in marine or air navigation" (*American Practical Navigator*)
- Most popular map projection for marine navigation: **Mercator projection**
  - Developed by Gerardus Mercator in 1569
  - Conformal: uses function that locally preserves angles, i.e., preserves direction
  - Cylindrical: maps sphere onto cylinder



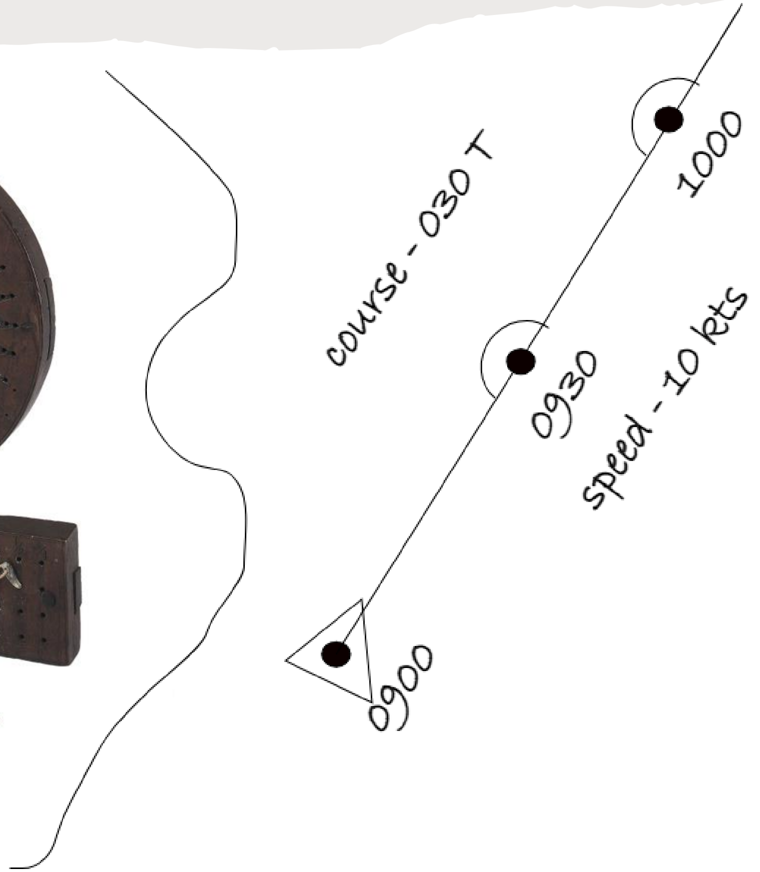
RHUMB LINES





- Since the Mercator projection preserves direction, all lines of constant bearing (i.e., lines that intersect meridians at the same angle) appear as straight lines called **rhumb lines** or **loxodromes**
- Great circles are harder to follow because one must constantly change course

DEAD RECKONING ON MERCATOR CHART



- **Dead reckoning:** "determines a predicted position by advancing a known position for courses and distances" *(APN);* dead reckoning plots are made on a chart, often a Mercator chart
- Uses only course and speed; subject to steering error, wind & ocean currents
- Primary means of determining longitude before use of chronometer
    - Used a traverse board and hourglass
- Chronometers at least made this somewhat more accurate
- Method used in modern inertial navigation systems

AREAS FOR FURTHER READING & RESEARCH

- **Sumner lines / Circles of equal altitude:** Discovered in 1837 by Thomas Hubbard Sumner as a method of finding position of ship using altitude of celestial bodies and chronometer
  - Interesting math; basically just more spherical trigonometry
- **Satellite trilateration:** Method used in GPS to determine location
  - Three satellites needed to uniquely determine a point on the earth

THANK YOU!

RECRUIT BAY ENLARGED
GOOD FRIDAY BAY ENLARGED
WALLABI GROUP
EASTER GROUP
ZEEWYK CHANNEL
PELSART GROUP
GEELVINK CHANNEL
COAST OF AUSTRALIA
CHAMPION BAY
Moresby Range
AUSTRALIA WEST COAST
THE HOUTMAN ROCKS
1840
SOUNDINGS IN FATHOMS

IMAGE CREDITS

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