The 14th Workshop on Numerical Ranges and Numerical Radii

Schedule

June 13 (Wed). Registration

4:30 p.m. - 6:00 p.m. Max-Planck-Institute MPQ, Hans-Kopfermann-Str. 1.
6:30 p.m. – *group dinner* in Garching (for those who are interested).

June 14 (Thursday) Max-Planck-Institute MPQ, Hans-Kopfermann-Str. 1.

8:45 - 9:20 registration
9:20 - 9:30 opening
9:30 - 10:00 Choi (ExQM Speaker)
10:00 - 10:30 Spitkovsky
10:30 - 11:00 *coffee break*
11:00 - 11:30 Tam
11:30 - 12:00 Farenick
12:00 - 2:00 *lunch break*
2:00 - 2:30 Nakazato
2:30 - 3:00 Chien
3:00 - 3:30 *coffee break*
3:30 - 4:00 Osaka
4:00 - 4:30 Taheri

June 15 Friday Max-Planck-Institute MPQ, Hans-Kopfermann-Str. 1.

9:30 - 10:00 Życkowski
10:00 - 10:30 Schulte-Herbrüggen
10:30 - 11:00 *coffee break*
11:00 - 11:45 Gross
11.45 - 12:15 Psarrakos
12:15 - 2:15 *lunch break*
2:15 - 3:00 Schuch (to be confirmed)
3:00 - 3:45 Wolf
3:45 - 4:15 *coffee break*
4:15 - 4:45 Weis
4:45 - 5:30 Huckle
5:30 - 5.40 planning for Saturday
5.40 - 5.50 conference photo
6:30 *dinner in Garching* (on individual basis)

June 16 (Saturday), Social event and discussion

Afternoon: Visit in Munich downtown museums, e.g., *Blue Rider* in Lenbachhaus
6:30 optional dinner in a Munich beergarden downtown (indiv. basis)

June 17 (Sunday), Institute of Advanced Studies, IAS, Lichtenbergstrasse 2a, 85748 Garching

10:00 - 10:30 Bebiano
10:30 - 11:00 *coffee break*
11:00 - 11:30 Badea
11:30 - 12:00 vom Ende
12:15 - 13:45 *light lunch buffet* at IAS faculty club (included in fee)
2:00 - 2:30 Diogo
2:30 - 3:00 Crouzeix
3:00 - 3:30 *coffee break*
3:30 - 4:00 Sze
4:00 - 4:30 Bracic
6:00 - 8:30 *conference dinner* in IAS faculty club (included in fee)

June 18 (Monday), Institute of Advanced Studies, IAS, Lichtenbergstrasse 2a, 85748 Garching

9:45 - 10:30 Kressner
10:30 - 11:00 *coffee break*
11:00 - 11:30 Lau
11:30 - 12:00 Li
12:00 - 12:10 closing remarks
12:10 - 2:00 *lunch on campus* at IPP mensa (indiv. basis)

Titles and Abstracts

Name: Catalin Badea, catalin.badea@univ-lille.fr

Affiliation: University of Lille, France

Title: Applications of Banach algebra numerical ranges

Abstract: There are several possible generalizations of the numerical range of Hilbert space operators to numerical ranges of operators acting on Banach spaces and to numerical ranges of elements of Banach algebras. The aim of my talk is to discuss several applications of these generalizations in the Banach algebra setting, highlighting the differences with the classical (Hilbertian) case.

Name: Natália Bebiano, bebiano@mat.uc.pt

Affiliation: CMUC, University of Coimbra, Dept. Mathematics, P3001-454 Coimbra, Portugal

Title: Numerical ranges of non self-adjoint operators in quantum mechanics

Abstract The formulation of conventional quantum mechanics is based on the theory of self-adjoint operators which describe *observables* and whose eigenvalues are the possible results of the respective measurements. In particular, the Hamiltonian operator is self-adjoint, has a real set of eigenvalues and corresponding orthonormal eigenfunctions. Certain relativistic extensions of quantum mechanics lead to non self-adjoint Hamiltonian operators with real spectra, which motivated an intense research activity namely on the so called *PT-quantum mechanics*. (Here P and T are, respectively, the *parity* and the *time reversal* operators.)

Numerical range techniques are used to investigate spectral properties of these non-Hermitian operators.

