Some characterizations of generalized quantum operations and entanglement breaking channels

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1. Introduction

\[ \mathcal{B}(\mathcal{H}) \] is the set of all bounded linear operators on a separable Hilbert space \( \mathcal{H} \) (finite or infinite dimension).

\[ S(\mathcal{H}) \] is the set of all quantum states on \( \mathcal{H} \). That is, \( \rho \in S(\mathcal{H}) \) if and only if \( \rho \geq 0 \) and \( \text{tr}(\rho) = 1 \).

A completely positive map \( \Phi \) from \( S(\mathcal{H}) \) into \( S(\mathcal{K}) \) is a generalized quantum operation if \( \text{tr}(\Phi(A)) \leq \text{tr}(A) \) for positive trace-class operator \( A \).
1. Introduction

A quantum channel is a completely positive linear map $\Psi$ from $S(\mathcal{H})$ into $S(\mathcal{K})$ defined by the Kraus representation

$$\Psi(\rho) = \sum_{i=1}^{\infty} V_i \rho V_i^*, \text{ with } \sum_{i=1}^{\infty} V_i^* V_i = I_{\mathcal{H}},$$

where $V_i \in \mathcal{B}(\mathcal{H}, \mathcal{K})$. 
The dual of $\Phi$ is defined by

$$
\Phi^+(X) = \sum_{i=1}^{\infty} V_i^* X V_i \quad \text{for } X \in B(\mathcal{H}).
$$

It is clear that

$$
|Tr[\Phi^+(X)Y]| = |Tr[X \Phi(Y)]| \leq \|\Phi\|\|Y\|Tr(|X|),
$$

for $X \in B(\mathcal{H})$ and $Y \in S(\mathcal{H})$, so $\Phi^+$ is well defined on $B(\mathcal{H})$. 

A completely positive linear map $\Psi$ from $B(\mathcal{H})$ into $B(\mathcal{K})$ is unital if $\Psi(I) = I$. A quantum channel is double stochastic if it is both trace-preserving and unital. In this case, $\Psi$ is a map from $B(\mathcal{H})$ into $B(\mathcal{K})$.

As usual, $|A|$ is the absolute value operator of $A \in B(\mathcal{H})$ and $A^+$ and $A^-$ are the positive and negative parts of $A$. That is $A^+ = \frac{A + |A|}{2}$, and $A^- = \frac{|A| - A}{2}$. 
As the definition in [1], for $r$ and $s \in \mathbb{R}^d$, we say that $r$ is sub-majorized by $s$, written as $r \prec_w s$, if $\sum_{i=1}^{k} r_i^\downarrow \leq \sum_{i=1}^{k} s_i^\downarrow$, $(1 \leq k \leq d)$. If the condition $\sum_{i=1}^{d} (r_i^\downarrow) = \sum_{i=1}^{d} (s_i^\downarrow)$ is supplemented, then $r$ is majorized by $s$ (written as $r \prec s$).

We also denote $A \prec B$ if the vector consisted by all eigenvalues of $A$ is majorized by that of $B$, where $A$ and $B$ are $n \times n$ Hermitian matrices.
Theorem

(Uhlmann’s Theorem) Let $A$ and $B$ be Hermitian matrices. The following statements are equivalent:

1. $A \prec B$;
2. There exists a random unitary quantum operation $\Phi$ such that $A = \Phi(B)$;
3. There exists a doubly stochastic quantum operation $\Phi$ such that $A = \Phi(B)$. 
1. Introduction

Theorem

(Choi theorem, [2, Theorem 2]) Let $\mathcal{H}$ and $\mathcal{K}$ be finite dimensional Hilbert spaces. Then $\Phi : \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{K})$ is completely positive if and only if

$$\sum_{i,j=1}^{n} |e_i\rangle\langle e_j| \otimes \Phi(\langle e_i|e_j\rangle) \geq 0,$$

where $\{|e_i\rangle\}_{i=1}^{n}$ is an orthonormal basis of $\mathcal{H}$. 
A quantum channel \( \Phi \) is called entanglement breaking (EBT) if \( I \otimes \Phi(\rho) \) is always separable for any states \( \rho \in S(\mathcal{H} \otimes \mathcal{H}) \), i.e., any entangled density matrix \( \rho \) is mapped to a separable one (see[4])

Such channels can be written in the form \( \Phi(\rho) = \sum_{k=1} R_k tr(F_k \rho) \) where each \( R_k \) is a density matrix and the \( F_k \) form a positive operator valued measure POVM.
1. Introduction

A EBT map is classical-quantum (CQ) if each $F_k = |k\rangle\langle k|$ in the POVM is a one-dimensional projection.

