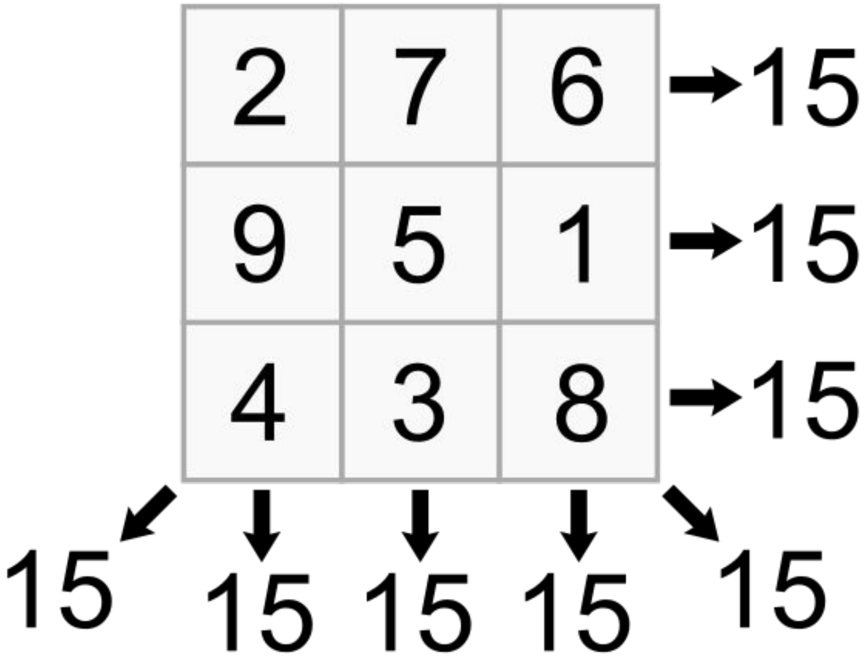
The Sagrada Família Magic Square

Tyler Pringle

Basilica de la Sagrada Família in Barcelona, Spain



So what are magic squares?

- A **magic square** is an n-by-n grid of numbers (generally positive integers) where the sums of the numbers in each row, each column, and both main diagonals are the same. This sum is the **magic constant** of the square.
- A **normal** magic square includes all the positive integers up to n^2.