Name: Janko Bračič, janko.bracic@fmf.uni-lj.si

Affiliation: Faculty of Natural Sciences and Engineering, University of Ljubljana, Slovenia

Title: Simultaneous zero inclusion property for spatial numerical ranges

Abstract: For a finite dimensional complex normed space X, we say that it has the simultaneous zero inclusion property if an invertible linear operator S on X has zero in its spatial numerical range if and only if zero is in the spatial numerical range of the inverse S^{-1}. Hilbert spaces have this property, but $\ell_p(n)$ do not have it if $p \neq 2$. We will present and discuss some related results.

Co-author(s): Cristina Diogo, ISCTE-IUL and IST Lisbon, Portugal.

Name: Mao-Ting Chien, mtchien@scu.edu.tw

Affiliation: Department of Mathematics, Soochow University, Taiwan

Title: An inverse numerical range problem for determinantal representations

Abstract: The numerical range of a matrix is the collection of quadratic forms over unit sphere vectors. The inverse numerical range problem aims to find a unit vector which corresponds to a given point of the numerical range. A hyperbolic ternary form associated to a matrix, according to Helton-Vinnikov theorem, admits a determinantal representation of a linear real symmetric pencil. A kernel vector function of the linear symmetric pencil is recognized as the inverse numerical range of the matrix. In this talk, we restrict our attention to hyperbolic ternary forms of elliptic curves. A relation between the kernel vector functions and the Riemann theta functions involved in Helton-Vinnikov theorem is proposed.

Co-author(s): Hiroshi Nakazato

Name: Man-Duen Choi, choi@math.toronto.edu

Affiliation: Department of Mathematics, University of Toronto, Canada

Title: Numerical Ranges in Modern Times

Abstract: Abstract: 100 years after the Toplitz-Hausdorff Theorem, we seek new meanings and new values of numerical ranges, in terms of quantum information.

Name: Michel Crouzeix, michel.crouzeix@univ-rennes1.fr

Affiliation: IRMAR, University of Rennes 1, France

Title: Spectral sets: numerical range and beyond

Abstract: Recall that, if a subset Ω of the complex plane contains the numerical range of a bounded operator A on a Hilbert space H, then Ω is a $C(\Omega)$-spectral set for A, i.e. $\|f(A)\| \leq C(\Omega)\sup_{z\in\Omega}|f(z)|$, for all rational functions f bounded in Ω. I have made the conjecture that $C(\Omega) \leq 2$ and nowadays the best estimate (due to César Palencia) is $C(\Omega) \leq 1+\sqrt{2}$. I will speak about this estimate and propose some variations allowing to consider non convex situations.

Co-author: Anne Greenbaum.

Name: Cristina Diogo, cristina.diogo@iscte-iul.pt

Affiliation: Department of Mathematics, University Institute of Lisbon, Portugal

Title: Faces of sets of operators with numerical range in a prescribed polyhedron

Abstract: Let $\mathcal{H}$ be a complex Hilbert space and $B(\mathcal{H})$ be the Banach algebra of all bounded linear operators on $\mathcal{H}$. For a non-empty closed convex set $K \subseteq \mathbb{C}$, let

$$\mathcal{W}^K = \{A \in B(\mathcal{H});\ \overline{W(A)} \subseteq K\}.$$

This is a convex set of operators and closed in the strong operator topology. When $\mathcal{H}$ is finite dimensional and K is a polyhedron, we are able to characterize faces of $\mathcal{W}^K$.

Name: Douglas Farenick, douglas.farenick@uregina.ca

Affiliation: Department of Mathematics and Statistics, University of Regina, Canada

Title: Classification of nonselfadjoint operators up to complete order isomorphism

Abstract: The 3-dimensional subspace $\mathcal{S}_T$ spanned by a Hilbert space operator T different from a scalar multiple of the identity, its adjoint T^*, and the identity operator carries the structure of a matricially ordered vector space with an Archimedean order unit – such spaces are more typically known as *operator systems*. In the category of operator systems, the natural morphisms are unital completely positive linear maps, and therefore an isomorphism in this category is a unital completely positive linear bijection in which the linear inverse is also completely positive. This is a much weaker notion than, say, unitary similarity – although unitary similarity is one example of an isomorphism in this category. This lecture examines the role of the numerical range determining when $\mathcal{S}_T$ and $\mathcal{S}_R$ are isomorphic operator systems, for operators T and R. Specific examples will be illustrated by considering certain weighted shift operators on finite- and infinite-dimensional Hilbert spaces.