A EBT map is quantum-classical (QC) if each density matrix $R_k = |k\rangle\langle k|$ is a one-dimensional projection with $\sum_{k=1}^{k=1} R_k = I$. 
Theorem

(Li-Poon, [5, Theorem 3.1]) There is a unital completely positive map \( \Phi : M_n \rightarrow M_m \) such that \( \Phi(A) = B \) if and only if there is a nonnegative column stochastic matrix \( D = (d_{pq}) \) such that \( (\mu_1, \mu_2 \cdots \mu_m) = (\lambda_1, \lambda_2 \cdots \lambda_n)D \), where \( \mu_i \) and \( \lambda_j \) are all eigenvalues of self-adjoint operators \( B \) and \( A \) respectively.
1. Introduction

Some properties such as the fixed points and interpolation of (generalized) quantum operations and its applications in quantum information attract much more attention of a number of researchers. In particular, conditions for the existence of a quantum channel $\Phi$ such that $\Phi(A_j) = B_j$ for $j = 1, 2 \cdots k$ are established in [5], where $\{A_1, A_2, \cdots A_k\}$ and $\{B_1, B_2, \cdots B_k\}$ are commuting families of Hermitian matrices, respectively.
2. GQO in finite dimension

In part 2, we mainly characterize the sets of

\[ \{ \Phi(B) : \Phi \text{ is a generalized quantum operation} \} , \]

\[ \{ \Phi(B) : \Phi^+ \text{ is a generalized quantum operation} \} , \]

and

\[ \{ \Phi(B) : \Phi \text{ and } \Phi^+ \text{ are generalized quantum operations} \} , \]

where \( \Phi^+ \) is a dual of \( \Phi \) and \( B \) is a self-adjoint operator. In particular, the third set is relevant to Uhlmann’s theorem in quantum information.
2. GQO in finite dimension

**Theorem**

If $A \in \mathcal{B}(\mathcal{H})$ is a self-adjoint operator, then \( \{ \sum_i A_i A A_i^* : A_i \in \mathcal{B}(\mathcal{H}, \mathcal{K}) \text{ with } \sum_i A_i^* A_i \leq I \} = \{ B - C : B, C \geq 0, tr(B) \leq tr(A^+), tr(C) \leq tr(A^-) \} \).

**Corollary**

If $A \in \mathcal{B}(\mathcal{H})$ is a self-adjoint operator, then \( \{ \sum_i A_i AA_i^* : A_i \in \mathcal{B}(\mathcal{H}, \mathcal{K}) \text{ with } \sum_i A_i^* A_i = I \} = \{ B - C : B, C \geq 0, tr(B) = tr(A^+), tr(C) = tr(A^-) \} \).
Lemma

Let $A \in B(H)$ be a positive operator.

(i) Then $\{ \sum_i A_i AA_i^* : A_i \in B(H,K) \text{ with } \sum_i A_i A_i^* \leq I \} = \{ B : 0 \leq B \in B(K) \text{ and } \|B\| \leq \|A\| \}$.

(ii) If $A$ is invertible, then $\{ \sum_i A_i AA_i^* : A_i \in B(H,K) \text{ with } \sum_i A_i A_i^* = I \} = \{ B : 0 \leq B \in B(K), \|B^{-1}\| \leq \|A^{-1}\| \text{ and } \|B\| \leq \|A\| \}$.

