Concepts in Quantum Cognition

Modeling Concepts as Unitary Hilbert Vectors and Contexts as Their Linear Operators

By Joshua Heaton

The Problem

How can we model concepts and schema in the brain?

Basic intuition gives sets with loose structure that are formed and brought up in cognition

The Problem: Prototype and Fuzzy Set Theory

Concepts can be modeled as a quadruple. (A, d, p, c)

A - the set or domain of readily envisionable objects, or conceptual domain

d - a function from $A \times A$ onto $\mathbb{R}$, the distance metric

p - a member of A that serves as the prototype for the domain

c - the characteristic function from A onto $[0, 1]$

The Problem: Prototype and Fuzzy Set Theory

For this problem we consider fuzzy inclusion in a set. I.e. Set membership is graded from 0 non-inclusion to 1 full inclusion.

This value is given by the characteristic function $c_A(x)$, with some rules

$$c_{A \cap B}(x) = min(c_A(x), c_B(x))$$

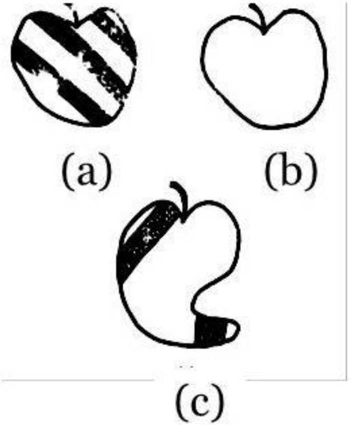
$$c_{A \cup B}(x) = max(c_A(x), c_B(x))$$

$$c_{A^c}(x) = 1 - c_A(x)$$

The Problem: Prototype and Fuzzy Set Theory

Let us consider the domain F of fruits and its subset A of Apples, and S of things that are striped. We consider,

$$c_{A \cap S}(a) = min(c_A(a), c_S(a))$$



But clearly,

$$c_{A \cap S}(a) > c_A(a)$$

$$c_{A \cap S}(a) > c_S(a)$$

The Problem

The Pet-Fish Problem

Take a sample of types of pets: bird, mouse, snake, dog, cat, parrot, hamster, guppy, goldfish

Pet	Bird	Mouse	Snake	Dog	Cat	Parrot	Hamst-er	Guppy	Gold-fish
Typicallity									

Dog, cat, goldfish, guppy, hamster, bird, parrot, mouse, snake

The Problem

The Pet-Fish Problem

Context: The pet is being taught

Pet	Bird	Mouse	Snake	Dog	Cat	Parrot	Hamst-er	Guppy	Gold-fish
Typicallity									

Context: The pet explains why he is so weird.

Pet	Bird	Mouse	Snake	Dog	Cat	Parrot	Hamst-er	Guppy	Gold-fish
Typicallity									

The Problem

The Pet-Fish Problem

Context: The pet is a fish

Pet	Bird	Mouse	Snake	Dog	Cat	Parrot	Hamst-er	Guppy	Gold-fish
Typicallity									

The Problem: Prototype and Fuzzy Set Theory Fail

This last one can also be viewed from the concept of fish, with the context of being a pet.

Similar to the problem of striped apples.

So how do we model concepts, their conjunctions and more?

The Problem in Hilbert Spaces

A Hilbert space H is a metric space with an inner product that is complete with respect to the norm induced by the inner product.

Before this we consider various sets

A concept S, and its set of states Σ with states $p, q, r \in \Sigma$

A set of contexts M^S, individual or pieces of contexts $e, f, g \in M^S$

A set of properties $\mathcal{L}$, individual properties $a, b, c \in \mathcal{L}$

The Problem in Hilbert Spaces

Consider a ground state $\hat{p}$, and respective states, contexts, properties, and functions

$$p_1, p_2, \ldots, p_n \in \Sigma \qquad \mu : \Sigma \times M \times \Sigma \to [0,1] \qquad \mu(q,e,p)$$

$$e_1, e_2, \ldots e_m \in M \qquad v : \Sigma \times \mathcal{L} \to [0,1] \qquad v(p,a)$$

$$a_1, a_2, \ldots, a_k \in \mathcal{L}$$

Where μ a probability function, how likely p becomes q under context e

v Is the weight of a given property in a particular state

The Problem in Hilbert Spaces

We consider the finite dimensional Hilbert space $\mathbb{C}^n$

A conceptual state p can either be pure which gives a unit vector $x_p \in \mathbb{C}^n$

Or a density state, which gives a self-adjoint linear operator ρ_p s.t. $Tr(\rho_p) = 1$

Properties and Contexts are given by orthogonal projection maps P

We define these as follows

The Problem in Hilbert Spaces

For some property $a \in \mathcal{L}$ we find its weight by $v(p, a) = x_p P_a x_p$ if p is a pure state and $v(p, a) = Tr(\rho_p P_a)$ if a density state.

Similarly for a context $e \in M$, we have the projector P_e which changes the state from p to q by

$$x_q = \frac{P_e x_p}{\sqrt{x_p P_e x_p}}$$ Or $$\rho_q = \frac{P_e \rho_p P_e}{Tr(\rho_p P_e)}$$

With respective probabilities

$$\mu(q, e, p) = x_p P_e x_p \quad \& \quad \mu(q, e, p) = Tr(\rho_p P_e)$$

The Problem in Hilbert Spaces

Given an orthonormal bases $B \in \mathbb{C}^n \quad u, v \in B$

$x = \sum_{u \in B} a_u u$ And $< u|x >= u \sum_{v \in B} a_v v = \sum_{v \in B} a_v < u|v >= a_u$

We can write $x = \sum_{u \in B} < u|x > u$

Given a projection map onto u $P_u \quad x = \sum_{u \in B} P_u x$

Given the above facts and the nature of projection ,

$xP_u x = a_u \bar{a}_u = |a_u|^2$

The Problem in Hilbert Spaces

This model is actually very accurate in predicting frequencies of exemplars as representations of concepts and specifically in the impact of contexts on those frequencies.

Use in artificial intelligence systems

How well does this model something as complex as self-concept?

Sources

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