Co-author(s): Martín Argerami

Name: David Gross, david.gross@thp.uni-koeln.de

Affiliation: Theoretical Physics, University of Cologne, Germany

Title: Convex Algorithms for Bilinear Reconstructions

Abstract: Over the past years, convex optimization-based algorithms for a number of important bilinear reconstruction problems have been found. Important examples of such problems inlcude "phase retrieval" (find a vector from the magnitude of linear measurements) and "blind deconvolution" (find a signal that has been blurred out by a structured convolution kernel - without knowing the kernel). The algorithms are based on convex methods for low-rank recovery.

I'll give an overview and highlight connections to quantum mechanics.

Name: Thomas Huckle.

Affiliation: Department of Computer Science, Technical University Munich, Germany

Title: Mathematicians 1933 - 1945

Abstract: In this talk we give a short description of Mathematics in Germany during the Nazi era. Based on an overview of the situation between 1933 and 1945, we sketch life and fate of mainly Jewish mathematicians all-too-often ending in being driven into suicide (or at best to emigration) if not into concentration camps and holocaust.

Especially, we dwell on Otto Toeplitz and Felix Hausdorff and the universities in Munich. Finally, we mention mathematicians involved in war-related research.

Name: Daniel Kressner

Affiliation: Department of Computer Science, EPFL Lausanne, Switzerland

Title: Subspace Methods for Approximating the Numerical Range and Associate Quantities

Abstract: This paper is concerned with subspace methods for computing the numerical range and associated quantities of large-scale matrices. Our approach is based on the connection between the numerical range and parameter-dependent eigenvalue problems. For example, the computation of the Crawford number corresponds to a Hermitian eigenvalue optimization problem. We establish local convergence of order $1+\sqrt{2} \approx 2.4$ for an existing subspace method applied to such and related eigenvalue optimization problems. For the particular case of the Crawford number, we show that the relevant part of the objective function is strongly concave. In turn, this enables us to develop a subspace method that only uses three-dimensional subspaces but still achieves global convergence and a local convergence that is at least quadratic. A number of numerical experiments confirm our theoretical results and reveal that the established convergence orders appear to be tight.

Co-author(s): Ding Lu and Bart Vandereycken

Name: Pan-Shun Lau, panlau@connect.hku.hk

Affiliation: Department of Applied Mathematics, The Hong Kong Polytechnic University, Hong Kong

Title: Convexity and star-shapedness of joint (p,q)-matricial range

Abstract: Let $\mathbf{A} = (A_1, \ldots, A_m)$ be an m-tuple of bounded linear operators acting on a Hilbert space $\mathcal{H}$. Their joint (p,q)-matricial range $\Lambda_{p,q}(\mathbf{A})$ is the collection of $(B_1, \ldots, B_m) \in \mathbf{M}_q^m$, where $I_p \otimes B_j$ is a compression of A_j on a pq-dimensional subspace. This definition covers various kinds of generalized numerical ranges for different values of p, q, m. In this talk, we will show that $\Lambda_{p,q}(\mathbf{A})$ is star-shaped if the dimension of $\mathcal{H}$ is sufficiently large. If $\dim \mathcal{H}$ is infinite, we extend the definition of $\Lambda_{p,q}(\mathbf{A})$ to $\Lambda_{\infty,q}(\mathbf{A})$ consisting of $(B_1, \ldots, B_m) \in \mathbf{M}_q^m$ such that $I_\infty \otimes B_j$ is a compression of A_j on a closed subspace of $\mathcal{H}$, and consider the joint essential (p,q)-matricial range

$$\Lambda_{p,q}^{ess}(\mathbf{A}) = \bigcap \{\mathbf{cl}(\Lambda_{p,q}(A_1 + F_1, \ldots, A_m + F_m)) : F_1, \ldots, F_m \text{ are compact}\}.$$

Both sets are shown to be convex, and the latter one is always non-empty and compact.

Co-author(s): Chi-Kwong Li, Yiu-Tung Poon, Nung-Sing Sze.

Name: Chi-Kwong Li (ckli@math.wm.edu

Affiliation: Department of Mathematics, College of William and Mary; Institute for Quantum Computing, University of Waterloo.