(iii) If $A$ is not invertible, then $\{ \sum_i A_i AA_i^* : A_i \in B(H,K) \text{ with } \sum_i A_i A_i^* = I \} = \{ B : 0 \leq B \in B(K) \text{ and } \|B\| \leq \|A\| \}$. 
2. GQO in finite dimension

**Theorem**

If $A \in \mathcal{B}(\mathcal{H})$ is a self-adjoint operator, then \[ \left\{ \sum_i A_i A A_i^* : A_i \in \mathcal{B}(\mathcal{H}, \mathcal{K}) \text{ with } \sum_i A_i A_i^* \leq I \right\} = \left\{ B - C : B, C \geq 0, \|B\| \leq \|A^+\|, \|C\| \leq \|A^-\| \right\}. \]

**Theorem**

If $A \in \mathcal{B}(\mathcal{H})$ is a self-adjoint operator and $A^+, A^- \neq 0$, then \[ \left\{ \sum_i A_i A A_i^* : A_i \in \mathcal{B}(\mathcal{H}, \mathcal{K}) \text{ with } \sum_i A_i A_i^* = I \right\} = \left\{ B - C : B, C \geq 0, 0 < \|B\| \leq \|A^+\|, 0 < \|C\| \leq \|A^-\| \right\}. \]
2. GQO in finite dimension

Theorem

Let $A, B \in \mathcal{B}(\mathcal{H})$ be positive operators and have eigenvalues $\lambda_1, \lambda_2 \cdots \lambda_n$ and $\mu_1, \mu_2 \cdots \mu_n$. Suppose $a = (\lambda_1, \lambda_2 \cdots \lambda_n)$ and $b = (\mu_1, \mu_2 \cdots \mu_n)$. The following are equivalent.

(a) There exists a contractive generalized quantum operation $\Phi$ such that $\Phi(A) = B$.

(b) There is a double sub-stochastic matrix $D$ such that $b = aD$.

(c) There exists a mixed partial isometries completely positive map $\Phi$ such that $\Phi(A) = B$.

(d) $b \prec_w a$. 

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Remark. It is well-known that Schur-Horn theorem give the characterization of \( \text{diag}\{U\text{diag}(\lambda_1, \lambda_2 \cdots \lambda_n)U^* : U \text{ is } n \times n \text{ unitary matrices}\} = \{(\mu_1, \mu_2 \cdots \mu_n) : (\mu_1, \mu_2 \cdots \mu_n) \prec (\lambda_1, \lambda_2 \cdots \lambda_n)\} \), where all \( \lambda_i \) and \( \mu_i \) are real numbers.

It is seem to hold that \( \text{diag}\{V\text{diag}(\lambda_1, \lambda_2 \cdots \lambda_n)V^* : V \text{ is } n \times n \text{ partial isometry matrices}\} = \{(\mu_1, \mu_2 \cdots \mu_n) : (\mu_1, \mu_2 \cdots \mu_n) \prec_w (\lambda_1, \lambda_2 \cdots \lambda_n)\} \) for nonnegative real numbers \( \lambda_i \) and \( \mu_i \).
2. GQO in finite dimension

However, this isn’t true. For example, \((1, \frac{1}{3}) \not\in \text{diag}\{V \text{diag}(1, \frac{1}{2})V^* : V \text{ is } 2 \times 2 \text{ partial isometry matrices}\}\). Indeed, suppose that \(V = (a_{ij})\) is a partial isometry and \((1, \frac{1}{3}) \in \text{diag}(V \text{diag}(1, \frac{1}{2})V^*)\), then \(VV^*\) is a rank one orthogonal projection, which implies that

\[
\sum_{i=1}^{2} \sum_{j=1}^{2} |a_{ij}|^2 = 1.
\]

And \((1, \frac{1}{3}) \in \text{diag}(V \text{diag}(1, \frac{1}{2})V^*)\) yields

\[
\sum_{i=1}^{2} |a_{i1}|^2 + \frac{1}{2} \sum_{i=1}^{2} |a_{i2}|^2 = \frac{4}{3}.
\]

It is a contradiction.
2. GQO in finite dimension

**Corollary**

If \( A \in \mathcal{B}(\mathcal{H}) \) is a self-adjoint operator, then \( \{ \sum_i A_i A_i^* : A_i \in \mathcal{B}(\mathcal{H}) \text{ with } \sum_i A_i A_i^* \leq I, \sum_i A_i^* A_i \leq I \} = \{ B - C : B, C \geq 0, B \prec_w A^+, C \prec_w A^- \}. \)

**Corollary**

Let \( A \in \mathcal{B}(\mathcal{H}) \) be a self-adjoint operator and \( \Phi \) be a quantum channel from \( \mathcal{B}(\mathcal{H}) \) into \( \mathcal{B}(\mathcal{K}) \). If \( \text{tr}(\Phi(A)^+) \geq \text{tr}(A^+) \), then \( \Phi(A^+) = \Phi(A)^+ \) and \( \Phi(A^-) = \Phi(A)^- \).
In part 3, we always assume that $\dim \mathcal{H} = m$ and $\dim \mathcal{K} = n$.

