$$U_{OR}|x,y,0\rangle = |\neg x, \neg y, x\vee y\rangle$$

Note on Homework 6

(4.8)

4.8 Check the effect of the matrix on $|000\rangle, |010\rangle, |100\rangle, |110\rangle$.

$|00\rangle\langle 11| \otimes X \to A$

$+ |01\rangle\langle 01| \otimes X \to B$

$+ |10\rangle\langle 10| \otimes X \to C$

$+ |11\rangle\langle 00| I \to D$

4.9 Apply CCNOT to $|x,y,1\rangle$.

4.10 Follow the suggestion in the problem.

4.11 Organize the vector as: $\frac{1}{\sqrt{2}}|0\rangle(c_1|0\rangle + c_2|1\rangle) + \frac{1}{\sqrt{2}}|1\rangle(c_3|0\rangle + c_4|1\rangle)$.

4.12 Assume the first column of U is $(a,b,c,d)^t$. Let U_1, U_2, U_3 be ...

4.13 Routine calculation.

4.14 Just follow the circuit in Fig. 4.7.

4.15 Show that the matrices are $|0\rangle\langle 0| \otimes I_2 + |1\rangle\langle 1| \otimes V_j$ for $j = 1, 2$, and ...

4.16 and **4.17** Write down the matrices in tensor form, the bra and ket vectors carefully, and verify.

$$U_{OR} = \begin{bmatrix} O & & X \\ & X & \\ X & & \\ I & & O \end{bmatrix}$$

(1) $U_{OR}|000\rangle$

$= (A+B+C+D)|000\rangle$

$= A|000\rangle + B|000\rangle + \cdots + D|000\rangle =$

e.g. $A|000\rangle = |00\rangle\langle 11|00\rangle \otimes X|0\rangle$

$= |110\rangle$

$$(A_1 \otimes X)|000\rangle = A_1|00\rangle \otimes X|0\rangle$$

$B|000\rangle$

(4.9) $U_{CCNOT} \to U$

$= (|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10|) I$

$+ |11\rangle\langle 11| X$

$$= \begin{bmatrix} I_6 & 0 \\ 0 & \begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix} \end{bmatrix}$$

$U|001\rangle =$

$U|011\rangle =$

$U|101\rangle =$

$U|111\rangle =$

$|x, y, \neg(x\wedge y)\rangle$

$U|00,1\rangle = |001\rangle$

$U|011\rangle = \cdots ?$

$U|101\rangle = \cdots ?$

(4.10) ~~Assume U exists s.t.~~ $|\psi\rangle, |\phi\rangle$

$|\Psi\rangle = U|\psi, 0\rangle = |\psi\psi\rangle$

$|\Phi\rangle = U|\phi, 0\rangle = |\phi\phi\rangle$

(1) $\langle\Psi|\Phi\rangle = \langle\psi 0|U^* U|\phi 0\rangle$

$= \langle\psi\psi|U|\phi 0\rangle$

$= \langle\psi 0|U^*|\phi\phi\rangle$

$= \langle\psi\psi|\phi\phi\rangle$

(2)

(2) $|\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|\phi\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(4.11)

$|\psi\rangle = a|00\rangle + b|11\rangle$

$(H \otimes I_2)|\psi\rangle = a\frac{(|0\rangle + |1\rangle)}{\sqrt{2}}|0\rangle + b\frac{(|0\rangle - |1\rangle)}{\sqrt{2}}|1\rangle$

$= \frac{|0\rangle}{\sqrt{2}}(c_1|0\rangle + c_2|1\rangle) + \frac{|1\rangle}{\sqrt{2}}(c_3|0\rangle + c_4|1\rangle)$

(4.12) $\mathcal{U} = \begin{pmatrix} a & \\ b & * \\ c & \\ d & \end{pmatrix}$. assume $a, b, c, d \neq 0$

Want: $U_3 U_2 U_1 \mathcal{U} = \begin{bmatrix} 1 & \\ 0 & * \\ 0 & \\ 0 & \end{bmatrix}$

Let $V_1 = \dfrac{\begin{bmatrix} \bar{c} & \bar{d} \\ -d & c \end{bmatrix}}{\sqrt{|c|^2+|d|^2}}$ $\quad V_1 \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} \sqrt{|c|^2+|d|^2} \\ 0 \end{bmatrix}$