Title: Preservation of the joint essential matricial range

Abstract: Let $\mathbf{A} = (A_1, \ldots, A_m)$ be an m-tuple of bounded linear operators acting on an infinite dimensional Hilbert space H. The qth matricial range of $\mathbf{A}$ is the collection of $(B_1, \ldots, B_m) \in M_q^m$ such that $B_j = X^* A_j X$ for some partial isometry X such that $X^*X = I_q$. The essential q-matricial range of $\mathbf{A}$ is the intersection of the closure of the q-matricial range of $\mathbf{A} + \mathbf{K}$, where $\mathbf{K}$ ranges over all m-tuple of compact operators. We show that for any positive integer N there is an m-tuple of compact operators $\mathbf{F}$ such that the closure of the qth matricial range of $\mathbf{A} + \mathbf{F}$ equals the essential qth matricial range of $\mathbf{A}$ for all $q \leq N$. Moreover, if $A_1, \ldots, A_m$ are self-adjoint and the essential qth matricial range of $\mathbf{A}$ is a simplex in $\mathbf{R}^m$, then there is an m-tuple of self-adjoint compact operators $\mathbf{F}$ such that the closure of the qth matricial range of $\mathbf{A} + \mathbf{F}$ equals the essential qth matricial range of $\mathbf{A}$ for all positive integer q.

Co-author(s): Vern Paulsen (University of Waterloo) and Yiu-Tung Poon (Iowa State University).

Name: Hiroshi Nakazato, nakahr@hirosaki-u.ac.jp

Affiliation: Department of Mathematics and Physics, Hirosaki University, Japan

Title: Determinantal representations and Numerical ranges

Abstract: In 1981, M. Fiedler posed a question concerning the characterization of the numerical range of a matrix via the hyperbolicity of the Kippenhahn curve. This question was partly solved by himself and generaly affirmatively solved by Helton, Vinnikov in 2007. Some related recent results are introduced in this talk.

Co-author(s): Mao-Ting Chien (Soochow University, Taiwan).

Name: Hiroyuki Osaka, osaka@se.ritsumei.ac.jp

Affiliation: Department of Mathematical Sciences, Ritsumeikan University, Japan

Title: Maps preserving $\mathcal{AN}$-operators

Abstract: Let H_1, H_2 be complex Hilbert spaces and $T : H_1 \to H_2$ be a bounded linear operator. Then T is said to be *norm attaining* if there exists a unit vector $x_0 \in H_1$ such that $\|Tx_0\| = \|T\|$. If for any closed subspace M of H_1, the restriction $T|M : M \to H_2$ of T to M is norm attaining, then T is called an *absolutely norm attaining* operator or $\mathcal{AN}$-operator. – In this talk, we discuss linear maps on $\mathcal{B}(H)$, which preserve the class of absolutely norm attaining operators on H.

Co-author(s): Ramesh Golla (IIT Hyderabad).

Name: Panayiotis J. Psarrakos, ppsarr@math.ntua.gr

Affiliation: Department of Mathematics, School of Applied Mathematical and Physical Sciences, National Technical University of Athens, Greece.

Title: Birkhoff-James ε-orthogonality sets of vectors and vector-valued polynomials

Abstract: Consider a complex normed linear space $(\mathcal{X}, \|\cdot\|)$, and let $\chi, \psi \in \mathcal{X}$ with $\psi \neq 0$. Motivated by recent works on rectangular matrices and on normed linear spaces, we study the Birkhoff-James ε-orthogonality set of χ with respect to ψ, give an alternative definition for this set, and explore its rich structure. We also introduce the Birkhoff-James ε-orthogonality set of polynomials in one complex variable whose coefficients are members of $\mathcal{X}$, and survey and record extensions of results on matrix polynomials to these vector-valued polynomials.

Co-authors: Vasiliki Panagakou, Nikos Yannakakis.

Name: Thomas Schulte-Herbrüggen, tosh@tum.de

Affiliation: TU Munich

Title: Some Remarks on Quantum Systems Theory as Reflected by Numerical Ranges

Abstract:

We illustrate how quantum engineering relates to numerical ranges, C-numerical ranges and more recent derived notions determined by orbits of proper subgroups of the unitaries. Quantum systems theory in terms of Lie groups and Lie semigroups with their symmetries provides a united framework to pinpoint the dynamic behaviour of closed and open quantum systems under all kinds of controls. Within this picture, expectation values not only connect naturally to numerical ranges, they also put challenges for the future.