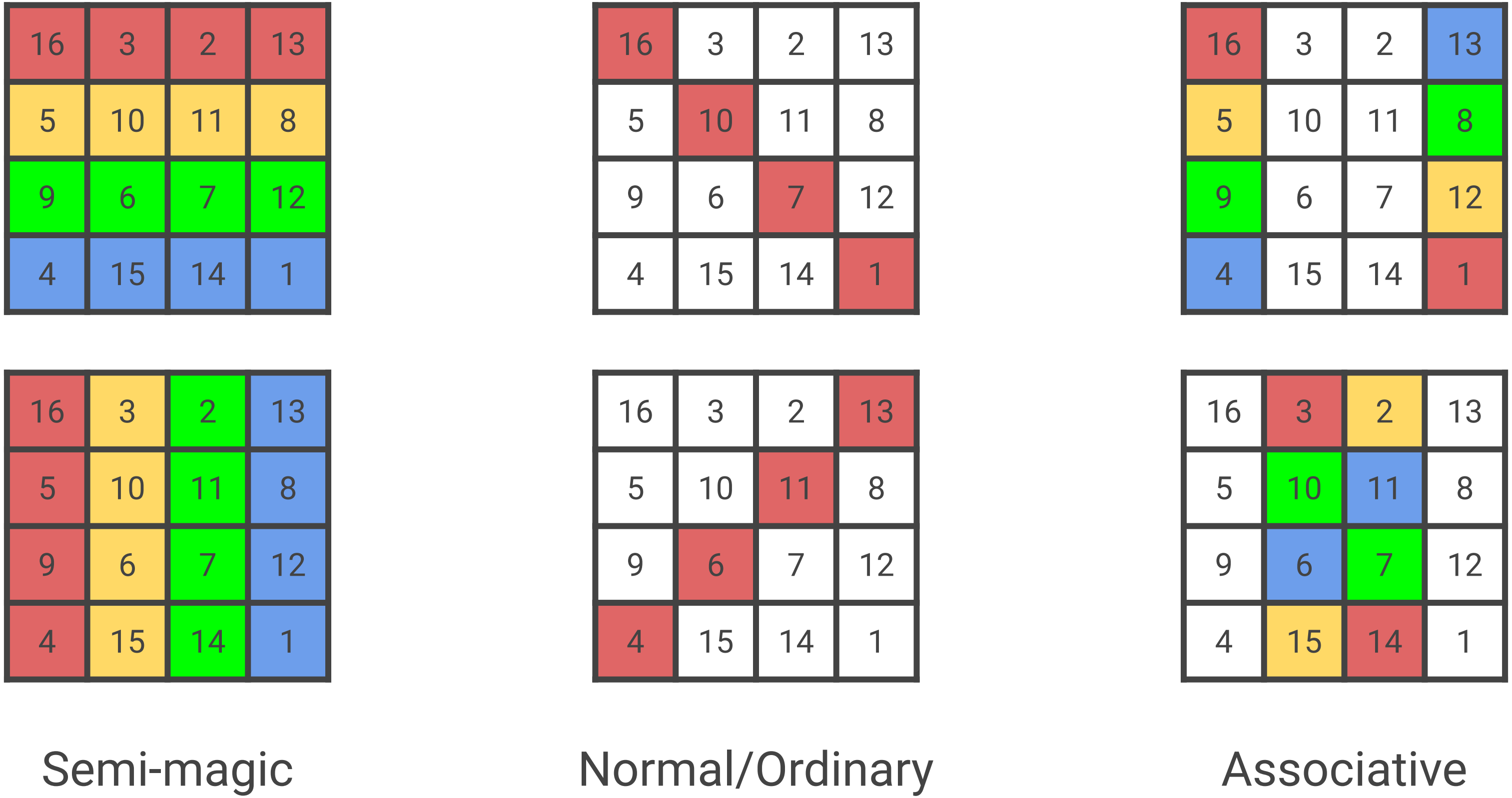
1	14	14	4	33
11	7	6	9	33
8	10	10	5	33
13	2	3	15	33

33

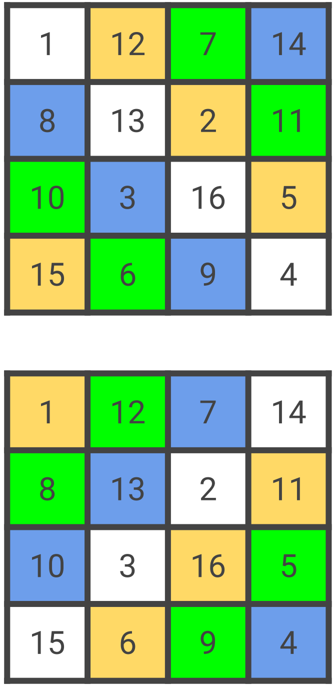
33	33	33	33

33

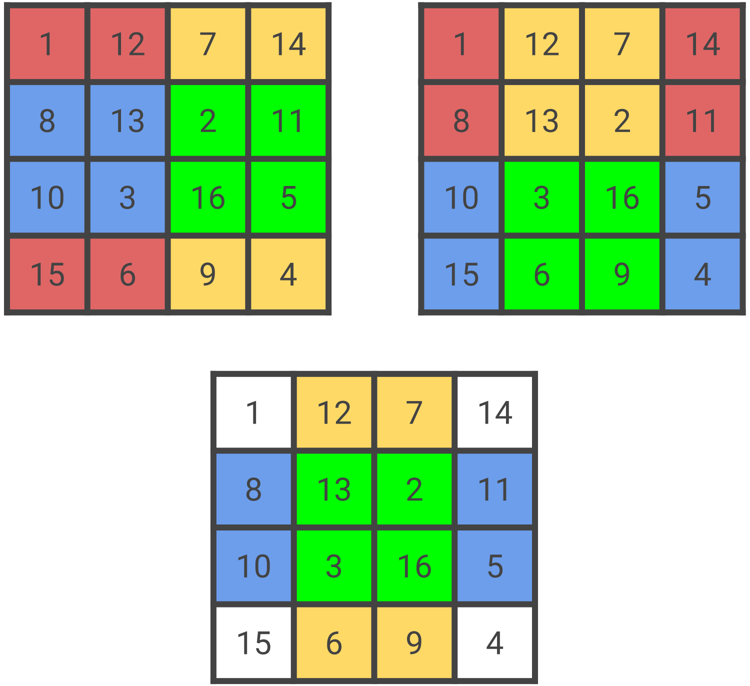
Are there any other groupings that add up to the magic constant?