$U_1 = \begin{bmatrix} I_2 & 0 \\ 0 & V_1 \end{bmatrix}$ Then $U_1 \mathcal{U} = \begin{bmatrix} a & x & x & x \\ b & x & x & x \\ \gamma_1 & x & x & x \\ 0 & x & x & x \end{bmatrix}$

$\gamma_1 = \sqrt{|c|^2+|d|^2}$

(!!) Find $V_2 \begin{bmatrix} b \\ \gamma_1 \end{bmatrix} = \begin{bmatrix} \sqrt{|b|^2+|\gamma_1|^2} \\ 0 \end{bmatrix}$

Let $U_2 = \begin{bmatrix} 1 & & \\ & V_2 & \\ & & 1 \end{bmatrix}$ Then $U_2 U_1 \mathcal{U} = \begin{bmatrix} x & x & x & x \\ x & x & x & x \\ 0 & x & x & x \\ 0 & x & x & x \end{bmatrix}$

(4.13) $\mathcal{U} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \xrightarrow{U_3 U_2 U_1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & x & x & x \\ 0 & x & x & x \\ 0 & x & x & x \end{pmatrix}$

$\xrightarrow{W_2 W_1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & x & x \\ 0 & 0 & x & x \end{pmatrix} \xrightarrow{V_1} \begin{pmatrix} 1 & & & \\ & 1 & & 0 \\ & 0 & 1 & \\ & & & 1 \end{pmatrix}$

$V_1 W_2 W_1 U_3 U_2 U_1 \mathcal{U} = I \quad \therefore \mathcal{U} = U_1^{\dagger} \cdots W_2^{\dagger} V_1^{\dagger}$

(4.14) $\begin{pmatrix} a & & & & & & & b \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ c & & & & & & & d \end{pmatrix}$

Figure 6.7

$p_1 \; p_2 \; p_3$

$|000\rangle$

$G_1 \; |100\rangle$

$G_2 \; |110\rangle$

$G_3 \; |111\rangle$

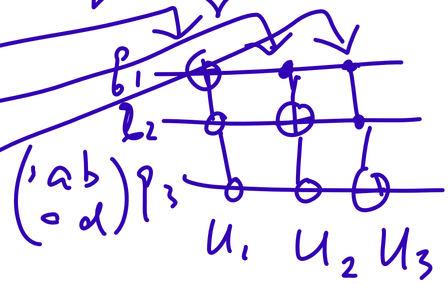
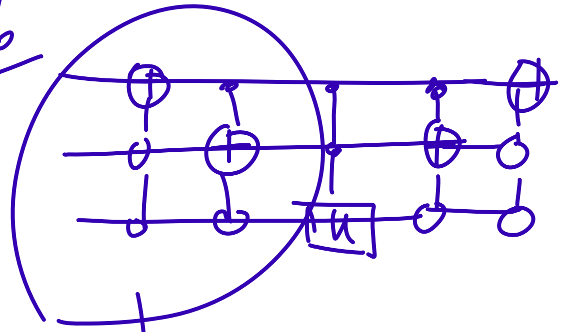


Fig 6.6



$$\begin{array}{cc} |000\rangle & |111\rangle \end{array}$$
$$\begin{bmatrix} a & & b \\ & 1 & \\ & \ddots & \\ & & 1 \\ c & & d \end{bmatrix} \quad \begin{matrix} X_0 \ |000\rangle \\ X_1 \\ \vdots \\ X_6 \\ X_7 \ |111\rangle \end{matrix}$$

n

(4.12)

$$\left[\begin{array}{c|c} I_6 & 0 \\ \hline 0 & \begin{matrix} a & b \\ c & d \end{matrix} \end{array}\right] \quad \begin{bmatrix} 1 \\ \vdots \\ X_6 \\ X_7 \end{bmatrix}$$

$$\hat{u} = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \quad \begin{pmatrix} a & & & c \\ & 1 & & \\ & & \ddots & \\ & & & 1 \\ b & & & d \\ & & & & 1 \end{pmatrix} \quad \begin{matrix} |000\rangle \\ \vdots \\ |110\rangle \\ |111\rangle \end{matrix}$$

$q_1 \ q_2 \ q_3$

$|000\rangle$

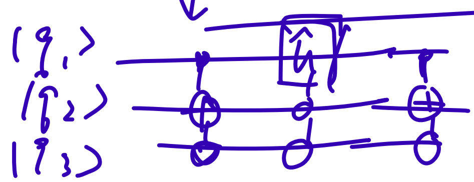
$|100\rangle$

$|110\rangle$

1

5

(4.4)(2)