For two $m \times n$ matrices $A, B$, the Hadamard product (Schur product) $A \circ B$ is a $m \times n$ matrix with elements given by $(A \circ B)_{i,j} = (a_{i,j}b_{i,j})$, for $1 \leq i \leq m, 1 \leq j \leq n$.

The following Schur product theorem and the Schmidt decomposition theorem are well-known in quantum information theory.
Lemma

(Schur product theorem) The Hadamard product of two positive semi-definite matrices is positive semi-definite.

Lemma

(The Schmidt decomposition theorem) If $\varphi \in B(\mathcal{H} \otimes \mathcal{K})$ is a pure state, then there exist orthonormal states $|e_i\rangle$ of $\mathcal{H}$ and orthonormal states $|f_i\rangle$ of $\mathcal{K}$ such that

$$|\varphi\rangle = \sum_{i=1}^{k} \lambda_i |e_i\rangle|f_i\rangle,$$

where $\lambda_i$ are non-negative real numbers with $\sum_i \lambda_i^2 = 1$. 
The following two theorems tell us that the entanglement breaking maps are enough much.

**Theorem**

For any separable state \( \rho \in S(\mathcal{H} \otimes \mathcal{K}) \), there exists an entanglement breaking map \( \Phi \) and the pure state \( |\varphi\rangle \in B(\mathcal{H} \otimes \mathcal{H}) \) such that

\[
(I \otimes \Phi)(|\varphi\rangle\langle\varphi|) = \rho.
\]
A bipartite quantum state $\rho \in S(\mathcal{H} \otimes \mathcal{K})$ is separable if it can be written as $\rho = \sum_{i=1}^{s} q_i \rho_i \otimes \sigma_i$ where $\rho_i \in S(\mathcal{H})$ and $\sigma_i \in S(\mathcal{K})$, $q_i > 0 \sum_{i=1}^{s} q_i = 1$.

As usual, for $\rho \in S(\mathcal{H} \otimes \mathcal{K})$, $\rho^A := \text{tr}_2(\rho)$ denotes the reduced density operators of $\rho$ in the subspace $\mathcal{H}$. That is $\rho^A$ is the partial trace of the quantum state $\rho$ over the subspace $\mathcal{K}$. 
The following corollary characterize the equivalent condition for that there exists an entanglement breaking map $\Phi$ such that 
$$(I \otimes \Phi)(|\phi\rangle\langle\phi|) = \rho,$$ 
where $\rho \in B(\mathcal{H} \otimes \mathcal{K})$ and $|\phi\rangle\langle\phi| \in B(\mathcal{H} \otimes \mathcal{H})$ are given.

**Corollary**

Let $\rho \in B(\mathcal{H} \otimes \mathcal{K})$ be a separable state and $|\phi\rangle\langle\phi| \in B(\mathcal{H} \otimes \mathcal{H})$ a pure state. Then there exists an entanglement breaking map $\Phi$ such that $(I \otimes \Phi)(|\phi\rangle\langle\phi|) = \rho$ if and only if $tr_2(\rho) = tr_2(|\phi\rangle\langle\phi|)$. 
The following corollary is a little extension of [4, Theorem 4].

**Corollary**

$\Phi$ is an entanglement breaking map if and only if $(I \otimes \Phi)(|\varphi\rangle\langle\varphi|)$ is a separable state, for some $|\varphi\rangle = \sum_{i=1}^{m} \sqrt{\lambda_i} |e_i\rangle |f_i\rangle$, $\lambda_i > 0$ and $\sum_{i} \lambda_i = 1$, where $\{|e_i\rangle\}_{i=1}^{m}$ and $\{|f_i\rangle\}_{i=1}^{m}$ are orthonormal bases of $\mathcal{H}$. 
3. EBT in finite dimension

Similarly, we have the following corollary.