16 3 2 13
5 10 11 8
9 6 7 12
4 15 14 1
Semi-magic
Normal/Ordinary
Associative



Pandiagonal



Most-perfect

1	14	14	4
11	7	6	9
8	10	10	5
13	2	3	15

Trivial magic squares

A magic square with repeated numbers is considered **trivial**, as they're usually not mathematically interesting.

Trivial magic squares

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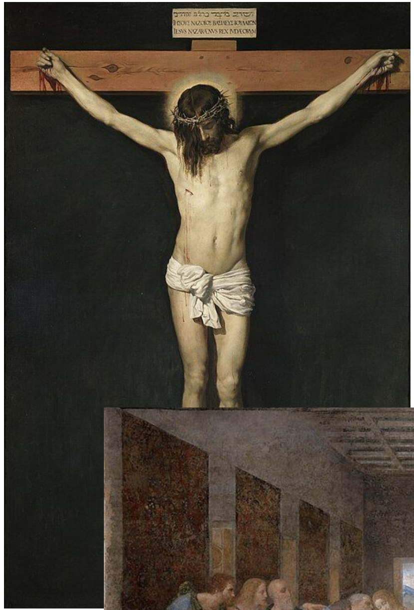
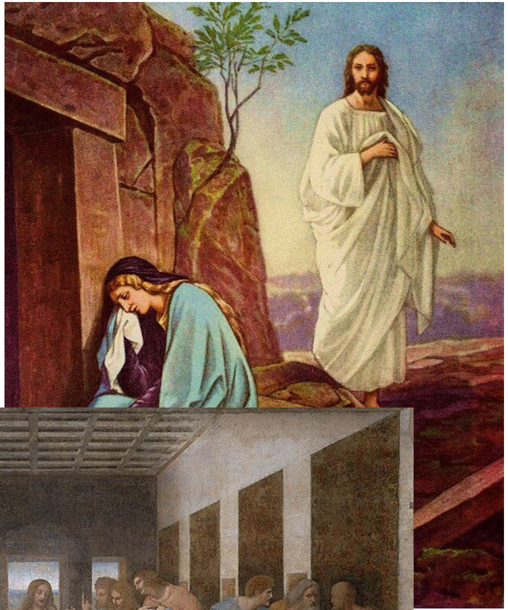
7	10	16	0
15	1	6	11
2	18	8	5
9	4	3	17

This one is <u>nontrivial</u> and still has a magic constant of 33

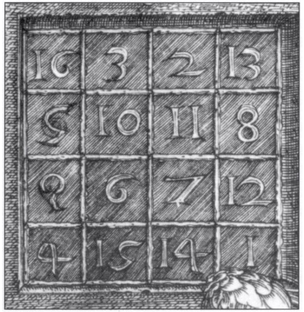
Important numbers in the Sagrada Familia square

- **33, the magic constant**
  - Also the traditional age Jesus is believed to have been crucified
  - The number **3** also has huge importance within Christianity
- **The repeated numbers: 14, 14, 10 & 10**
  - When added, you get 48. Divide 48 by 4 and you get the number **12** (12 tribes of Israel, 12 apostles)