$|100\rangle$

$|111\rangle$

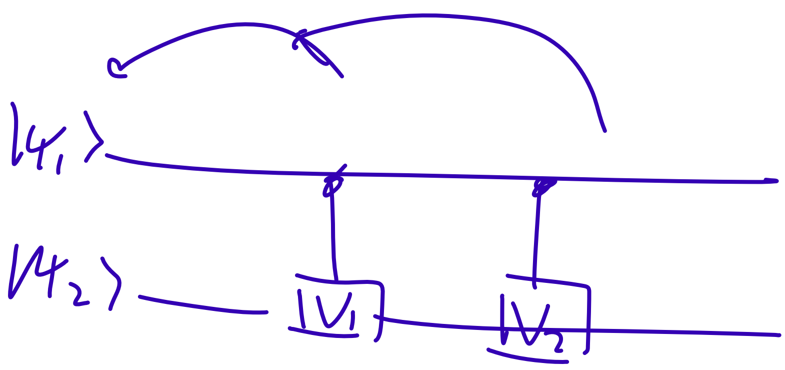
$|000\rangle$

$\downarrow$

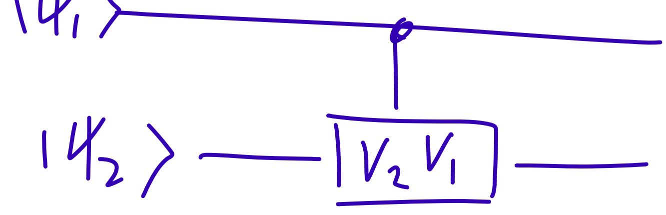
$|100\rangle$

$|110\rangle$

(6.13)



$$\underbrace{\begin{pmatrix} I_2 & \\ & V_1 \end{pmatrix} \begin{pmatrix} I_2 & \\ & V_2 \end{pmatrix}}$$



$|00\rangle$

$|01\rangle$

$|10\rangle$

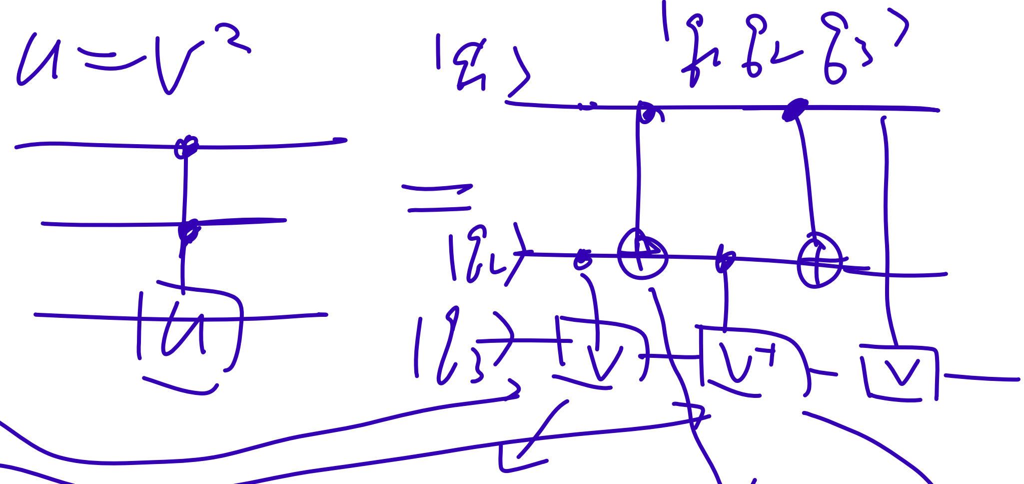
$|11\rangle$

$$V_1 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$V_2 = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

(4.16)

$U = V^2$



A B C D E

$EDCBA = \begin{bmatrix} I_6 & \\ & U \end{bmatrix}$

$I_2 \otimes \left[\begin{array}{c|c} I & 0 \\ \hline 0 & V \end{array}\right] * \begin{bmatrix} I & 0 \\ 0 & X \end{bmatrix} \otimes I_2$

$I_2 \otimes \begin{bmatrix} I & 0 \\ 0 & V^{-1} \end{bmatrix}$

$I_2 \otimes \left(|0\rangle\langle 0| \otimes I_2 + |1\rangle\langle 1| \otimes V \right)$

(4.17)