Co-author: Gunther Dirr, Robert Zeier.

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Affiliation: New York University Abu Dhabi (NYUAD), UAE
The College of William and Mary, USA (Emeritus)

Title: On the maximal numerical range

Abstract: We show that the maximal numerical range $W_0(A)$ of an operator A has a non-empty intersection with the boundary of its numerical range $W(A)$ if and only if A is normaloid. A description of this intersection is given.

In the finite dimensional setting, we also establish when $W(A) = W_0(A)$, describe $W_0(A)$ explicitly for A unitarily similar to direct sums of (at most) 2-by-2 blocks, and provide some insight into the behavior of $W_0(A)$ when A^*A has two distinct eigenvalues only, the smaller of them being simple.

Co-author: Based partially on a capstone project by Ali N. Hamed under supervision of the author.

Name: Raymond Nung-Sing Sze, raymond.sze@polyu.edu.hk

Affiliation: Department of Applied Mathematics, The Hong Kong Polytechnic University, Hong Kong

Title: The generalized numerical range of a set of matrices

Abstract: For a given set of $n \times n$ matrices $\mathcal{F}$, we study the union of the C-numerical ranges of the matrices in the set $\mathcal{F}$, denoted by $W_C(\mathcal{F})$. In this talk, we present some basic algebraic and topological properties of $W_C(\mathcal{F})$, and show that there are connections between the geometric properties of $W_C(\mathcal{F})$ and the algebraic properties of C and the matrices in $\mathcal{F}$. Furthermore, we consider the starshapedness and convexity of the set $W_C(\mathcal{F})$. In particular, we show that if $\mathcal{F}$ is the convex hull of two matrices such that $W_C(A)$ and $W_C(B)$ are convex, then the set $W_C(\mathcal{F})$ is star-shaped. We also investigate the extensions of the results to the joint C-numerical range of an m-tuple of matrices.

Co-author(s): P.S. Lau (HK PolyU), C.K. Li (William & Mary), Y.T. Poon (Iowa State U)

Name: Fatemeh Esmaeili Taheri, fattaheri@math.uc.pt

Affiliation: Department of Mathematics, University of Coimbra, Portugal

Title: Characterize distribution of Rayleigh quotients in the numerical range of matrix.

Abstract: The numerical range of a matrix A endowed with an inner product $(.,.)$ is the set of all complex numbers of the form (Ax, x), where x varies over all vectors on the unit sphere.
The ratio $\frac{(Ax,x)}{(x,x)}$ is well defined for any nonzero vector $x \in \mathbb{C}^n$ and any matrix $A \in \mathbb{M}_n$, and is called the Rayleigh quotient of x with respect to A.
Thus, the numerical range comprises all the Rayleigh quotients of the matrix.
Since the publication of the seminal paper of Kippenhahan, many authors have developed the theory of numerical range in several directions. One of these directions is statistical and topological data Analysis. Let us build up Rayleigh quotients from normal vectors randomly chosen in the appropriate sphere for a matrix and get set of points in the numerical range. This talk tries to describe the location of Rayleigh quotients in the numerical range of matrix, density of the points and probability distribution inside the shape of numerical range.

Name: Tin-Yau Tam, tamtiny@auburn.edu, ttam@unr.edu

Affiliations: Department of Mathematics and Statistics, Auburn University, USA & Department of Mathematics and Statistics, University of Nevada, Reno, USA

Title: Toeplitz-Hausdorff Theorem - Convexity and Connectedness

Abstract: The celebrated Toeplitz-Hausdorff Theorem asserts that the classical numerical range is convex, which is a very nice geometric result. We will discuss its relation with some connectedness property. Similar relation will be given for some generalized numerical ranges.