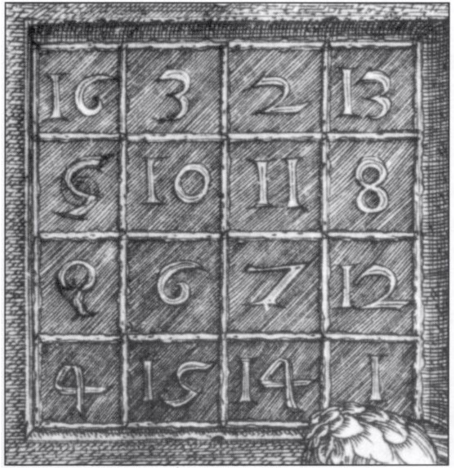
Origins of the Sagrada Familia square



- Sculpted by **Josep Maria Subirachs** sometime after 1987 as a part of the Passion Facade of the Basilica
- Inspired by Albrecht Durer’s magic square in the engraving *Melencolia I*

Albrecht Dürer

Melencolia I (1514)

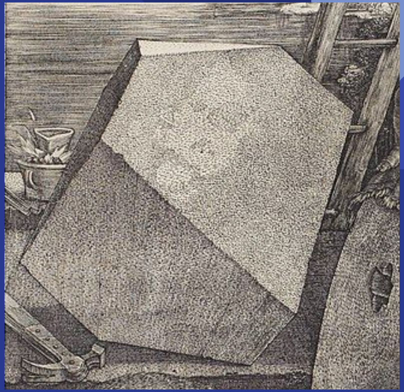


Closeup of the square

Why did Subirachs put the magic square there in the first place, though?