**Corollary**

Suppose that \( \{|e_i\rangle\}_{i=1}^m \) and \( \{|f_i\rangle\}_{i=1}^m \) are orthonormal bases of \( \mathcal{H} \). If \( \Phi \) is a linear map from \( B(\mathcal{H}) \) into \( B(\mathcal{K}) \), then \( \Phi \) is completely positive if and only if
\[
(I \otimes \Phi)(|\varphi\rangle\langle\varphi|) \geq 0
\]
for the vector \( |\varphi\rangle = \sum_{i=1}^m \sqrt{\lambda_i} |e_i\rangle|f_i\rangle \in \mathcal{H} \otimes \mathcal{H} \), where all \( \lambda_i > 0 \).
Theorem

For any $\rho \in S(\mathcal{H})$, $\sigma \in S(\mathcal{K})$, there exist classical-quantum (CQ) $\Phi$ and quantum-classical (QC) $\Psi$, respectively such that $\Phi(\rho) = \sigma$ and $\Psi(\rho) = \sigma$. 
Corollary

Let $\rho_1, \rho_2 \in B(H), \sigma_1, \sigma_2 \in B(K)$ satisfy $\rho_1 \rho_2 = 0$, then there exists an entanglement breaking map $\Phi$ such that $\Phi(\rho_1) = \sigma_1$ and $\Phi(\rho_2) = \sigma_2$.

Corollary

Let $A \in B(H)$ and $B \in B(K)$ be self-adjoint operators. Then there exists a completely positive trace preserving map $\Phi$ such that $\Phi(A) = B \iff$ there exists an entanglement breaking map $\Psi$ such that $\Psi(A) = B$.
Corollary

Let $A, B \in B(\mathcal{H})$ be self-adjoint operators. If there exists an entanglement breaking map $\Phi(\bullet) = \sum_{i=1}^{m} |e_i\rangle\langle e_i| \text{tr}(|\varphi_i\rangle\langle \varphi_i| \bullet)$, where $\{|\varphi_i\rangle\}_{i=1}^{m}$ and $\{|e_i\rangle\}_{i=1}^{m}$ are orthonormal bases of $\mathcal{H}$, respectively, such that $\Phi(A) = B$ if and only if $B \prec A$. 
4. infinite dimensional case

The purpose of part 4 is mainly considered some results of part 2 in the infinite dimensional case.

**Theorem**

Let $\mathcal{H}$ and $\mathcal{K}$ be infinite dimensional Hilbert spaces. For $\rho \in S(\mathcal{H})$ and $\sigma \in S(\mathcal{K})$, there exists a quantum channel $\Psi$ from $S(\mathcal{H})$ into $S(\mathcal{K})$ such that $\Psi(\rho) = \sigma$. 

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Corollary

Let $\mathcal{H}$ and $\mathcal{K}$ be infinite dimensional Hilbert spaces. If $A \in B(\mathcal{H})$ and $B \in B(\mathcal{K})$ are positive trace class operators, then there exists a sequence $C_i \in B(\mathcal{H}, \mathcal{K})$ such that $\sum_{i=1}^{\infty} C_i AC_i^* = B$ and $\sum_{i=1}^{\infty} C_i^* C_i \leq I$ if and only if $tr(B) \leq tr(A)$.

Remark 1. Let $\mathcal{H}$ be an infinite dimensional Hilbert space. If $\rho \in S(\mathcal{H})$ is invertible and $\sigma \in S(\mathcal{H})$ is a pure state such that $\sum_{i=1}^{\infty} A_i \rho A_i^* = \sigma$ and $\sum_{i=1}^{\infty} A_i^* A_i = I$ then $\sum_{i=1}^{\infty} A_i A_i^*$ does not exist.
Corollary

Let $A \in \mathcal{B}(\mathcal{H})$ and $B \in \mathcal{B}(\mathcal{K})$ be self-adjoint trace class operators with eigenvalues $\lambda_1, \lambda_2 \cdots \lambda_n \cdots$ and $\mu_1, \mu_2 \cdots, \mu_n \cdots$ respectively. Then there exists a quantum channel $\Phi$ such that $\Phi(I)$ exists and $\Phi(A) = B$ if and only if

$$(\mu_1, \mu_2, \cdots \mu_n, \cdots) = (\lambda_1, \lambda_2, \cdots, \lambda_n, \cdots)(a_{ij}),$$

where $a_{i,j} \geq 0$, $\sum_{i=1}^{\infty} a_{i,j} \leq M$ ($M$ is a fixed positive values) for all $j = 1, 2 \cdots$ and $\sum_{j=1}^{\infty} a_{i,j} = 1$ for all $i = 1, 2 \cdots$.
4. infinite dimensional case