Name: Frederik vom Ende, frederik.vom-ende@tum.de

Affiliation: Department of Chemistry, TU Munich, 85747 Garching, Germany

Title: The C-Numerical Range in Infinite Dimensions

Abstract: For trace-class operators $C \in \mathcal{B}_1(\mathcal{H})$ and bounded operators $T \in \mathcal{B}(\mathcal{H})$ on a separable infinite-dimensional Hilbert space $\mathcal{H}$, the closure of the C-numerical range $W_C(T) := \{\operatorname{tr}(CU^\dagger TU) \,|\, U \in \mathcal{B}(\mathcal{H}) \text{ unitary}\}$ is star-shaped with respect to the set $\operatorname{tr}(C)W_e(T)$, where $W_e(T)$

denotes the essential numerical range of T. Moreover, the closure of $W_C(T)$ is convex if either C is normal with collinear eigenvalues or if T is essentially self-adjoint. This naturally generalizes the previously known star-shapedness and convexity result invoking convergence of complex sets when going from matrices to operators.

Moreover, in the case of compact normal operators we will see that if the eigenvalues of C are collinear, then the closure of $W_C(T)$ coincides with the closure of the convex hull of the C-spectrum of T. This talk is based on arXiv:1712.01023.

Co-author(s): Dr. Gunther Dirr (Department of Mathematics, University of Würzburg, 97074 Würzburg, Germany).

Name: Stephan Weis, maths@weis-stephan.de

Affiliation: Centre for Quantum Information and Communication, Université libre de Bruxelles, Belgium

Title: Classification of joint numerical ranges of three hermitian matrices of size three

Abstract: The possible shapes of the numerical range

$$\{(\langle\psi|\mathrm{Re}\,A|\psi\rangle, \langle\psi|\mathrm{Im}\,A|\psi\rangle) : |\psi\rangle \in C^3, \langle\psi|\psi\rangle = 1\} \subset R^2$$

of a 3-by-3 matrix A were described by Kippenhahn, Math. Nachr. 6 (1951), 193, see also Keeler et al., LAA 252 (1997), 115. One dimension higher, in dimension three, only very little was know about the joint numerical range

$$\{(\langle\psi|F_1|\psi\rangle, \langle\psi|F_2|\psi\rangle, \langle\psi|F_3|\psi\rangle) : |\psi\rangle \in C^3, \langle\psi|\psi\rangle = 1\} \subset R^3$$

of three hermitian 3-by-3 matrices F_1, F_2, F_3 apart from the upper bound of at most four filled ellipses which can lie in the boundary, see Chien and Nakazato, LAA 430 (2009), 204. We give a complete classification of the configurations formed by segments and ellipses in the boundary of the joint numerical range of three hermitian 3-by-3 matrices.

Co-authors: Konrad Szymański and Karol Życzkowski, Marian Smoluchowski Institute of Physics, Jagiellonian University, Kraków, Poland

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Name: Michael Wolf, m.wolf@tum.de

Title: Undecidability of the Spectral Gap and Related Problems

Abstract: – will follow –

Co-authors: Toby S. Cubitt, David Perez-Gracia.

Name: Karol Życzkowski, karol@cft.edu.pl

Affiliation: Institute of Physics, Jagiellonian University, Cracow and Center for Theoretical Physics, PAS, Warsaw

Title: On restricted numerical range

Abstract: Restricted numerical range of an operator X is formed by the set of all possible Hilbert-Schmidt inner products, $z = \mathrm{Tr}\rho X$, where ρ denotes a quantum *state* – convex combination of rank one projectors which is hermitian, normalized and satisfies certain additional properties. In particular, for operators X acting on a space $\mathcal{H}_{N^2} = \mathcal{H}_N \otimes \mathcal{H}_N$, we analyze numerical range restricted to a) *product states*, $\sigma^{\otimes} = \sigma_A \otimes \sigma_B$, b) *separable states*, i.e. convex combinations of

product states, c) states related to *stochastic maps*, $\sigma_{\text{stoch}} \; : \; \text{Tr}_B \sigma_{\text{stoch}} = \mathbb{I}/N$, and d) *bistochastic maps*, $\sigma_{\text{bist}} \; : \; \text{Tr}_A \sigma_{\text{bist}} = \text{Tr}_B \sigma_{\text{bist}} = \mathbb{I}/N$, where Tr_A and Tr_B denote partial traces with respect to both subspaces of $\mathcal{H}_{N^2}$. Two latter sets yield possible projections of the set of quantum operations on a plane. We investigate the problem for which operators of a given dimension the ratio of the volume of separable numerical range and standard numerical range is minimal.

Co-authors: Konrad Szymański and Jakub Czartowski (Jagiellonian University).