1 14 14 4
11 7 6 9
8 10 10 5
13 2 3 15

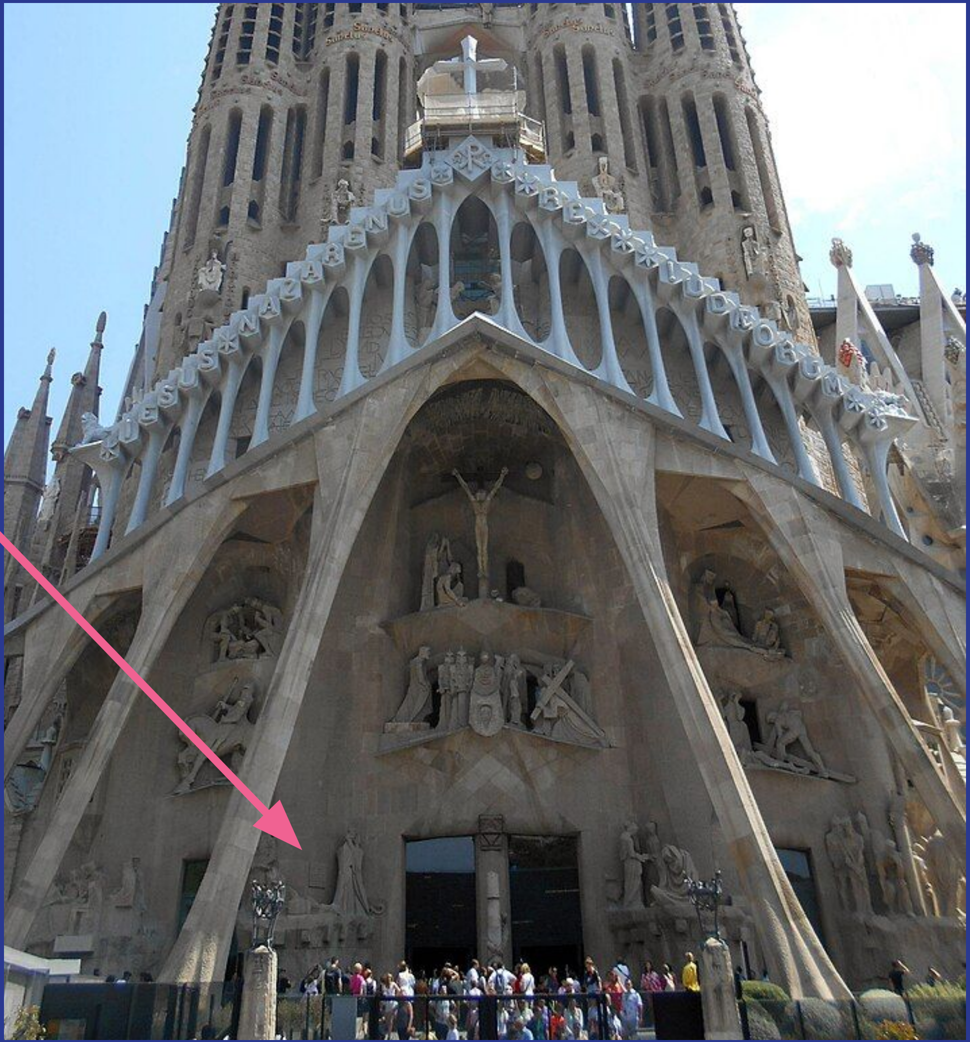
Mathematical Objects in *Melencolia I*



Dürer's Solid

Perfect sphere