Corollary

Let $A \in \mathcal{B}(\mathcal{H})$ and $B \in \mathcal{B}(\mathcal{K})$ be self-adjoint trace class operators with eigenvalues $\lambda_1, \lambda_2 \cdots \lambda_n \cdots$ and $\mu_1, \mu_2 \cdots \mu_n \cdots$ respectively. Then there exists a double stochastic quantum operation $\Phi$ such that $\Phi(A) = B$ if and only if

$$ (\mu_1, \mu_2, \cdots \mu_n, \cdots) = (\lambda_1, \lambda_2, \cdots, \lambda_n, \cdots)(a_{ij}), $$

where $a_{i,j} \geq 0$, $\sum_{i=1}^{\infty} a_{i,j} = 1$ for all $j = 1, 2 \cdots$ and $\sum_{j=1}^{\infty} a_{i,j} = 1$ for all $i = 1, 2 \cdots$. 
Corollary

If $\rho \in S(\mathcal{H})$ is a pure state, then for all $\sigma \in S(\mathcal{H})$ there exists a double stochastic quantum operation $\Phi$ such that $\Phi(\rho) = \sigma$. 
Corollary

Let $\Phi$ be the quantum channel on $S(\mathcal{H})$. If $A$ is a self-adjoint trace-class operator, then $\Phi(A) = A$ if and only if $\Phi(A^+) = A^+$ and $\Phi(A^-) = A^-$. In this case, $\Phi(|A|) = |A|$.

Remark 2. In general, $\Phi(|A|) = |A|$ do not imply $\Phi(A) = A$, for a quantum channel $\Phi$ and a self-adjoint trace class operator $A$. For example, let $U = \text{diag}(1, -1)$ be a unitary operator on two dimension Hilbert space and $A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. Then $U|A|U^* = |A|$ and $UAU^* \neq A$. 
In part 5, we will consider mainly the extension of [2, theorem 2] in the infinite dimension Hilbert space. Our main result is the following theorem.

**Theorem**

Let $\mathcal{H}$ and $\mathcal{K}$ be infinite dimensional separable Hilbert spaces. Suppose that $\Phi$ is a normal bounded linear map from $B(\mathcal{H})$ into $B(\mathcal{K})$. Then $\Phi$ is a completely positive map if and only if

$$\sum_{i,j=1}^{\infty} \sqrt{\lambda_i \lambda_j} \langle e_i \, | \, e_j \rangle \otimes \Phi(|e_i \rangle \langle e_j|) \geq 0,$$

where $\{|e_i\rangle\}_{i=1}^{\infty}$ is an orthonormal basis of $\mathcal{H}$ and all $\lambda_i > 0$, with $\sum_{i=1}^{\infty} \lambda_i < \infty$. 

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Lemma

([3, Proposition 20.1]) (1) The W*-topology and the weak operator topology (WOT) agree on bounded subsets of $B(H)$. 
(2) A sequence in $B(H)$ converges in the W*-topology if and only if it converges in the weak operator topology.

Lemma

Let $P_n, A \in B(H), n = 1, 2, \ldots$. If $P_n$ are orthogonal projections such that $P_n$ converges to the unit operator $I$ in the weak operator topology, then $A \geq 0$ if and only if $P_nAP_n \geq 0$ for all $n = 1, 2, \ldots$. 
5. Choi theorem in infinite dimension case

Lemma

Let $P_n, A_n \in B(\mathcal{H})$, $n = 1, 2, \ldots$. If $A_n$ converges to $A$ in the weak operator topology, $P_n$ are orthogonal projections such that $P_n$ converges to the unit operator $I$ in the weak operator topology, then $P_n A_n P_n$ converges to $A$ in the weak operator topology.


6. Further results on entanglement breaking channels and completely positive maps, (Preparation)


Many Thanks to your attention!