Compass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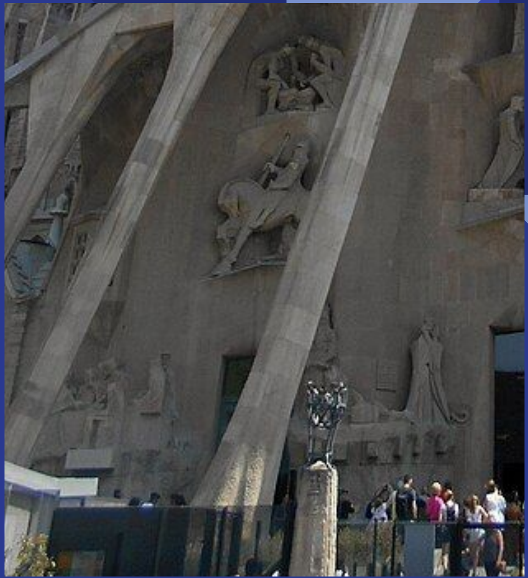
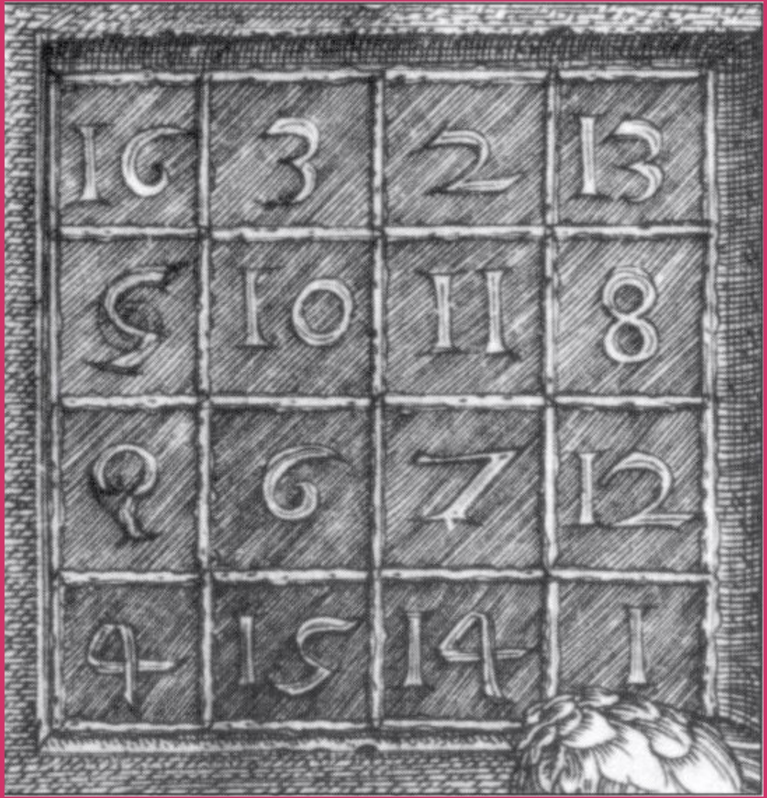
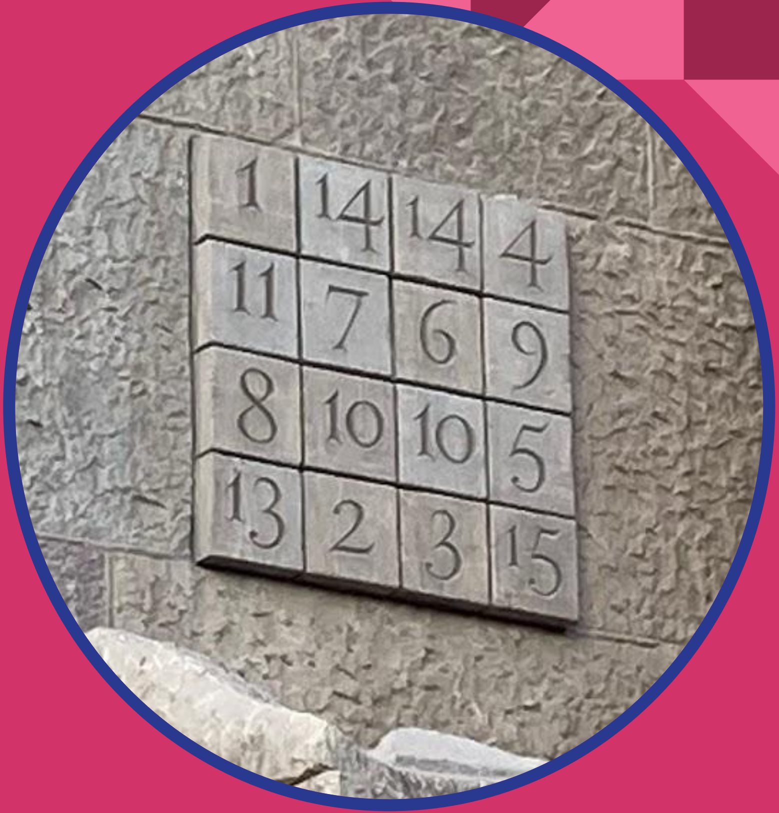



16 3 2 13
5 10 11 8
9 6 7 12
4 15 14 1


1 14 14 4
11 7 6 9
8 10 10 5
13 2 3 15

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Melencolia I square

1	14	14	4
11	7	6	9
8	10	10	5
13	2	3	15

Sagrada Família square

0. Start with the *Melencolia* square

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

1. Rotate square clockwise

4	9	5	16
15	6	10	3
14	7	11	2
1	12	8	13

How Subirachs transformed the *Melencolia* square

2. Rotate square clockwise again

4	9	5	16
15	6	10	3
14	7	11	2
1	12	8	13

1	14	15	4
12	7	6	9
8	11	10	5
13	2	3	16

How Subirachs transformed the *Melencolia* square

3. Subtract 1 from specific cells, 1 in each row and column

1	14	15	4
12	7	6	9
8	11	10	5
13	2	3	16

1	14	14	4
11	7	6	9
8	10	10	5
13	2	3	15

How Subirachs transformed the *Melencolia* square

1	14	14	4
11	7	6	9
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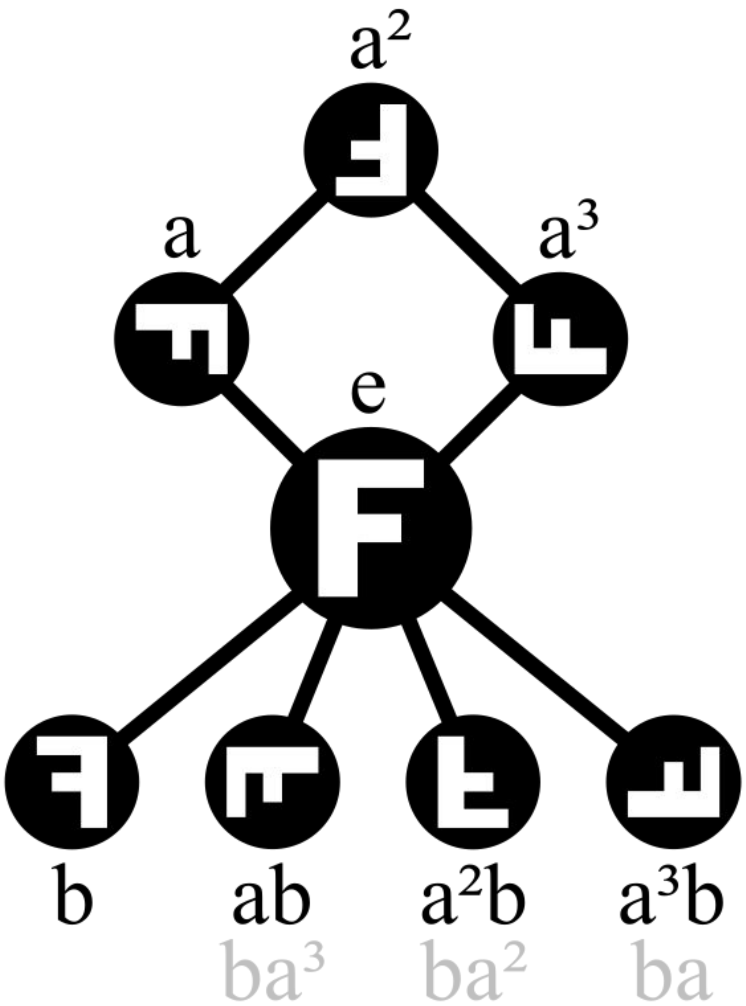


A magical property

Magic squares remain magic when you rotate them by 90 degrees one or more times, when you reflect them horizontally or vertically, or any combination of those two actions.

In other words, magic squares remain magic when transformed by any of the 8 elements of D_4.

8	1	6
3	5	7
4	9	2

4	3	8
9	5	1
2	7	6

2	9	4
7	5	3
6	1	8

6	7	2
1	5	9
8	3	4

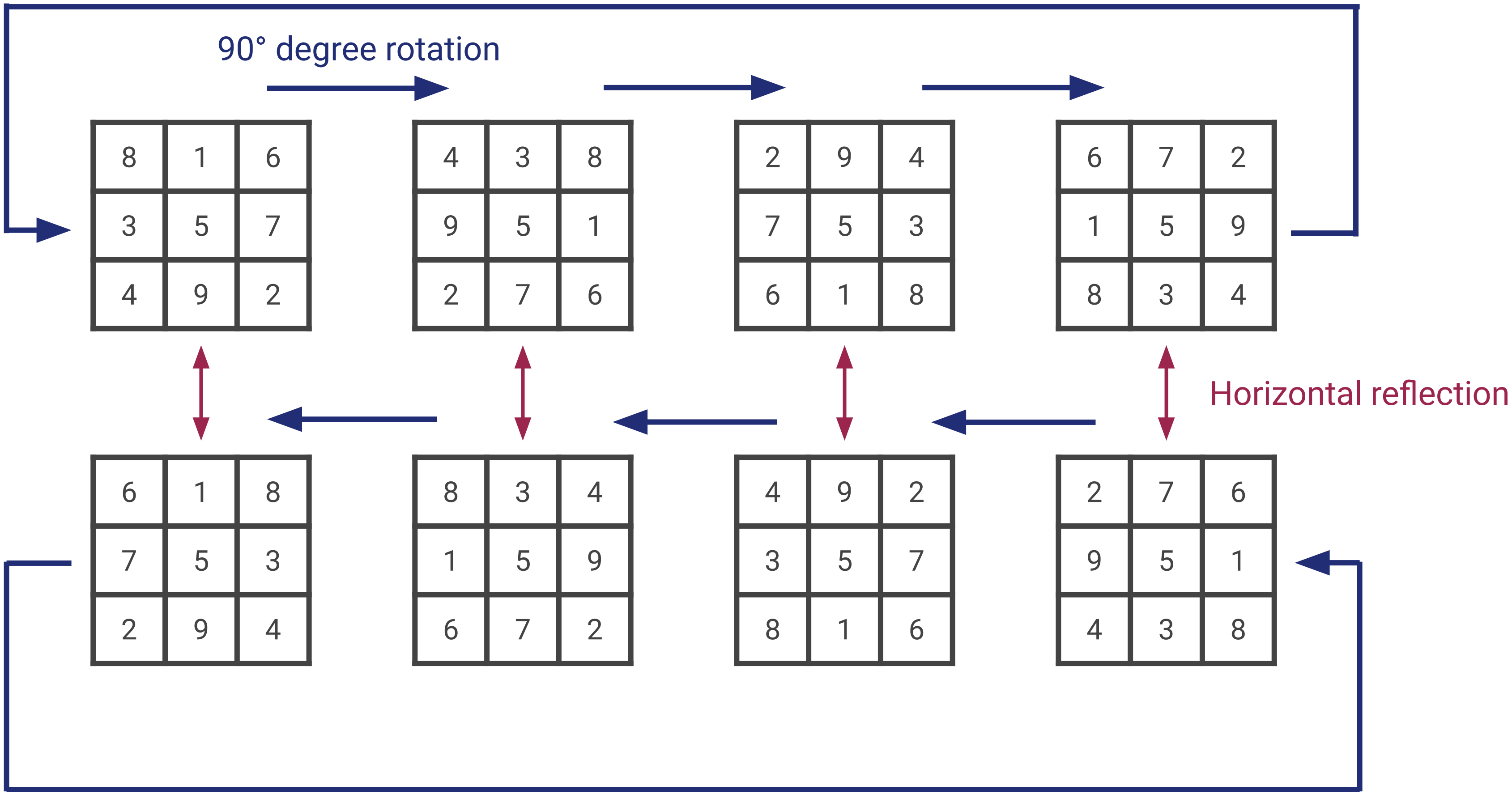
6	1	8
7	5	3
2	9	4

8	3	4
1	5	9
6	7	2

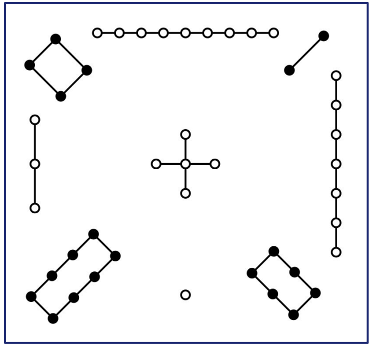
4	9	2
3	5	7
8	1	6

2	7	6
9	5	1
4	3	8

These 8 squares are considered to be in the same equivalence class.


90° degree rotation
Horizontal reflection
8 1 6
3 5 7
4 9 2
4 3 8
9 5 1
2 7 6
2 9 4
7 5 3
6 1 8
6 7 2
1 5 9
8 3 4
6 1 8
7 5 3
2 9 4
8 3 4
1 5 9
6 7 2
4 9 2
3 5 7
8 1 6
2 7 6
9 5 1
4 3 8

Magic squares have been around for a while.



Luoshu Square
(as early as 4th century BCE)

4	9	2
3	5	7
8	1	6

Parshvanatha temple
square (12th century CE)

a^2	b^2	c^2
d^2	e^2	f^2
g^2	h^2	i^2

Open problem: a 3x3 magic square of squares?
($100 prize offered in 1996)

"...the keys to mathematics are beauty and elegance and not dullness and technicality."

Jerry P. King

Sources

- https://blog.sagradafamilia.org/en/divulgation/the-magic-square-the-passion-facade-keys-to-understanding-it/
- https://en.wikipedia.org/wiki/Magic_square
- https://en.wikipedia.org/wiki/Luoshu_Square
- https://en.wikipedia.org/wiki/Sagrada_Fam%C3%ADlia
- https://en.wikipedia.org/wiki/Melencolia_I
- S. Cammann, "The Magic Square of Three in Old Chinese Philosophy and Religion," *History of Religions*, vol. 1, no. 1, pp. 37–80, Jul. 1961, doi: 10.1086/462439.
- J. P. King, *The Art of Mathematics*. Ballantine Books, 1993.
- L. Sallows, "Letters to the editor," *The Mathematical Intelligencer*, vol. 25, no. 4, pp. 6–7, Sep. 2003, doi: 10.1007/BF02984856.